CS 621: Assignment 4

Due: Friday, November 8, 2002 (In Lecture or 4130 Upson by 4pm)

Scoring on each problem is on a 0-1-2-3 scale. 3 = complete success, 2 = overlooked a small point, 1 = germ of the right idea, 0 = missed the point of the problem. One point will be deducted for insufficiently commented code. Test drivers and related material will be posted on the course website. For each problem submit output and a listing of all the scripts/functions that you had to write/modify in order to produce the output. You are allowed to discuss *background* issues with other students, but the codes you submit must be your own.

Problem 1. Suppose $A \in \mathbb{R}^{m\times n}$ has full column rank and that $b \in \mathbb{R}^m$. In the TLS problem we minimize

$$\phi(E,r) \;=\; \| E \|_F^2 + \| r \|_2^2 \qquad E \in \mathbb{R}^{m\times n},\; r \in \mathbb{R}^m$$

subject to the constriant that $b + r$ is in the range of $A + E$. Implicit is the assumption that there are errors throughout the data matrix A. The ordinary LS problem results if we assume that A is exact and we minimize ϕ subject to the constraint that $b + r \in \text{ran}(A + E)$ and $E = 0$. Now suppose that just the first p columns of A are exact. The *partial TLS* problem results if we minimize ϕ subject to the constraint that $b+r \in \text{ran}(A+E)$ and $E(:,1{:}p) = 0$. If such a minimizing E_{opt} and r_{opt} can be found then the PTLS solution satisfies $(A+E_{opt})x_{PTLS} = b+r_{opt}$. Complete the following MATLAB function so that it performs as specified

```
  function x = PTLS(A,b,p)
% A is m-by-n and has rank n, b is m-by-1, and 0 <= p <= n
% (A + E)x = b+r where norm(E,'fro)^2 + norm(r,2)^2 is minimized and E(:,1:p) = 0.
% Assume that such an x exists.
```

Make effective use of the MATLAB functions `qr` and `svd`. Test your implementation with the script `P1` available on the website. Hint. Start by computing the QR factorization of $A(:,1{:}p)$.

Problem 2. Complete the following function so that it performs as specified

```
  function [c,s] = ComplexGivens(y,z)
% y an z are complex.
% c is real and s is complex so that Q = [c conj(s); -s c] is unitary and
% the second component of Q'*[y;z] is zero.
```

Test your implementation with the script `P2` available on the website.

Problems 3 and 4. Refer to the discussion of ridge regression and cross-validation in §12.1.3 and complete the following function so that it performs as advertised.

```
  function [x,lambda_opt] = RidgeCV(A,b)
% A is m-by-n and has rank n and b is m-by-1
% x satisfies (A'*A + lambda_opt*I)x = A'*b  where
% lambda_opt minimizes the 2-norm of r(lambda) = b - A*((A'*A + lambda*I)\(A'*b))
```

Make effective use `svd`. Use a MATLAB minimizer like `fmin` to determine the optimum lambda. Justify your choice of termination criteria. Test your implementation with the script `P3` available on the website.