CS 621: Assignment 6

Due: Friday, December 6, 2002 (In Lecture or 4130 Upson by 4pm))

Scoring on each problem is on a 0-1-2-3 scale. 3 = complete success, 2 = overlooked a small point, 1 = germ of the right idea, 0 = missed the point of the problem. One point will be deducted for insufficiently commented code. Test drivers and related material will be posted on the course website. For each problem submit output and a listing of all the scripts/functions that you had to write/modify in order to produce the output. You are allowed to discuss *background* issues with other students, but the codes you submit must be your own.

Problem 1. The matrix

$$T = \begin{bmatrix} T_{11} & T_{12} & \cdots & T_{1m} \\ 0 & T_{22} & \cdots & T_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & \cdots & T_{mm} \end{bmatrix}$$

is a *normalized upper quasi-triangular matrix* if (a) the diagonal blocks are either 1-by-1 or 2-by-2, (b) the 2-by-2 blocks correspond to T's complex eigenvalues, and (c) each 2-by-2 diagonal block T_{ii} satisfies $T_{ii}(1,1) = T_{ii}(2,2)$. Note that if $C \in \mathbb{R}^{2\times 2}$ and

$$Q^T C Q = \begin{bmatrix} \lambda & \beta \\ \alpha & \lambda \end{bmatrix} \qquad \alpha\beta < 0$$

then $\lambda(C) = \{\lambda + \sqrt{-\alpha\beta},\ \lambda + \sqrt{-\alpha\beta}\}$. Complete the following functions

```
  function [Q,T] = normalSchur2(C)
% C is a 2-by-2 real matrix with complex eigenvalues
% Q is orthogonal so that Q'*C*Q = T with T(1,1) = T(2,2);

  function [Q,T] = normalSchur(A)
% A is a real matrix.
% Q is orthogonal such that Q'*A*Q = T is a normalized upper quasi-triangular
% matrix.
```

You are allowed to use the MATLAB `schur` function. Your implementation of `normalSchur` should make effective use of `normalSchur2`. Test your implementations with the script `P1` available on the website.

Problem 2. Develop a closed form expression for

$$F = \exp\left(\begin{bmatrix} \lambda & \beta \\ \alpha & \lambda \end{bmatrix}\right) \qquad \alpha\beta < 0.$$

Implement the following function

```
  function F = MyExpM(T)
% T is a normalized upper quasitriangular matrix with distinct eigenvalues.
% F = exp(T)
```

Make effective use of the ideas in §11.1.4. You may use \ to solve the Sylvester systems that arise in (11.1.5). Test your implementation with the script `P2` available on the website.

Problem 3. Subject to the coupling of complex conjugate eigenvalue pairs, the eigenvalues can be arbitrarily ordered in the real schur form. For example, if

$$T_0 = \begin{bmatrix} \lambda & a & b \\ 0 & c & d \\ 0 & e & f \end{bmatrix}$$

then it is possible to find an orthogonal Q such that

$$Q^T T_0 Q = T = \begin{bmatrix} \tilde{c} & \tilde{d} & \tilde{a} \\ \tilde{e} & \tilde{f} & \tilde{b} \\ 0 & 0 & \lambda \end{bmatrix}$$

with $\lambda(T(1{:}2, 1{:}2)) = \lambda(T_0(2{:}3, 2{:}3))$. Implement the following function

```
   function [Q,T] = Swap3(T0)
% T0 is a 3-by-3 matrix with the property that T0(2:3,1) = 0 and T0(2:3,2:3)
% has complex eigenvalues.
% Q is orthogonal with the property that Q'*T0*Q = T and T(3,1:2) = 0.
```

Test your implementation with the script `P3` available on the website.

Problem 4. Suppose $A \in \mathbb{R}^{n \times n}$ has r real eigenvalues. If $Q^T A Q = T$ is its real schur form and

$$T(n-r+1{:}n, n-r+1{:}n)$$

is upper triangular, then the columns of $Q(:, 1{:}n-r)$ form an orthonormal basis for the invariant subspace defined by A's complex eigenvalues. Implement the following function

```
   function [Q,T] = OrderedSchur(A)
% A is an n-by-n real matrix with r real eigenvalues.
% Q is orthogonal such that Q'*A*Q = T is upper quasi-triangular with that
% property that T(n-r+1:n,n-r+1) is upper triangular.
```

Test your implementation with the script `P4` available on the website.