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# Gaussian elimination

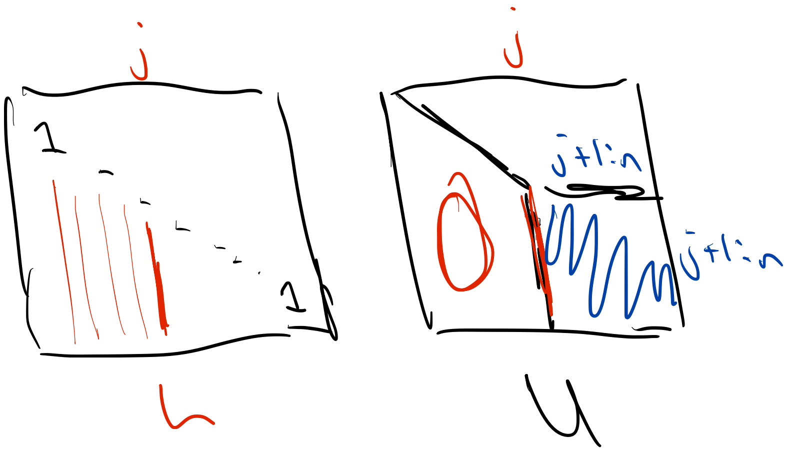
$L = I$

for $j = 1:n-1$

Gauss transf.: $(I + \tau_j e_j^T) A$

- $L(j+1:n, j) = A(j+1:n, j) / A(j,j)$
- $A(j+1:n, j) = 0$
- $A(j+1:n, j+1:n) \mathrel{-}= L(j+1:n, j)\, A(j, j+1:n)$

$U = A$




Potential problem: $A(j,j) = 0$

$\begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$ Solution: pivoting

$A$ nonsingular $\Rightarrow$ at least one non-zero in the first col

$$PA = \begin{pmatrix} a_{11} & a_{12}^T \\ a_{21} & A_{22} \end{pmatrix}$$

$a_{11} \neq 0$

$$PA = \underbrace{\begin{pmatrix} 1 & 0 \\ a_{21}/a_{11} & I \end{pmatrix}}_{L_1} \underbrace{\begin{pmatrix} a_{11} & a_{12}^T \\ 0 & A_{22} - a_{21} a_{12}^T / a_{11} \end{pmatrix}}_{U_1}$$

$$A_{22} - a_{21} a_{12}^T / a_{11} = S$$

$S$ is the Schur complement

$$\det(A) = \det(PA)$$

$$\det(A) \neq 0$$

$$= \cancel{\det(L_1)}^{\,1} \det(U_1)$$

$$= \underbrace{a_{11}}_{\neq 0} \cdot \det(S) \Rightarrow \det(S) \neq 0$$

$$PA = \begin{pmatrix} 1 & 0 \\ a_{21}/a_{11} & I \end{pmatrix} \begin{pmatrix} a_{11} & a_{12}^T \\ 0 & S \end{pmatrix} \qquad P_1 S = L_1 U_1 \Rightarrow S = P_1^T L_1 U_1$$

$$= \begin{pmatrix} 1 & 0 \\ a_{21}/a_{11} & P_1^T L_1 \end{pmatrix} \begin{pmatrix} a_{11} & a_{12}^T \\ 0 & U_1 \end{pmatrix}$$

$$= \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & P_1^T \end{pmatrix}}_{\bar{P}} \begin{pmatrix} 1 & 0 \\ P_1 a_{21} & L_1 \end{pmatrix} \underbrace{\begin{pmatrix} a_{11} & a_{12}^T \\ 0 & U_1 \end{pmatrix}}_{U} \qquad A = LU$$

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$L = I, \quad P = I$

for $j = 1:n-1$

$A(j+1:n, j) = 0$

$A(j+1:n, j+1:n) -=$