CS 621: Matrix Computations
Fall 2001
Problem Set 1

Handed out: Mon., Sep. 10.

Due: Mon., Sep. 17 in lecture.

1. Consider solving $L\mathbf{x} = \mathbf{b}$ for $\mathbf{x}$ where L is a lower triangular matrix. Suppose L is singular and suppose $\mathbf{b} \in R(L)$ (i.e., the system is consistent). Develop an algorithm, based on forward substitution, that computes a solution $\mathbf{x}$ to this system. Note: you can make the simplifying assumption that there is just one 0 on the diagonal of L. For full credit, your algorithm should also have a test for whether $\mathbf{b} \in R(L)$ or not.

2. A "Toeplitz" matrix A is one whose entries are constant along all diagonals, i.e., $A(i,j) = A(i+1,j+1) = A(i+2,j+2) = \cdots$ for any entry (i,j). Toeplitz matrices sometimes correspond to recurrence relationships. Write down a linear system $L\mathbf{x} = \mathbf{b}$ such that L is unit lower triangular, Toeplitz, and sparse (i.e., most entries of L are zero), almost all the entries of $\mathbf{b}$ are zeros, and $\mathbf{x}$ is the well-known Fibonacci series (that is, the entries of $\mathbf{x}$ are $1, 1, 2, 3, 5, 8, 13 \ldots$).

3. Consider forward substitution to solve $L\mathbf{x} = \mathbf{b}$, where L is an $n \times n$ unit lower triangular matrix. Let β be the maximum absolute value of entries of $\mathbf{b}$. Let λ be the maximum absolute value of entries of L below the diagonal. Show that the maximum absolute value of entries in $\mathbf{x}$ is at most $\beta(1+\lambda)^{n-1}$. [Hint: Use induction on the forward substitution process. The trick is to formulate a good induction hypothesis.]

4. In Matlab, make a random $n \times n$ matrix with entries uniformly distributed in $[-1, 1]$. (Use `rand`.) Then compute its LU factorization using GEPP. (In matlab this is: `[l,u,p]=lu(a);`.) Next, compute the maximum absolute entry of L^{-1}. (Matrix inverse in Matlab is `inv`.)

   Then make a unit lower triangular matrix $\bar{L}$ by putting ones on the diagonal and entries uniformly distributed in $[-1, 1]$ below the diagonal. (Refer to the `eye()` and `tril()` functions.) Then compute the maximum absolute entry of $\bar{L}^{-1}$.

   For both of these experiments, make plots of the maximum absolute entry (on the y-axis) versus n (on the x-axis). Use the `semilogy` function for making the two plots.

   You should obtain very different plots for the experiment in the first paragraph versus the experiment in the second paragraph. Nobody knows exactly why the plot in the second case grows so much faster than the in the first case, although some partial results are known.

   Hand in your m-file and the requested plots.