CS 621: Matrix Computations
Fall 2001
Prelim 1

Handed out: Thurs., Sep. 27.

This is a timed 75-minute closed-book and closed-note exam. Write all answers in the exam booklet. There are a total of 75 points on this exam.

1. [**10 points**] The following posting appeared yesterday on the internet newsgroup `comp.soft-sys.matlab`. The author of this posting seems to be a bit confused about stability and conditioning. Please explain what advice you'd give him. Incidentally, `RCOND` in Matlab means the reciprocal of the condition number of a matrix.

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I get some warnings like RCOND=10e-17 matrix close to singular or badly
scaled.  The consequence is that the results of my simulation, which
uses matrixes of 400 order, are completely wrong.  The points I obtain
in the plot shouldn't be greater than '1', and they actually are bigger.
Is there a possibility of calling another program like mathematica or
C, to obtain better results with a similar computing time?  In that case
is there anybody who has a mex file in C language which solves the inverse
matrix without this problem.  Thanks in advance
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2. [**10 points**] Explicitly work out the condition number in the ∞-norm of

$$\begin{pmatrix} 1 & 1 \\ 1 & 1+a \end{pmatrix}$$

as a function a. [Note: Obtain A^{-1} any way you'd like. One way to obtain A^{-1} from A is to repeatedly solve linear systems of the form $A\mathbf{x} = \mathbf{e}_i$ for $\mathbf{x}$, where $\mathbf{e}_i$ is the ith column of the identity matrix.]

3. [**15 points**] Consider the division operation of two real numbers a/b with $b \neq 0$. Show that this operation is well-conditioned for all data.

4. [**15 points**] In class we showed that $\|AB\|_\infty \leq \|A\|_\infty \|B\|_\infty$. Find an example of an $A, B \in \mathbf{R}^{2\times 2}$ such that this inequality becomes an equation. On the other hand, find an example of $A, B \in \mathbf{R}^{2\times 2}$ such that the right-hand side is 1000 times larger than the left.

5. [**10 points**] Let A be an $m \times n$ matrix, let P be an $m \times m$ permutation matrix, and let Q be an $n \times n$ permutation matrix. Show that $\|A\|_p = \|PAQ\|_p$ for any matrix p-norm.

6. [**15 points**] Let A be an $n \times n$ matrix that is upper triangular except for the first column and last row, which are both full. In other words, $A(i, j) = 0$ for all i, j satisfying $1 < j < i < n$. How many flops, accurate to the leading term, are required to solve $A\mathbf{x} = \mathbf{b}$ using Gaussian elimination without pivoting, followed by forward and back substitution?