CS 621: Sample Quiz

Actual quiz is in-class, sixty minutes, closed book and notes, Monday, October 17, 2005.

P1.

If $A \in \mathbb{R}^{n \times n}$ is nonsingular and $u, v \in \mathbb{R}^n$, then

$$(A + uv^T)^{-1} = A^{-1} + \alpha A^{-1} uv^T A^{-1} \qquad \alpha = -1/(1 + v^T A^{-1} u)$$

Complete the following MATLAB function so that it performs as specified.

```
  function [x,y] = Solve(A,u,v,b)
% A n-by-n matrix
% u,v,b are column n-vectors
% A and C = A +u*v' are nonsingular
% x solves Ax = b and y solves (C^2)y = b
```

You may use `[L,U,P] = lu(·)` and `\` to solve triangular systems.

P2. Suppose $T \in \mathbb{R}^{n \times n}$ is a specific symmetric positive definite Toeplitz matrix and that `x = Tsolver(b)` solves the linear equation $Tx = b$ in $O(n^2)$ flops. Explain clearly how this function could be used to solve the linear system

$$\begin{bmatrix} T & v \\ v^T & 1 \end{bmatrix} \begin{bmatrix} z \\ \alpha \end{bmatrix} = \begin{bmatrix} c \\ \gamma \end{bmatrix}$$

where $v, z, c \in \mathbb{R}^n$, $\gamma, \alpha \in \mathbb{R}$, and the matrix of coefficients is positive definite.

P3. The matrix

$$A = \begin{bmatrix} .780 & .563 \\ .913 & .659 \end{bmatrix}$$

has determinant equal to 10^{-6}. Give an upper bound for its smallest singular value. What can you say about the relative accuracy of `x = [981 726; 529 384]\b` when solved on a computer with unit roundoff 10^{-17}?