Delaunay Mesh Generation and Parallelization

January 31st, 2006

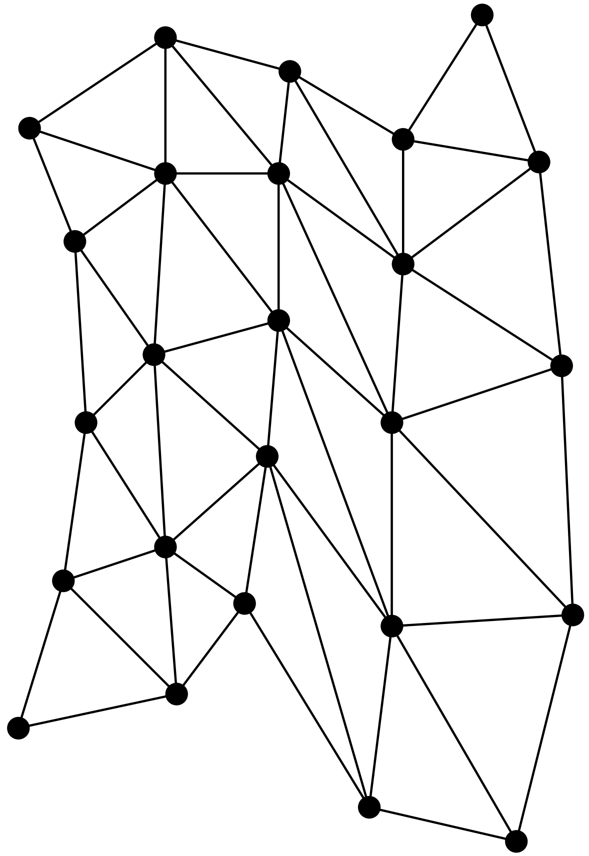
Context

- Mesh generation useful for
  - Finite element method
    - Determines basis functions
  - Graphics
    - Tessellation of surface into polygons for rendering

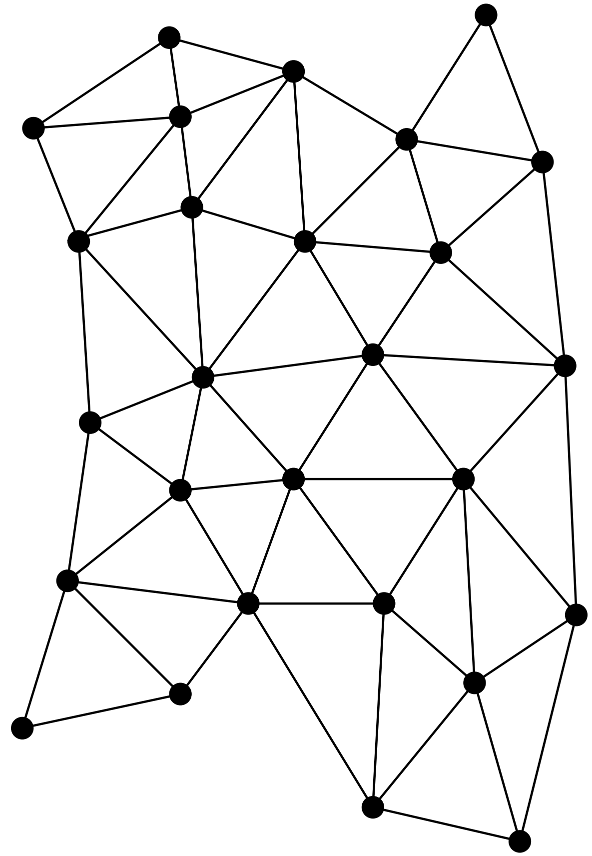
Mesh Quality

- Why is mesh quality important?
  - Can impact solution of linear systems
    - Angles too large can lead to errors
    - Angles too small lead to ill-conditioned systems
  - Tradeoffs regarding number of elements
    - More elements for better accuracy
    - Fewer elements for better speed

Structured vs. Unstructured



Structured Mesh

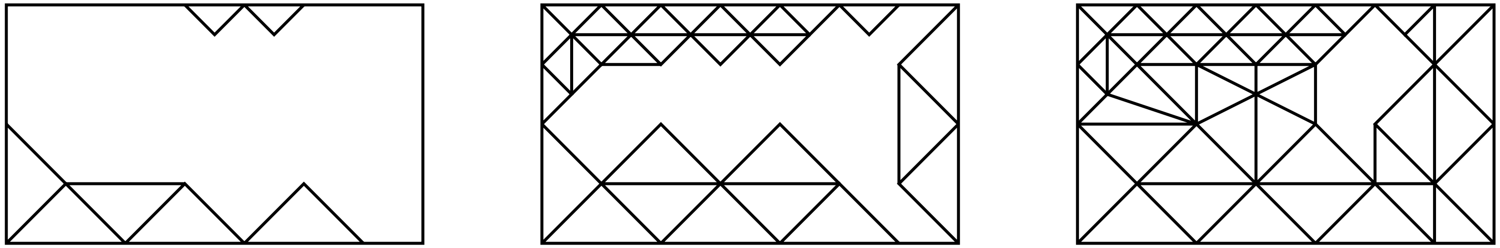


Unstructured Mesh

Some diagrams taken from Jonathan Shewchuk's lecture notes: http://www.cs.berkeley.edu/~jrs/mesh/

Different Types of Meshing

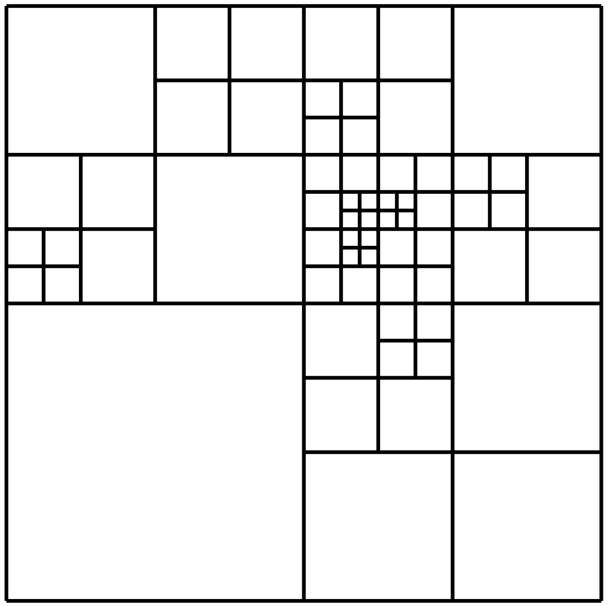
- Advancing Front
  - Starts at boundary of mesh
  - Uses heuristics to "grow" mesh inwards

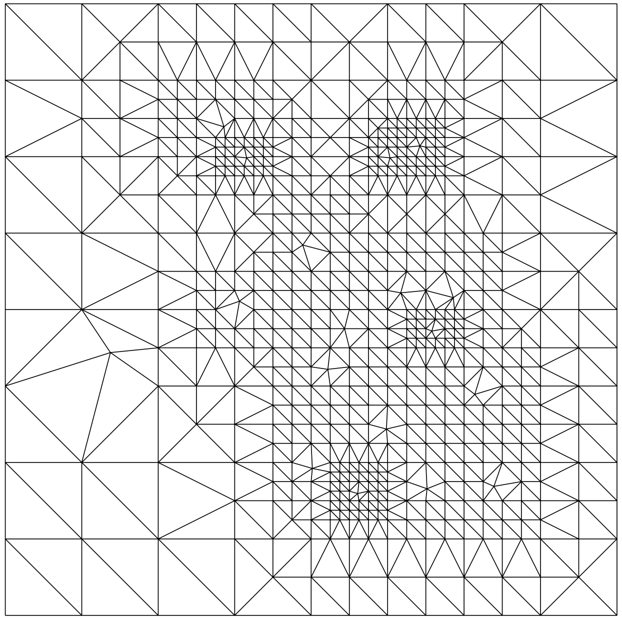


From JS lecture notes

Different Types of Meshing

- Quadtree based meshes
  - Subdivide plane using quadtree structure
  - Warp quadtree to match vertices
  - Triangulate resulting grid



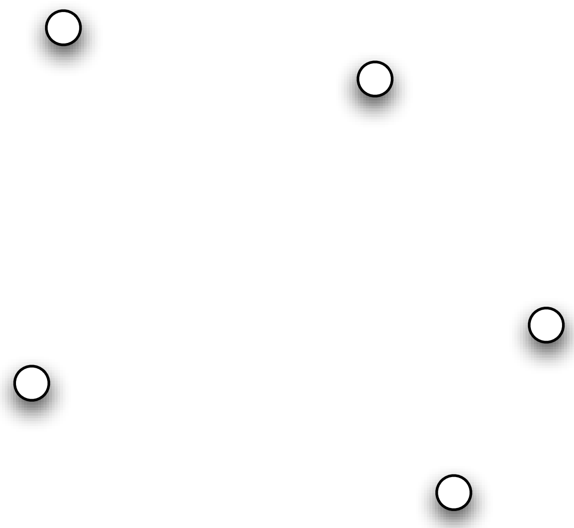


From JS lecture notes

Delaunay Meshes

Delaunay Mesh Generation

- What is a Delaunay mesh?
  - Tessellation of a surface given vertices



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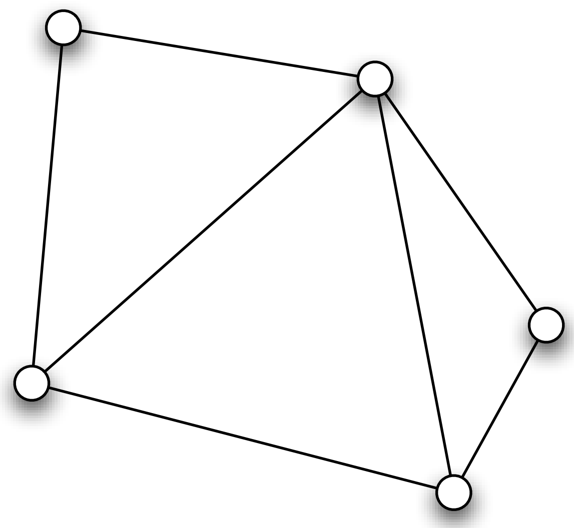
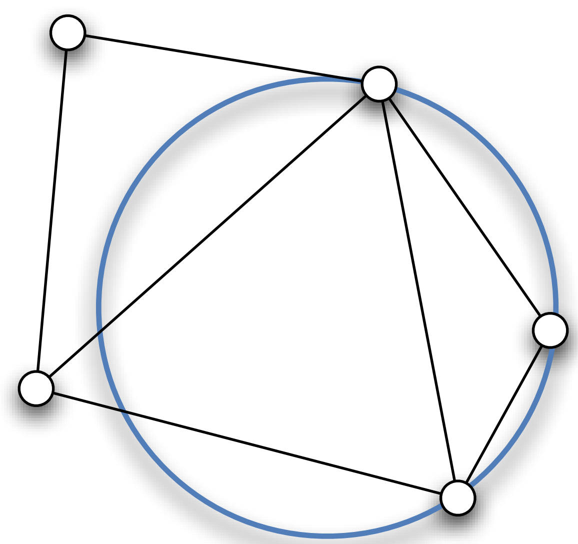


Delaunay Mesh Generation

- What is a Delaunay mesh?
  - Tessellation of a surface given vertices
  - Satisfies the Delaunay property
    - Circumcircle of any triangle does not contain another point in the mesh

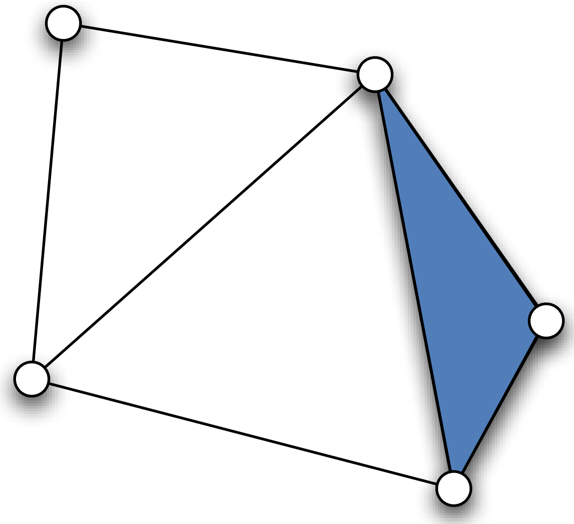


Why Delaunay Meshes?

- Provides specific guarantees
  - "The Delaunay triangulation of a point set minimizes the maximum angle over all possible triangulations"
    - Fewer skinny triangles
- Additional points can be inserted to meet certain quality constraints
  - Angle constraints
  - Triangle size constraints

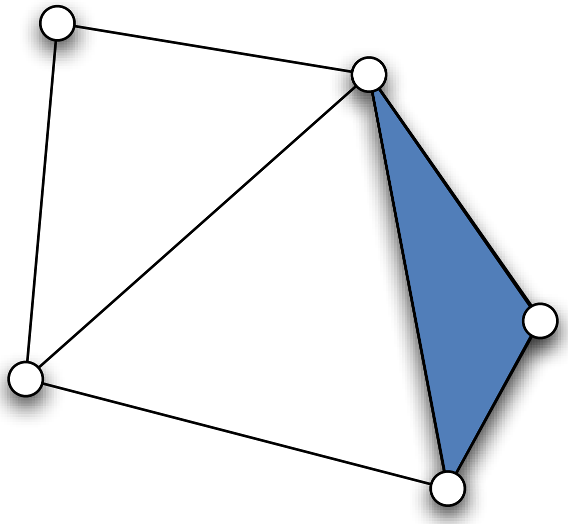
Delaunay Mesh Generation

- Want all triangles in mesh to meet quality constraints
  - No angle < 30°
- Fix bad triangles through iterative refinement
  - Add new vertices to mesh and retriangulate



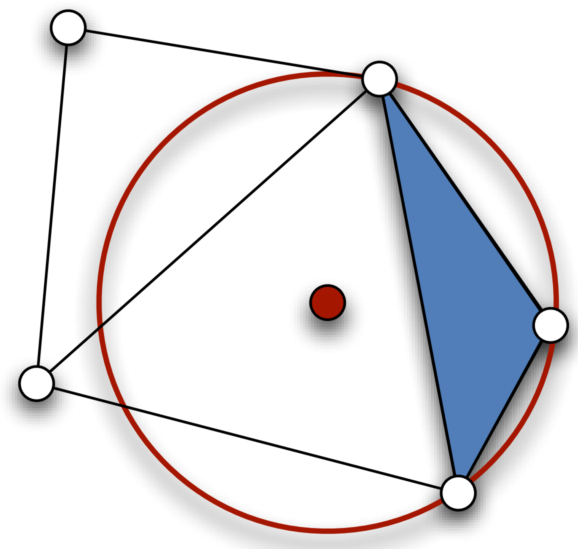
Mesh Refinement

- Choose "bad" triangle



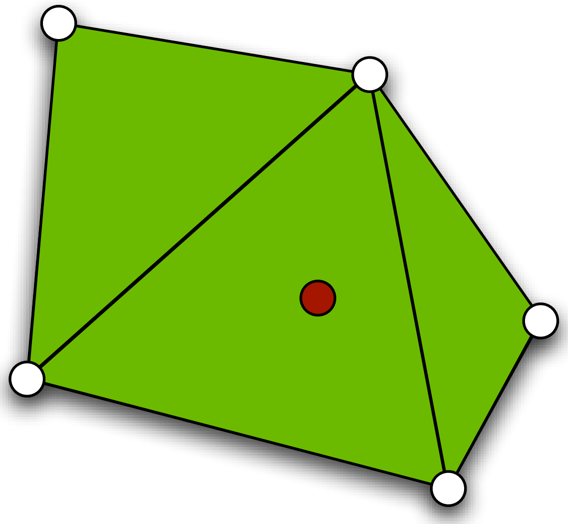
Mesh Refinement

- Choose "bad" triangle
- Add new vertex at center of circumcircle



Mesh Refinement

- Choose "bad" triangle
- Add new vertex at center of circumcircle
- Gather all triangles that no longer satisfy Delaunay property into cavity



Mesh Refinement

- Choose "bad" triangle
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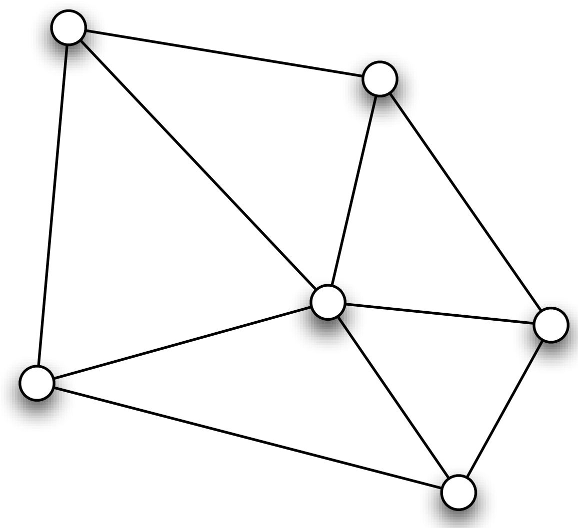
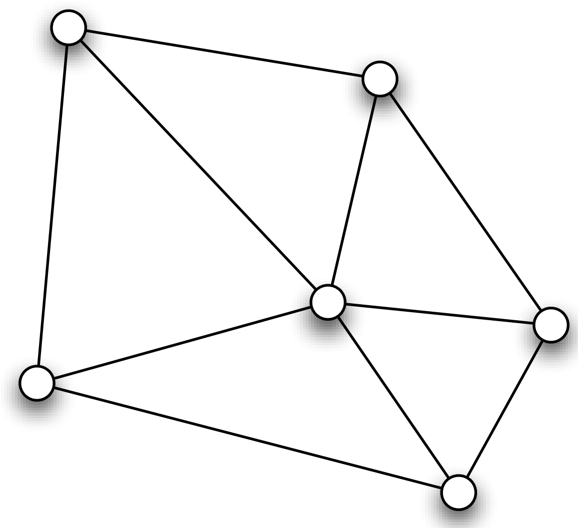
- Gather all triangles that no longer satisfy Delaunay property into cavity
- Re-triangulate affected region, including new point



Mesh Refinement

- Choose "bad" triangle
- Add new vertex at center of circumcircle
- Gather all triangles that no longer satisfy Delaunay property into cavity
- Re-triangulate affected region, including new point
- Continue until all bad triangles processed



Program

```
Mesh m = /* read in mesh */
WorkQueue wq;
wq.enqueue(mesh.badTriangles());

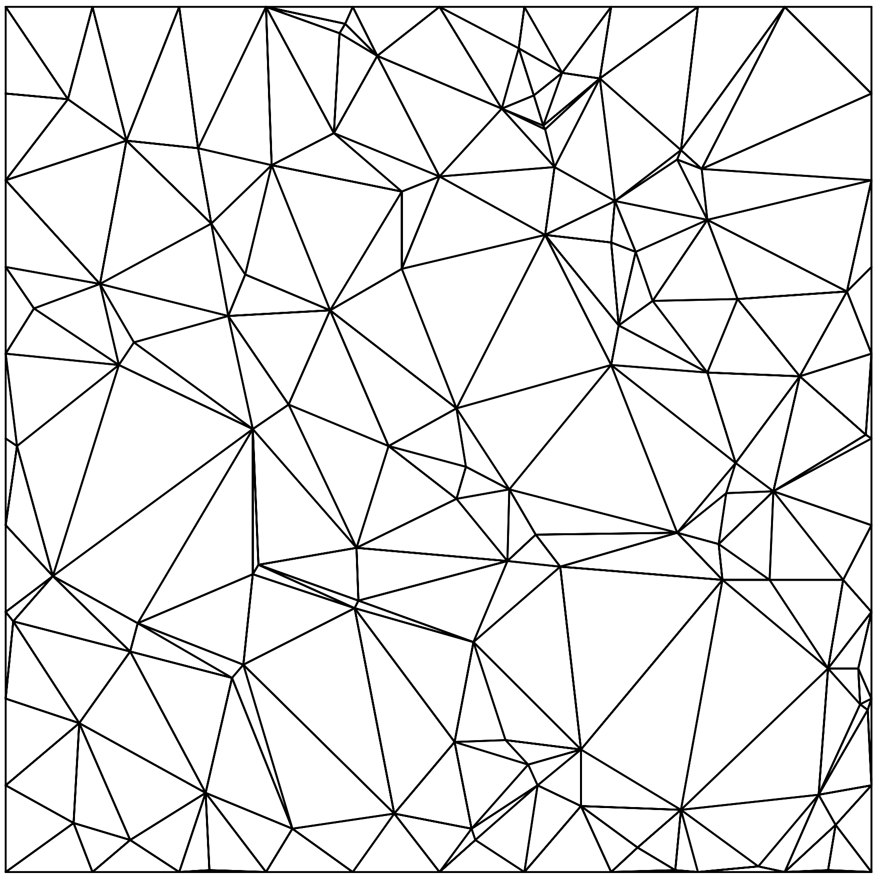
while (!wq.empty()) {
    Triangle t = wq.dequeue();    //choose bad triangle

    Cavity c = new Cavity(t);     //determine new vertex
    c.expand();                   //determine affected triangles
    c.retriangulate();            //re-triangulate region

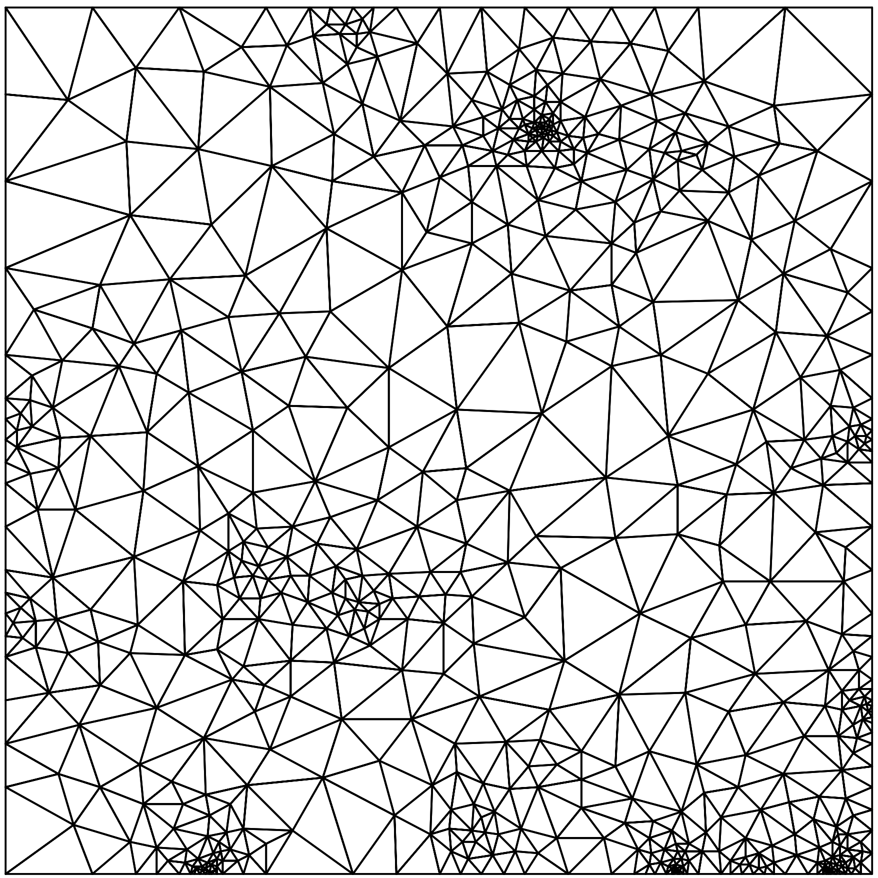
    m.update(c);                  //update mesh

    wq.enqueue(c.badTriangles()); //add new bad triangles to queue
}
```

Refinement Example



Original Mesh



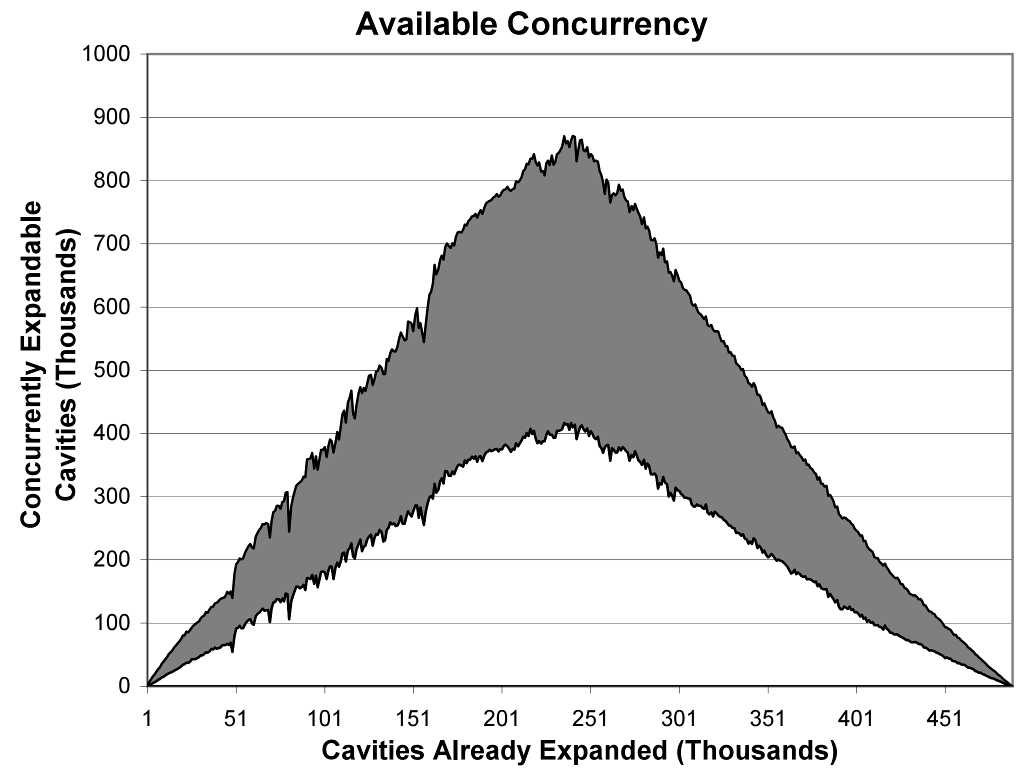
Refined Mesh

Parallelization Opportunities

- Multiple bad triangles to be processed
- Algorithm inherently non-deterministic
  - Order of processing irrelevant
- Effects of re-triangulation localized
  - Update operations mostly independent
- Can process multiple triangles in parallel
  - Triangles must be sufficiently far apart

Parallelization Opportunities

- Estimated available parallelism for mesh of 1M triangles
- Actual ability to exploit parallelism dependent on scheduling of processing
  - C. Antonopolous, X. Ding, A. Chernikov, F. Blagojevic, D. Nikolopolous and N. Chrisochoides *Multigrain parallel Delaunay Mesh generation*, ICS05



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    c.retriangulate();            //re-triangulate region

    m.update(c);                  //update mesh

    wq.enqueue(c.badTriangles()); //add new bad triangles to queue
}
```

Abstractions

- WorkSet abstraction
  - Replaces queue of bad triangles
    - Queue introduces loop carried dependence
- getAny operation
  - Does not make ordering guarantees
    - Removes dependence between iterations
    - In the absence of interference, iterations can execute in parallel

Abstractions

- Graph abstraction
  - Mesh can be viewed as an undirected graph
    - Nodes in Graph represent triangles in mesh
    - Edges in Graph capture triangle adjacency
- Subgraph abstraction
  - Collection of affected triangles is a Subgraph of overall Graph
- replaceSubgraph operation
  - Updating mesh after re-triangulation is replacing one Subgraph with another

Rewritten Program

```
Graph g = /* read in mesh */
WorkSet ws;
ws.add(g.badNodes());

while (!ws.empty()) {
    Node n = ws.getAny();                 //choose bad node

    Subgraph s1 = expandCavity(n);    //determine affected nodes
    Subgraph s2 = reTriangulate(s1);  //re-triangulate region

    g.replaceSubgraph(s1, s2);        //update graph

    ws.add(s2.badNodes());            //add new bad nodes to set
}
```

Automatic Parallelization

- Can now exploit "getAny" to parallelize loop
  - Can try to expand cavities in parallel
  - Expansions can still conflict
- How do we deal with this?
  - Ideally, automatically

Automatic Parallelization

- Most automatic parallelization focuses on “regular” data structures
  - arrays, matrices
- Parallelization for irregular data structures much trickier
  - Why?

Automatic Parallelization

- Must ensure no dependences between parallel code
  - Static detection of possible dependences
  - If dependence exists, parallel execution not possible

Automatic Parallelization

- Must ensure no dependences between parallel code
  - Static detection of possible dependences
  - If dependence exists, parallel execution not possible
- Analysis harder for irregular data structures
- Static determination of dependences intractable

Optimistic Parallelization

- In practice, most cavities can be expanded in parallel safely
  - No way to know this *a priori*
  - Only guaranteed safe approach is serialization
- What if we perform parallelization without prior guarantee of safety?

Optimistic Parallelization

- Expand cavities in parallel
- Perform run time checks to ensure that expansions do not conflict
  - If check fails, roll back expansion process, try again

Parallelization Issues

- Must ensure that run-time checks are efficient
- How to perform roll backs
- Would like to minimize conflicts
  - Scheduling becomes important
    - Number of available cavities for expansion exceeds computational resources
    - Choose cavities to expand to minimize conflicts
      - Empirical testing: ~30% of cavity expansions conflict

Distributed Memory

- Elements of mesh now distributed among processors
  - Accessing elements on different processors high latency
    - Expanding cavity whose elements are on multiple processors is slow
    - Want to hide latency
- Perform multiple expansions *per processor.*

Summary

- Mesh generation is an important algorithm
  - Delaunay mesh generation is a particularly useful variant of this algorithm
- Delaunay mesh generation intuitively parallelizable
  - In practice (especially automatically) not so easy
    - Must determine that loop can be parallelized
    - Must ensure that cavities do not conflict
- Several challenges to effective parallelization
  - Dynamic dependence checking
  - Scheduling
  - Distributed memory model