More general formulation

$f: X \to Y$ relative error:

$$\frac{|f(x+h) - f(x)|}{|f(x)|} =$$

$$\frac{|h|}{|x|}$$

$$= \frac{\|A\| \|x\|}{\|Ax\|}$$

$f_A: \mathbb{R}^n \to \mathbb{R}^m$

$f_A(x) = Ax$

$$\frac{\partial (f_A)_i}{\partial x_j} = A_{ij}$$

Note 2: upper bound

$\|\delta x\| =$ $\|\delta A\| \|x\|$

can show $\exists\, \delta A$ s.t. equality

$= \sigma_1(A)\sigma_1(A^{-1})$

Note 3:

$$\Sigma^{-1} = \begin{pmatrix} 1/\sigma_1 & & \\ & \ddots & \\ & & 1/\sigma_n \end{pmatrix}$$

SVD of A^{-1}:

$$A^{-1} = V \Pi \Pi^T \Sigma^{-1} \Pi \Pi^T U^T$$

$$\|A^{-1}\|_2 \geq 1/\sigma_n(A)$$

$$R_2(A) = \frac{\sigma_1(A)}{\sigma_n(A)}$$