Oct 23, 2020

QR iteration

$$T_0 = A$$
$$Q_k R_k = T_k$$
$$T_{k+1} = R_k Q_k$$

$\Rightarrow$

Shifted QR iteration

$$T_0 = A$$

σ_k approx eigval

$$Q_k R_k = T_k - \sigma_k I$$
$$T_{k+1} = R_k Q_k + \sigma_k I$$

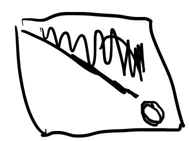
① $R_k = Q_k^T T_k - \sigma_k Q_k^T$

② $Q_k = T_k R_k^{-1} - \sigma_k R_k^{-1}$

$$T_{k+1} = R_k Q_k + \sigma_k I = \left(Q_k^T T_k - \cancel{\sigma_k Q_k^T}\right) Q_k + \cancel{\sigma_k I} = Q_k^T T_k Q_k$$

What if σ_k is eval of T_k? (and of A)

$\Rightarrow T_k - \sigma_k I$ singular $\Rightarrow R_k$ singular



e_n^T

$$\Rightarrow T_{k+1} = \left(\begin{array}{c|c} T_{11} & t_{12} \\ \hline 0 & \sigma_k \end{array}\right)$$

to get rest of evals

or 2x2

2x2 blocks correspond to conjugate pairs of complex eigenvalues

$$\det\left[\begin{pmatrix} a-\lambda & b \\ c & d-\lambda \end{pmatrix}\right] = (a-\lambda)(d-\lambda) - bc$$

Double shifts

Bottom 2×2 block

$\sigma_k, \ \bar{\sigma}_k$

$Q_k R_k = T_k - \sigma_k I$

$T_{k+1} = R_k Q_k + \sigma_k I$

$Q_{k+1} R_{k+1} = T_{k+1} - \bar{\sigma}_k I$

$T_{k+2} = R_{k+1} Q_{k+1} + \bar{\sigma}_k I$

Can show:

$T_{k+2} = RQ$

$QR = (T_k - \sigma_k I)(T_k + \bar{\sigma}_k I)$

$= T_k^2 + |\sigma_k|^2 I - 2\,\mathrm{Real}(\sigma_k)\, T_k$