Sep 23, 2020

GEPP

$L = I, \quad P = I$

for $j = 1 : n-1$

  pick $k = \underset{\ell > j}{\arg\max} |A(\ell, j)|$

  swap rows k and j of A, L

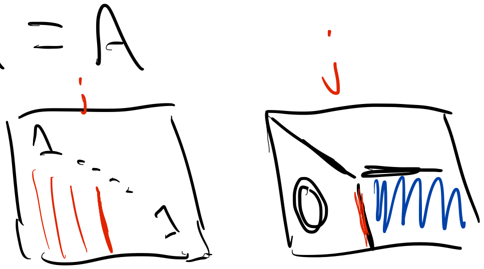
  update P

  $L(j+1:n, j) = A(j+1:n, j) / A(j,j)$

  $A(j+1:n, j) = 0$

  $A(j+1:n, j+1:n) -= L(j+1:n, j) A(j, j+1:n)$

$U = A$



$PA = LU$

$Ax = b \Rightarrow$ ① $c = P^T b$ ② $Ly = c$ ③ $Ux = y$

Error analysis

$A + E = \hat{L}\hat{U} \quad |E| \le n \varepsilon |\hat{L}||\hat{U}|$

$\hat{u}_{jk} = a_{jk}$

step 1: $\hat{u}_{jk} = \hat{u}_{jk} - \hat{\ell}_{j1} \hat{u}_{1k}$

⋮

step $j-1$: $\hat{u}_{jk} = \hat{u}_{jk} - \hat{\ell}_{j,j-1} \hat{u}_{j-1,k}$

$\hat{u}_{jk} = fl\left(a_{jk} - \sum_{i=1}^{j-1} \hat{\ell}_{ji} \hat{u}_{ik}\right)$

$= a_{jk}(1+\delta) - \left(\sum \hat{\ell}_{ji} \hat{u}_{ik} (1+\delta_i)\right)(1+\delta)$

$|\delta| \le \varepsilon, \quad |\delta_i| \le (j-1)\varepsilon + O(\varepsilon^2)$ HW1

$a_{jk} = \frac{1}{1+\delta} \hat{u}_{jk} \hat{\ell}_{jj} - \sum \hat{\ell}_{ji} \hat{u}_{ik} (1+\delta_i)$ ($\hat{\ell}_{jj} = 1$)

$+ e_{jk}$

$|e_{jk}| \le \left| \sum_{i=1}^{j} \hat{\ell}_{ji} \hat{u}_{ik} \delta_i \right|$

$$\|\delta A\|_2^2 = \max_{\|y\|_2^2} \|u_n e_n^T y\|_2^2 = \max_{\|y\|_2^2} y_n^2 \|u_n\|_2^2 \leq 1$$

$$y_n = 1 \quad (y = e_n)$$

$$\|x\|_2^2 = 1$$

$$\|\Sigma^{-1} \underbrace{U^T u_n}_{e_n} \underbrace{e_n^T e_n}_{1}\|_2 = \|\Sigma^{-1} e_n\|_2 = \frac{1}{\sigma_n}$$

$$\|A^{-1} \delta A x\|_2 = \frac{1}{\sigma_n}$$

$$= \|A^{-1}\|_2 \cdot \|\delta A\|_2 \cdot \|x\|_2$$

$$\|\delta A\|_2 = 1, \quad \|x\|_2 = 1$$