CS 621: Assignment 1 (With Corrections)

Due: Wednesday, September 12, 2007 (In Lecture or 5153 Upson by 4pm)

Scoring for each problem is on a 0-to-3 scale (3 = complete success, 2 = overlooked a small detail, 1 = germ of the right idea, 0 = missed the point of the problem.) One point will be deducted for insufficiently commented code. Unless otherwise stated, you are expected to utilize fully MATLAB'S vectorizing capability subject to the constraint of being flop-efficient. Test drivers and related material are posted on the course website `http://www.cs.cornell.edu/courses/cs621/2007fa/`. For each problem submit output and a listing of all scripts/functions that you had to write in order to produce the output. You are allowed to discuss *background* issues with other students, but the codes you submit must be your own.

P1. (A Structured Matrix-Matrix Product)

We say that a matrix $A \in \mathbb{R}^{2n\times 2n}$ has *property J* if

$$J^T AJ = -A^T$$

where

$$J = \begin{bmatrix} 0 & I_n \\ -I_n & 0 \end{bmatrix}.$$

Determine what this says about the blocks in

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}.$$

(a) Write a function `A = RandJ(n)` that generates a random $2n$-by-$2n$ J-matrix. (Use `randn`.) Submit a listing of your implementation. (b) Write a function `B = SquareJ(A)` that takes a J-matrix A and returns A^2. Your implementation should make full use of Matlab's vectorizing capabilities subject to the constraint that it is flop efficient. Include a comment that indicates how many flops are requred, i.e., $(???)\times n^3$. (c) Test your code on the script `P1.m` that is available on the website. Submit output and a listing of `RandJ` and `SquareJ`.

P2. (A Structured Matrix-Matrix Product)

Suppose $Q \in \mathbb{R}^{n\times n}$ is orthogonal and $v \in \mathbb{R}^n$. Consider the matrix

$$M = \left[\, v \,|Qv \mid \cdots \mid Q^{n-1}v \,\right]$$

Implement the following MATLAB function so that it performs as specified:

```
   function alpha = KrylovProd(Q,v,x)
%  Q is and n-by-n orthogonal matrix that is block diagonal with 2-by-2 blocks
%  v and x are column n-vectors
%  alfa = x'*M'*M*x  where M = [ v | Qv | Q^{2}v | ... | Q^{n-1}v ]
```

Test your code on the script `P2.m` that is available on the website. Submit output and a listing of `KrylovProd`.

P3. (The Haar Wavelet Transform)

Define the matrix $W_1 = (1)$ and $W_2,\ W_4,\ W_8, \ldots$ by

$$W_{2m} \;=\; \begin{bmatrix} W_m & W_m \\ W_m & -W_m \end{bmatrix}$$

Complete the following MATLAB function so that it performs as specified:

```
  function y = Haar(x)
% x is a column n-vector and n is a power of 2.
% y = W_{n}x
```

Hint. Look at the product

$$\begin{bmatrix} f \\ g \end{bmatrix} = \begin{bmatrix} W_m & W_m \\ W_m & -W_m \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$

Your implementation should be recursive and involve $O(n \log n)$ flops. Take a look at the Strassen algorithm in GVL4 §1.3 for guidance on the design of a recursive matrix procedure. Test your code on the script `P3.m` that is available on the course website. Submit output and a listing of `Haar`. How many flops are required? Explain.

Book Problems

From §1.5 in GVL4, work these brief "pencil-and-paper" problems: P1.5.1, P1.5.2, P1.5.3, P1.5.5, P1.5.6. The solutions you submit should be concise and well-written.