CS6180 Lecture 21 Supplement

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If $\Gamma \vdash_{PA} E$ then $\Gamma^0 \vdash_{HA} E^0$.

Definition of F^0:

(1) P is an atomic formula P^0 is P

(2) $(A \Rightarrow B)^0$ is $A^0 \Rightarrow B^0$

(3) $(A \ \& \ B)^0$ is $A^0 \& B^0$

(4) $(A \ \vee B)^0$ is $\sim(\sim A^0 \ \& \sim B^0)$

(5) $(\sim A)^0$ is $\sim A^0$

(6) $(\forall x.A(x))^0$ is $\forall x.A^0(x)$

(7) $(\exists x.A(x))^0$ is $\sim \forall x. \sim A^0(x)$.

e.g. $((\forall x.A(x) \Rightarrow B) \Rightarrow \exists x.(A(x) \Rightarrow B))^0$ is $((\forall x.A(x) \Rightarrow B) \Rightarrow \sim \forall x. \sim (A(x) \Rightarrow B))$

For arithmetic : $\underset{\textbf{PA}}{\vdash} G \Rightarrow \underset{\textbf{HA}}{\vdash} G^0$

Note classically $E \Leftrightarrow E^0$