

Oct 7, 2020 LLS: $\hat{x} = \arg \min_x \|Ax - b\|_2^2$ A $m \times n$ $m > n$
 A full rank

$$\hat{x} = A^+ b \quad \hat{x} = R^{-1} Q^T b \quad r = b - A\hat{x} = (I - QQ^T)b$$

① conditioning: how sensitive problem is to perturbations

② stability: how accurate our algorithm is

backward stable alg \Rightarrow relative error is $O(k \cdot \epsilon_{mach})$

$$\text{LLS: } (A, b) \rightarrow (\hat{x}, A\hat{x}) = (A^+ b, AA^+ b)$$

perturb A or b , how does \hat{x} change? ($A\hat{x}$ change?)

$m = n \Rightarrow$ should get $k_2(A) = \|A\|_2 \|A^{-1}\|_2 = \sigma_1 / \sigma_n$

$$\text{Ex: } A = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad b = \begin{pmatrix} \epsilon \\ 1 \end{pmatrix} \quad A^T A = [1] \quad A^T b = \epsilon \\ \hat{x} = (A^T A)^{-1} A^T b = \epsilon$$

$$b + \delta b = \begin{pmatrix} 2\epsilon \\ 1 \end{pmatrix} \Rightarrow \tilde{x} = 2\epsilon \quad \frac{\|\tilde{x} - \hat{x}\|}{\|\hat{x}\|} = 1 \\ \frac{\|\delta b\|}{\|b\|} = O(\epsilon)$$

Problem! Projection of b onto $\text{range}(A)$ is small ($O(\epsilon)$)
relative to the size of b

$$A \hat{x} = \begin{pmatrix} \epsilon \\ 0 \end{pmatrix} \quad A \tilde{x} = \begin{pmatrix} 2\epsilon \\ 0 \end{pmatrix} \quad \frac{\|A \tilde{x} - A \hat{x}\|}{\|A \hat{x}\|} = O(1)$$

$$\hat{x} = R^{-1} Q^T b$$

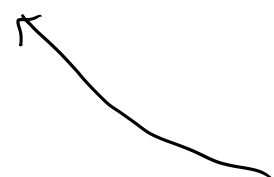
$$\kappa_2(A) = \sigma_1 / \sigma_n$$

$$[A] = [U] \begin{bmatrix} \diagup \\ \Sigma \\ \diagdown \end{bmatrix} [V^T]$$

problems

$$\|A\|_2 =$$

small perturbations to A can cause relatively large changes to


$$r = Ax - b$$

$$\text{range}(A) \Leftrightarrow A^T b = 0 \Leftrightarrow u^T b = 0$$

not much to do