CS 621: Assignment 3

Due: Friday, October 12, 2007 (In Lecture or 5153 Upson by 4pm)

Scoring for each problem is on a 0-to-3 scale (3 = complete success, 2 = overlooked a small detail, 1 = germ of the right idea, 0 = missed the point of the problem.) One point will be deducted for insufficiently commented code. Unless otherwise stated, you are expected to utilize fully MATLAB'S vectorizing capability subject to the constraint of being flop-efficient. Test drivers and related material are posted on the course website `http://www.cs.cornell.edu/courses/cs621/2007fa/`. For each problem submit output and a listing of all scripts/functions that you had to write in order to produce the output. You are allowed to discuss *background* issues with other students, but the codes you submit must be your own.

P1. (I-plus-Skew Toeplitz Systeme)

Linear systems of the form $(I + S)x = b$ where S is skew-symmetric ($S = -S^T$) are positive definite. Complete the following function so that it performs as specified:

```
  function [x,condest] = ToepEyeSkew(v,b)
% v and b are column vectors of length n-1 and n respectively.
% x solves the linear system (I + S)x = b  where S is a skew
% Toeplitz matrix whose first column is [0;v].
% condest is an O(n) estimate of the condition of (I+S).
```

Be sure to explain your method for computing x and include the rationale behind your condition estimator.

P2. (A Structured Block-Cyclic Markov Problem)

We say that a matrix M is *Markov* if its entries are nonnegative and it has unit column sums. For such a matrix there is a unique x (called the *Markov vector*) with positive entries satisfying `sum(x)` $= 1$ so that $Mx = x$. (Actually, you need to assume a little more about M for this to be the case, but we suppress this detail in this problem.) The *LU method* for computing x is to compute $P(M - I) = LU$ and observe that U is singular. It turns out that because of M's special structure, $U(1{:}n-1, 1{:}n-1)$ is nonsingular and $u_{nn} = 0$. If $Uw = 0$ then $(M - I)w = 0$ and $x = w/(w_1 + \cdots + w_n)$.

We say that a Markov matrix M has block cyclic structure if it has the form

$$M = \begin{bmatrix} 0 & 0 & 0 & M_4 \\ M_1 & 0 & 0 & 0 \\ 0 & M_2 & 0 & 0 \\ 0 & 0 & M_3 & 0 \end{bmatrix} \qquad p = 4$$

where each M_i is r-by-r and Markov. Complete the following function so that it performs as specified.

```
  function x = BlockCyclicMarkov(M)
% M is a length-p cell array with the property that M{1},...,M{p} are each
% r-by-r Markov matrices.
% x is a nonnegative column pr-vector with sum(x) = 1 having the property
% that Ax = x where A is the block cyclic Markov matrix defined by the M{i}.
```

Use the LU method.

P3. (Diagonal of the Cholesky Factor)

Complete the following function so that it performs as specified:

```
  function d = DiagChol(u)
% Suppose u is a  column n-vector and
% I + u*u' = G*G' is the Cholesky factorization of I + u*u'.
% d = diag(G), i.e., d(i) = G(i,i), i=1:n
```

Your implementation should be $O(n^2)$. Hint. Look at the blocking

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} = \begin{bmatrix} G_{11} & 0 \\ G_{21} & G_{22} \end{bmatrix} \begin{bmatrix} G_{11} & 0 \\ G_{21} & G_{22} \end{bmatrix}^T$$

where $A = I + uu^T$ and A_{11} is 1-by-1.

GTD3. (The Block Diagonal Part of the Cholesky Factor)

Complete the following function so that it performs as specified and is efficient:

```
  function D = DiagChol(U)
% Suppose I + U*U' = G*G' is the Cholesky factorization of I + U*U'
% where U is N-by-r. Assume that N = nr where n is an integer.
% D is N-by-r and D(i,1:r) = G(i,i) for i = 1:r, r+1:2r, ..., N-r+1:N.
% In other words, D is a stacking of G's diagonal blocks.
```