

Oct 9, 2020 LLS: $\hat{x} = \arg \min_x \|Ax - b\|_2^2$ $A = QR$ $R\hat{x} = Q^T b$

Sensitivity:

- ① A nearly singular ② b not be close to range(A)

Want: backward stable

$$R\hat{x} = Q^T b$$

$$A = QR \begin{cases} \text{Is } \hat{\hat{Q}}\hat{\hat{R}} \approx A? \\ \text{Is } \hat{\hat{Q}}^T \hat{\hat{Q}} \approx I? \end{cases}$$

CGS/MGS

Householder / Givens

Householder $Q_j = I - 2v_j v_j^T$

$$x = a_j \quad v_j = \frac{\text{sign}(x_1)\|x\|_2 e_1 + x}{\|x\|_2} \quad v_j = v_j / \|v_j\|_2$$

$$\tilde{Q} = Q + E, \quad \|E\| = O(\epsilon)$$

$$f(\tilde{Q}A) = f((Q+E)A) = (Q+E)A + F = QA + EA + F$$

$$= Q(A + \underline{Q^T E A} + \underline{Q^T F}) \quad (QQ^T = I)$$

$$\|Q^T E A\|_2 \leq \|Q^T\| \|E\| \|A\| = O(\epsilon \|A\|)$$

$$\|Q^T F\|_2 \leq \|F\|_2 \quad |F_{ij}| = |q_i^T a_j| |\delta| |\delta| = O(\epsilon) \quad \|F\|_2 \leq O(\epsilon \cdot \|A\|_2)$$

$$|q_i^T a_j| \leq \|q_i\|_2 \|a_j\| = \|a_j\|_2$$

$$Q(A + G) \quad \|G\|_2 = O(\varepsilon \cdot \|A\|_2) \quad G = Q^T E A + Q^T F$$

$$= F + (\tilde{Q}_2(Q_1 A + G_1)) \quad \|G_1\|_2$$

$$\|G_2\|_2 = O(\varepsilon \cdot \|Q_1 A + G_1\|_2) \geq O(\varepsilon \|Q_1\|_2) \\ = O(\varepsilon \|A\|_2) + O(\varepsilon^2)$$

$$Q_2(Q_1 A + G_1) + G_2 = Q_2 Q_1 A + \underbrace{Q_2 G_1 + G_2}_{O(\varepsilon \|A\|_2)} \quad O(\varepsilon \|A\|_2)$$



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$$\hat{x}_2 = V_2 \gamma \quad A$$

$$V_2 \gamma = \hat{x}_2$$


$$A \begin{pmatrix} \hat{x}_1 \\ \hat{x}_2 \end{pmatrix} = U \Sigma \begin{pmatrix} V_1^T \hat{x}_1 \\ V_2^T \hat{x}_2 \end{pmatrix}$$

