Oct 5, 2020 LLS: $\hat{x} = \arg\min_x \|Ax-b\|_2^2$ $A = QR$ $\hat{x} = R^{-1}Q^T b$

LU derivation

$$A = \begin{pmatrix} x & x & x \\ x & x & x \\ x & x & x \end{pmatrix} \xrightarrow{L_1 A} \begin{pmatrix} x & x & x \\ 0 & x & x \\ 0 & x & x \end{pmatrix} \xrightarrow{L_2 L_1 A} \begin{pmatrix} x & x & x \\ 0 & x & x \\ 0 & 0 & x \end{pmatrix} = U$$

$$L_2 L_1 A = U \Rightarrow A = (L_1^{-1} L_2^{-1}) U = LU$$

Now for QR today: Q_i are $m \times m$

$$A = \begin{pmatrix} x & x \\ x & x \\ x & x \end{pmatrix} \xrightarrow{Q_1 A} \begin{pmatrix} x & x \\ 0 & x \\ 0 & x \end{pmatrix} \xrightarrow{Q_2 Q_1 A} \begin{pmatrix} x & x \\ 0 & x \\ 0 & 0 \end{pmatrix}$$

$$Q_2 Q_1 A = \begin{pmatrix} R \\ 0 \end{pmatrix} \Rightarrow A = Q_1^T Q_2^T \begin{pmatrix} R \\ 0 \end{pmatrix} = \underbrace{\boxed{Q}}_{m \times m} \underbrace{\begin{pmatrix} R \\ 0 \end{pmatrix}}_{m \times n}$$

Things to worry about:

1. multiply by $m \times m$ matrices. expensive?
2. form Q efficiently?
3. $\underset{m \times n}{\boxed{A}} = \underset{m \times n}{\boxed{Q}}\ \underset{n \times n}{\boxed{R}}$ $Q^T Q = I$ R upper tri.

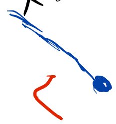
Householder reflectors

$$A = \begin{pmatrix} x & x \\ x & x \\ x & x \end{pmatrix} \xrightarrow{Q_1 A} \begin{pmatrix} x & x \\ 0 & x \\ 0 & x \end{pmatrix}$$

$$Q_1 a_1 = \begin{pmatrix} x \\ 0 \\ 0 \end{pmatrix} = \alpha e_1$$

$$\|Q_1 a_1\|_2 = \|a_1\|_2 \Rightarrow Q_1 a_1 = \|a_1\|_2 e_1$$

$x = a_1$



$-x$

orthogonally to v

2

Onto proj. onto H: $I - \frac{v v^T}{v^T v}$

$$\begin{array}{cc} & \begin{array}{ccccc} & i & & j & \end{array} \\ \begin{array}{c} \\ i \\ \\ j \\ \\ \end{array} & \left(\begin{array}{ccccc} 1 \ddots 1 & & & & \\ & c & & -s & \\ & & 1 \ddots 1 & & \\ & s & & c & \\ & & & & 1 \ddots 1 \end{array}\right) \end{array}$$

$A_{ik}, A_{jk} \Rightarrow G$

$(GA)_{jk} = 0 \qquad (GA)_{ik} \neq 0$

if $A_{ik'}, A_{jk'} = 0$

$(GA)_{ik'} = (GA)_{jk'} = 0$

if one is non-zero,
can both fill in

$$\begin{pmatrix} X & X & X & X & X \\ O & X & X & X & X \\ O & O & X & X & X \\ O & O & X & X & X \\ O & O & \cancel{X} & \cancel{O} & X \end{pmatrix}$$

O

O

X