CS 621: Assignment 6

"Due": Friday, November 30, 2007 (In Lecture or 5153 Upson by 4pm)

Scoring for each problem is on a 0-to-3 scale (3 = complete success, 2 = overlooked a small detail, 1 = germ of the right idea, 0 = missed the point of the problem.) One point will be deducted for insufficiently commented code. Unless otherwise stated, you are expected to utilize fully MATLAB'S vectorizing capability subject to the constraint of being flop-efficient. Test drivers and related material are posted on the course website `http://www.cs.cornell.edu/courses/cs621/2007fa/`. For each problem submit output and a listing of all scripts/functions that you had to write in order to produce the output. You are allowed to discuss *background* issues with other students, but the codes you submit must be your own.

P1. (An Iteration for Unsymmetric Positive Definite Systems)

We want to investigate the solution of $Au = f$ where $A \neq A^T$. For a model problem, consider the finite difference approximation to

$$-u'' + \sigma u' = 0 \qquad 0 < x < 1$$

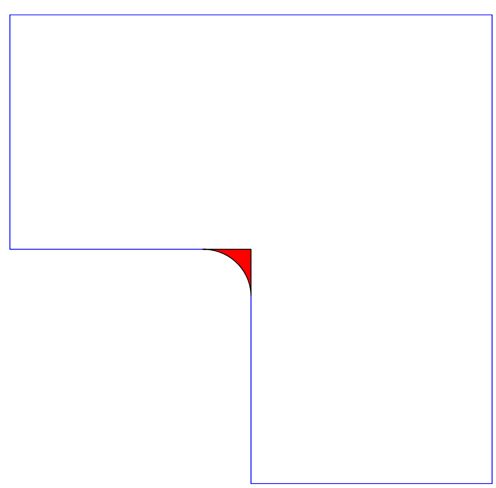
where $u(0) = 10$ and $u(1) = 10\exp^{\sigma}$. This leads to the difference equation

$$-u_{i-1} + 2u_i - u_{i+1} + R(u_{i+1} - u_{i-1}) = 0 \qquad i = 1:n$$

where $R = \sigma h/2$, $u_0 = 10$, and $u_{n+1} = 10\exp^{\sigma}$. The number R should be less than 1. What can you say about the convergence rate for the iteration $Mu^{(k+1)} = Nu^{(k)} + f$ where $M = (A + A^T)/2$ and $N = (A^T - A)/2$?

P2. (A Modified L-Shaped Domain)

Recall that `A = delsq(numgrid('L',2*N+1)` assigns to `A` the discrete Laplacian operator associated with the L-shaped region defined by removing the third quadrant portion of the square having vertices $(\pm 1, \pm 1)$. Write a function `A = DisLapModL(N,r)` that generalizes this for the modified L-shaped region



Here, we have smoothed the sharp re-entrant corner at (0,0) with a quarter-circle having center at $(-r, -r)$. We assume that $0 < r < 1$. Order the unknowns so that those associated with `numgrid('L',2*N+1)` are followed by those that are associated with the gridpoints in the shaded region

$$S \ = \ \{(x, y) \ : \ -r \leq x \leq 0, \ -r + \sqrt{r^2 - x^2} \leq y \leq 0\}.$$

Make effective use of `numgrid`. The shaded region unknowns should be ordered in the top-to-bottom-left-to-right style used by `numgrid`.

P3. (Preconditioned Congugate Gradient)

Solve the equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

on (our usual) L-shaped region with the constraint that u is zero on the boundary except along the right edge where $u(1,y) = \sin(4\pi y)$. Use pcg with a preconditioner that is the tridiagonal part of the matrix of coefficients. Turn in a contour plot of the solution for a reasonably chosen grid dimension. Write a paragraph on the connections between grid dimension, solution accuracy, and the number of pcg iterations.

GTD6. (An Incomplete Block Cholesky Preconditioner)

Assume that

$$A = \begin{bmatrix} A_{11} & \cdots & A_{1p} \\ \vdots & \ddots & \vdots \\ A_{p1} & \cdots & A_{pp} \end{bmatrix} \qquad A_{ij} \in \mathbf{R}^{r\times r}$$

is block sparse, symmetric, and positive definite and that

$$U = \begin{bmatrix} U_1 \\ \vdots \\ U_p \end{bmatrix} \qquad U_i \in \mathbf{R}^{r\times r}$$

is dense with full column rank. Develop a symmetric positive definite preconditioner M whose Cholesky factor G has the form $G = LD$ where

$$L = \begin{bmatrix} L_{11} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ L_{p1} & \cdots & L_{pp} \end{bmatrix}$$

and $D = \text{diag}(D_1, \ldots D_p)$. The following properties must hold:

1. The L_{ii} and D_i are lower triangular.
2. If A_{ij} is zero then the (i,j) block of $A - LL^T$ is zero.
3. If K is the Cholesky factor of $LL^T + UU^T$ then its diagonal blocks are the same as G's diagonal blocks.

Write a function `[L,D] = BlkIncChol(A,U)` that does this. Store all matrices in conventional full format. You may ignore the possibility of breakdown during the incomplete factorization process.