Foreword

Michael Beeson

Stephen Kleene was one of the greatest logicians of the twentieth century, and had an enormous influence on the subject. The book in your hands is the textbook that spread that influence far beyond his own students, to an entire generation of logicians. It was first published in 1952, some twenty years after the publication of Gödel's paper on the incompleteness of arithmetic, which marked, if not the beginning of modern logic, at least a turning point after which "nothing was ever the same." Kleene was one of the leading developers of mathematical logic, and lived a long full life of scholarship and teaching; here I will tell you something about his life and work, as well as about this important book.

The 1930s was a time of creativity and ferment in the subject, when the notion of "computable" moved from the realm of informal mathematical use of the word to the realm of exact mathematics. This was accomplished by the work of Kurt Gödel, Alan Turing, and Alonzo Church, who gave three apparently different precise definitions of "computable function". Church put forward the thesis that all intuitively computable functions are lambda-definable. Almost a decade later, Kleene gave this thesis the name it bears today, "Church's thesis". This thesis was not at first accepted, even by Gödel (who received it in a private communication in 1934). Gödel's own notion of "general recursive function" was given in lectures two or three months later in 1934. Then in 1936, Turing proved that every Turing-computable function is lambda-definable, and in that same year, Kleene proved that all these notions are equivalent.[1] Then there was a collective realization that this was indeed the "right notion"; but it is perhaps too little appreciated that Kleene made a *further* contribution by introducing the notion of *partial* recursive function; the equivalence proofs of 1936 had been for *total* functions, i.e. those defined for every integer input. In [1], Kleene notes that Gödel asked him in 1940, "What is a partial recursive function?", and tells us that he (Kleene) was the first to use Turing machines to compute partial rather than total functions. He says that he first did this in his seminar in 1941, and that was the origin of the way he presents the material in his book. Today, partly thanks to the influence of this textbook, most people do not realize that this important notion, now considered absolutely basic, was not present for the first several years of the subject!

Kleene made many other contributions to logic, some of which are mentioned below. Aside from these direct contributions, his textbooks played an invaluable part in educating the logicians of the 1950s and 1960s, who have in turn taught the young logicians of the present. All those with whom I spoke about this foreword emphasized how important Kleene's book had been in their own logical education.

The Wikipedia article about Kleene says that Kleene's standing among logicians is suggested by the witticism "Kleeneliness is next to Gödeliness".[2] Among the achievements that earned Kleene this reputation are the following: he invented partial recursive functions, the arithmetic hierarchy, the notion of hyperarithmetic set and the hyperarithmetic hiearchy and the related theory of recursive well-orderings, the notion of recursive realizability (a major tool in the metamathematics of intuitionism), the notion of recursive functional (he gave schemata S1-S9 for defining them), and the Kleene

[1] See for example [2], and further references cited there.

[2] For non-native English speakers: this is a pun on "Cleanliness is next to godliness".

star, the basis for today's theory of "regular expressions", a fundamental tool in computer science. He also proved some fundamental theorems about the permutability of inferences in Gentzen's cut-free proofs, and (with his student Richard Vesley) wrote *Foundations of Intuitionistic Mathematics*, a book containing a thorough logical analysis of intuitionism.

Stephen Cole Kleene was born in Hartford, Connecticut, in 1909. He was the son of poet Alice Lena Cole and economics professor Gustav Kleene. Although the family lived in Hartford, summers were spent in Hope, Maine, where the family farm was located. Examples of his early mathematics education, in his mother's handwriting, have survived. If baby Stephen cries for five minutes and then for seven minutes, for how long has baby Stephen cried? He collected butterflies as a youngster, a pastime he returned to as an adult. He was proud to have a variant of a butterfly named after him (*Beloria Todde Ammiralis Ba Kleenei*, which he discovered in the mid-1950s.) Any adventures of his childhood have not come down to us; the next we know is that in 1927, inspired by Thoreau's *The Maine Woods*, he made a bicycle trip to Mt. Katahdin, a journey of some 120 miles. At night he sneaked into barns to sleep. For food he took a large bag of fig newtons. Fig newtons remained a favorite treat all his life. As an undergraduate, he was noticed, and teased a bit, because he did not smoke, drink, or swear, according to one of his college roommates.

Kleene's Ph. D. advisor was Alonzo Church, creator of the lambda calculus. While still a student, he attended lectures by Gödel at the nearby Institute for Advanced Study, so he was taught by two masters of the subject. Kleene's thesis (1934) developed the theory of the integers in lambda calculus, modifying Church's original definitions using ideas of his own and of his fellow student J. Barkley Rosser. His work on the equivalence of various notions of computability came a bit later. In 1935, he became an instructor of mathematics at the University of Wisconsin, which remained his home institution all his life. He started at the bottom rung, but he climbed the academic ladder successfully, serving as chair of the mathematics department, and as Dean for five years, during the Vietnam War era. During his service as Dean, the Army Mathematics Research Center at the University of Wisconsin was bombed, killing a graduate student. Kleene was very much opposed to the Vietnam war himself, but had no sympathy with violence or with disruptive protests.

In 1939 he spent a year at the Institute for Advanced Study in Princeton, where he invented the new field of "recursion theory", which remained one of his main areas of interest all his life.

At that point World War II intervened. He enlisted in the U.S. Navy, in which he rose to the rank of Lt. Commander, and served as a Navy instructor of navigation. He made one sea trip, and had a minor adventure: his ship chased a light that the captain thought was an enemy ship for some hours, until Kleene, as navigation officer, made the determination that they were chasing a star. Eventually Kleene's talents were more fully utilized as a project director at the Naval Research Laboratory in Washington, D. C. In 1942 he married Nancy Elliot. After World War II, he visited Amsterdam, where he explored his mutual interests with the Dutch intuitionists L. E. J. Brouwer, Arend Heyting, and their students. The intuitionistic school of mathematical philosophy believes that if one proves that something exists with certain properties, then one ought to be able to "construct" such a something. The word "construct" was not precisely defined (hence the need for "intuition"); but Kleene saw that he could translate it as "compute", which he *had* precisely defined. This fruitful idea lead to his notion of "recursive realizability", which one finds carefully explained in the last chapter of his textbook.

After the war, Kleene returned to Wisconsin and the teaching of logic. That effort resulted in this classic textbook. It is divided into three parts; roughly speaking, Part I corresponds to the period 1900–1920, Part II corresponds to the period 1920-1930, and Part III (except part of the last chapter) to the 1930s. Part I lays the foundation for the technical studies in the rest of the book, by introducing the student to the paradoxes and the different philosophies of mathematics (logicism, intuitionism, and formalism) that developed in response to the paradoxes. Later textbook authors, apparently feeling the need to use every possible page for technical results, generally do not present these ideas so thoroughly. Since these ideas are quite necessary for a student to understand the initial driving aims of the subject, many a logic teacher has referred students to Kleene for supplemental reading.

Part II develops the basic machinery of the propositional and predicate calculus, and formal number theory. Because of its formal nature, this material is difficult for students. Kleene's classroom experience must have helped him to present it in an optimal fashion. Every detail is explained, but without losing sight of the big picture. At the end of Part II, the student is in a position to understand the statement of Gödel's theorem; it is as if, having climbed a mountain (of technicalities) one can now see a new peak looming ahead. Kleene presents Gödel's theorem at the very end of Part II (Theorem 28). Again, his presentation is optimal: "We can give the rigorous metamathematical proof now, by borrowing one lemma from results of the next two chapters." He then presents the main idea (the diagonal argument), postponing the technical details of Gödel numbering to the next chapter.

In Part III, Kleene presents the tools of the 1930s: Gödel numbering and its use for the arithmetization of metamathematics, general recursive functions, partial recursive functions, the recursion theorems, and Turing machines. Having completed Part III, the student finally stands on the peak, with a 360 degree view of the subject; of course there are many interesting parts of the mountain left unexplored, but Kleene does at least *mention* many of these along the way, with references to the literature.

The 1930s saw great advances in logic, not only with respect to Gödel's incompleteness theorems and the foundations of computability theory, but also in at least two other areas: In Germany, Gerhard Gentzen laid the foundations of proof theory, and the Polish school led by Łukasiewicz and Leśniewski, and their student Tarski, carried out numerous logical investigations. The war brought all these activities to a halt, and resulted in the transplantation of Gödel and Tarski to the United States, and the deaths of Gentzen and of Tarski's student Presburger. Kleene studied Gentzen's work, and in 1951 (just a year before publishing this textbook) he made important contributions to the study of Gentzen's rules. This textbook contains, in the same last chapter that presents recursive realizability, a careful proof of Gentzen's fundamental "cut-elimination" theorem, the thrust of which is that it is possible to find proofs that only involve subformulas of the final theorem and of the axioms–it is not necessary to take detours through very long formulas (unless some long axioms are needed). Even in the twenty-first century, Kleene's book is still a good place to learn about cut-elimination; although we have excellent modern textbooks on proof theory, they are of course longer and more detailed, so Kleene's chapter is still a good starting point.

The work of the Polish school is less well-represented in Kleene's book, although Kleene met Tarski on many occasions, and the bibliography shows that Kleene was well aware of the work of Łukasiewicz and Tarski. He does mention Presburger's decidability result for a fragment of arithmetic, by way of introduction to Gödel's theorem. Since his book has about 550 pages as it is, he was faced with many decisions about what to omit. He had several years of teaching to help with those decisions, but that jam-packed last chapter (with both realizability and cut-elimination) shows how difficult it must have been to bring the book to a close. There is a fascinating article [1] in which Kleene tells us, among many other things, that the book was nearly not published: the publisher (van Nostrand) said that it would be 600 pages and would have to sell several thousand copies in the next few years to recoup the cost of printing. Kleene then persuaded North-Holland and Nordhoff to share the risk and publish it jointly, and van Nostrand brought out an American edition, from sheets printed in the Netherlands. My copy of the book is from the fifth reprinting (1967), and there was a ninth reprinting in 1988. Kleene says [1], "According to incomplete records, about 17,500 copies were sold through 1986, not counting sales of a reprint in Taiwan and of two in Japan, of two printings of the Russian translation (the first consisting of 8000 copies), of one of the Spanish translation, and of one of the Chinese translation." It seems that the book was a commercial as well as an academic success.

Kleene had a lifelong love of the outdoors. He was a member of the Sierra Club and a strong supporter of environmental causes in general. In his later years, he gave yearly "garden parties" in the late summer, with all the vegetables coming out of his garden. When he was the chair of the Department of Mathematics at the University of Wisconsin, there was an annual picnic for graduate students and faculty on Picnic Point, a small peninsula projecting into Lake Mendota. His son Bruce recalls that Kleene cooked hot dogs over an oak fire, "bellowing all the time." He was a tall man with a booming voice, which never needed amplification even in the largest halls. Bruce also recalls the time when Kleene took down a dead oak tree in their yard with a handsaw: "It was a very tall and very large tree. He climbed to the top of the tree, and roped himself in. Limb by limb, trunk section by section, the tree came thudding down. It took a long time, many days of work. During that time, the whole neighborhood resounded with his voice booming out a steady stream of groans and 'damnation!'s, his all purpose and single curse word." Bruce also recalls being around his father at mathematics conferences, where he understood nothing "except the stuff about a touring machine, which of course was a special kind of fast car." Stephen and Nancy Kleene had two more sons, Ken and Paul, and a daughter Pamela.

Kleene was an avid climber and, until well into his seventies, led the biannual logic picnic at Madison (now the Kleene Memorial Logic Picnic) on hikes up the cliffs at Devil's Lake. According to Saunders MacLane, Kleene's knowledge of mushrooms was legendary. MacLane also recounts [3]

> In 1949 he and I, about to attend a meeting at Dartmouth of the American Mathematical Society, got together on a project to climb all the peaks in the Presidential range, and we were joined by a third climber, a vacationing bellhop. Then, as we three stood finally on top of the last peak (Mount Madison), a thunderstorm struck. Kleene bounded down from the peak shouting, "Get down; it's lightning." I stepped down a bit, searched for our third companion only to find him flat on the ground, unconscious. Steve (quicker by way of his height) went down to the Madison Pass hut for help ... [The victim] later recovered. Steve and I continued to climb assorted mountains and Steve kept on rock climbing.

Another logician, Sol Feferman, told me of going hiking in Trömso, Norway (north of the Arctic circle) with Kleene and a Norwegian logician in 1977 (when Kleene would have been 68, and his companions much younger). Feferman said, "By then, Kleene was limping with one leg, but he surged ahead of us all the way. No fig newtons to my recollection."

Kleene's research was recognized not only by the informal respect of his colleagues, but also by several prize awards. In 1983 he was awarded the Leroy P. Steele Prize by the American Mathematical Society in 1983, with the citation, "for three important papers which formed the basis for later developments in generalized recursion theory and descriptive set theory." These papers presented his work on the arithmetical and hyperarithmetical hierarchies, a way that Kleene had discovered to classify real numbers according to the complexity of their simplest definitions. In 1990, President H. W. Bush presented Kleene with the National Medal of Science at a ceremony in the White House East Room. The medal was awarded for "his leadership in the theory of recursion and effective computability and for developing it into a deep and broad field of mathematical research." Earlier honours included election to the National Academy of Sciences (1969), election as President of the Association for Symbolic Logic (1956–58), President of the International Union of the History and the Philosophy of Science (1961) and of the Union's Division of Logic, Methodology and Philosophy of Science (1960–62). He was editor of the Journal of Symbolic Logic for twelve years. Kleene is also widely considered to be one of the grandfathers of computer science, since his work on finite automata and regular expressions is now a part of every computer science major's undergraduate education, and underlies some of the pattern-matching algorithms used in computer searching.

Kleene influenced the subject of modern logic not only through his textbooks and his research, but also through his Ph. D. students, many of whom went on to become well-known logicians. Kleene's thirteen doctoral students were Nels David Nelson (1946), Gene F. Rose (1952), John Addison (1955), Clifford Spector (1955), Paul Axt (1958), Richard Vesley (1962), Yiannis Moschovakis (1963), Douglas Clarke (1964), Shih-Chao Liu (1965), Joan Rand Moschovakis (1965), Robert L. Constable (1968), Dick de Jongh (1968), and David Kierstead (1979). These students, of course, had students of their own: the Mathematics Genealogy Project counts a total of 438 mathematical descendants of Stephen Kleene.

Kleene says [1] that "all my spare time for seven and a half years went into the composition of the book." That takes us back into World War II. In fact even more time was involved, because "the earlier preparation of my logic course was determining for the first ten chapters, which essentially followed that course. Subsequent chapters largely contain material used in seminars given in the spring semesters following the course beginning with 1938–39, except for the last two chapters." The last chapter is the one with cut elimination and realizability, and in [1] Kleene tells us that he first became aware of Gentzen's paper in 1947, which explains why it wasn't used in his pre-war seminars. Kleene also tells us that in 1942, when he entered the Navy, he realized that publication of his book was a long way off, and "the uncompleted manuscript reposed in Saunders MacLane's office" at Harvard. The exact point at which notes for his course (which began in 1936) turned into a book manuscript may be hard to pinpoint, but probably it was before 1939. It must have been an auspicious day when he picked up the manuscript from MacLane's office again.

Today Kleene's book is rarely used as the textbook in logic courses, but is still an important reference work. It presents only the part of logic developed before 1952, which makes it unsuitable as a sole textbook; but it presents that part very well, which makes it an excellent reference work. There is essentially no model theory in Kleene's book– and as Kleene himself explains in [1], "Little of what is presented now in an introductory model theory course was known in

1952." Gödel's completeness theorem is presented in Part IV, and is essentially the only model theory in the book. There is no chapter on formal set theory, despite the mention of Gödel's papers on the subject in the bibliography, and we have Kleene's explanation [1]: "Gödel's work on the consistency of the continuum hypothesis came out in 1938, which was after the basic choices had been made as to the direction my course, seminars, and book would take."

These omissions are not good reasons to consider the book to be of only historical interest. The material it does cover is still at the foundations of the subject, and the exposition is as good as it always was. For all but the very few who intend to pursue the subject to the frontiers of research, Kleene's book would still be a good textbook, and for those few, it would still be a good starting point. The present affordable edition ought to put this classic book on the bookshelf of every student of logic, where it should gather no dust.

References

[1] Kleene, S. C., The writing of *Introduction to Metamathematics*, pp. 161–168 in *Perspectives on the History of Mathematical Logic*, edited by Thomas Drucker, Birkhäuser, Basel (1991).

[2] Kleene, S. C., Introductory note to 1934, pp. 338–345 in Feferman, S, *et. al.* (eds.), *Kurt Gödel Collected Works, Volume I*, Oxford University Press (New York) and Clarendon Press (Oxford), 1986.

[3] MacLane, S., Stephen Cole Kleene, *Biographical Memoirs National Academy of Sciences* **75** (1998), 154–163.