Interprocedural Dataflow Analysis

Propagating information across procedure boundaries is useful.

- Optimize caller using information about callee

```
x := 2
CALL f(x) //call by reference
...x...   //is x equal to 2 here?? Need to look at f
```

- Optimize callee using information about callers

```
...                   PROCEDURE f(a,b)
CALL f(x,5)              ......
...
```

  - We might generate specialized code for f in which b is 5.
  - We might *clone* f and specialize clone.
  - We might inline f.

Where do such opportunities arise? (i) calling library code (ii) object-oriented programs.

Facts about interprocedural dataflow analysis

- Significantly harder asymptotically and to implement than intraprocedural dataflow analysis
  - Intraprocedural analysis: unknowns are lattice values.
  - Interprocedural analysis: unknowns are functions on lattice values.
  - Aliasing: different program names for same location
- Strategies for dealing with complexity:
  - invent special-purpose algorithms that work for important special cases (eg, if the lattice is finite or of bounded height)
  - use general-purpose techniques that compute approximate but conservative solutions

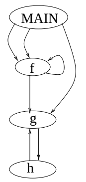
Game plan for inter-procedural analysis lectures:

- Start with call-by-value language
  Key problem:solving dataflow function equations
- Call-by-value + global variables
  Some problems have structure we can exploit to speed up analysis.
- Call-by-reference language
  Additional problem: aliasing

Key data structure: Call (multi)graph

- Structure:
  Nodes: one for each procedure
  Edges: from node f to node g if procedure f may invoke procedure g

```
MAIN ()
    ... f(...)
    .... g(...)
    ...f(...)
    ...
f(..)
    ...f(...)
    ...g(...)
g(..)
    ...h(...)..
h(..)
    ...g(...)...
```



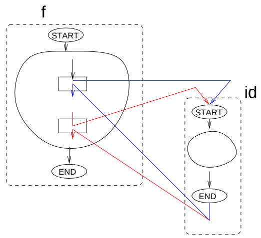
Program   Call Graph

Context-insensitive analysis

One obvious approach: reduce interproc case to intraproc case

forward dataflow problem:
- merge information from all call sites of procedure at START
- copy dataflow information coming out of END to all return sites

```
PROCEDURE f (..)
 ..... id(2) ....
 ......id(3).....

PROCEDURE id(n)
   return n;
```



Problem: information propagates along impossible interprocedural paths
such as blue edge into id and red edge out of id

This loses precision: in our example, we would not detect that id(2) is 2!

However, this reduction of inter-procedural analysis to the intra-procedural case is safe.

It is called context-insensitive analysis.

Let us begin with simple program model in which there is no aliasing

```
MAIN()
   var p,q;
   p = read();
   q = f(p,3);
   ...
PROCEDURE f(x,y)
   var z,a;
   if (x > 0) z = x;
   a = y;
   ...f(y,z)...
```

- no higher-order procedures
- no data structures
- no global variables
- call-by-value
- assignments in procedure modify locals and parameters
- recursion is allowed

Check: no aliasing

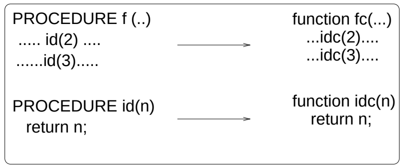
- Algorithm: Building call graph is trivial if there are no higher-order procedures.
- Use: Call graph plays a role sort-of like that of control flow graph in intraprocedural analysis, but not quite....

Context-sensitive analysis

Do not mix dataflow information from different call-sites of a procedure

One implementation of context-sensitive analysis:

- model each program procedure by a function on dataflow values
- to analyze dataflow effect of a call to procedure g, dataflow analyzer invokes the associated function, passing it some dataflow values and getting dataflow values back
- solves problem of avoiding dataflow mixing



Dataflow functions for constant propagation

Another implementation: build the composite graph as in context-insensitive analysis but propagate dataflow values together with a "tag" that identifies call sequence that generated that value (Sharir and Pnueli)

Main problem with context-sensitive analysis: termination

- Intuitively, we are doing something similar to a symbolic execution of program.
- Analysis must terminate even if program execution does not terminate!
- Difficulty with recursive procedures: analysis usually requires symbolic execution of both sides of conditionals, so how do we ensure termination? Here's an example where program terminates but analysis would not.


```
procedure main() {
   var p;
   p = read();
   return f(3,p);


procedure f(n,p) {

 if (n > p)
     then return f(n-1,p);
     else return 1;
}
```

```
function mainc() {
   p = bottom;
   p = T;
   return fc(3,p);
  }


function fc(n,p) {
   t1 = t2 = t3 = bottom;
   if (n > p)
      then t1 = fc(n-1,p);
      else t2 = 1
   t3 = join(t1,t2);
   return t3;
```


Source of termination problem:

The recursive definition of `fc` in previous slide is really an equational definition of `fc`: interpreting this definition as an "executable function" gets us into trouble in general.

Two important special cases:

- No recursion in program: no problem with non-termination in interpreting equational definitions as functions.
  Determining there is no recursion: call-graph should have no cycles.
  This idea does not work even in the presence of *static* recursion which even FORTRAN allows. Static recursion: text of program has recursive calls; Dynamic recursion: at runtime, two or more activations of a procedure coexist at some point in time.
- Domain is finite: solve equations iteratively by tabulating values of functions.

What do we do if we have recursion and domain is not finite?

Usual strategy: Replace recursive dataflow function with non-recursive conservative approximation

1. Identify recursive calls in call graph, and approximate their return values to T. Then, solve resulting acyclic problem.

```
function  fc(n,p){                      function fc(n,p) {
 t1 = t2 = t3 = bottom;                   t1 = t2 = t3 = bottom;
 if (n > p)                               if (n > p)
     then t1 = fc(n-1,p);   --->            then t1 = T;
     else t2 = 1;                           else t2 = 1
 t3 = join(t1,t2);                        t3 = join(t1,t2);
 return t3;}                              return t3;}
```

Recursive call identification: back edges in DFS of call-graph (not necessarily unique)

Adding global variables to program model:

```
GLOBAL G1, G2, G3
MAIN()
   var p,q
   p = f(G1,3); //S1 => MOD-S1 is {p,G2}
   ...
PROCEDURE f(x,y)
   var z,a
   if (x > 0) G2 := x;      // S2 => MOD-S2 is {G2}
   z = G1;                  // S3 => MOD-S3 is {z}
   ...f(G1,z)...
```

Still no aliasing.

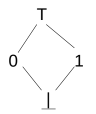
Inter-procedural dataflow analysis: simple extension of call-by-value case.

(eg) Context-sensitive inter-procedural constant propagation: dataflow function for procedure will have one additional parameter for each global variable.

Solving recursive function equations by tabulation

- necessary condition: finite lattice

- Example: constant propagation in which all values other than 0 and 1 are set to T.




```
function fc(n,p)
  {t1 = t2 = t3 = bottom;
   if (n>p) then t1 = fc(n-1,p);
            else t2 = 1;
   t3 = join(t1,t2);
   return t3;}
```

Compute a sequence of approximations to fc as follows:

fc[0] (n,p) = $\perp$

```
fc[i+1](n,p) =
      {t1 = t2 = t3 = bottom;
       if (n>p) then t1 = fc[i](n-1,p);
                else t2 = 1;
       t3 = join(t1,t2);
       return t3;}
```

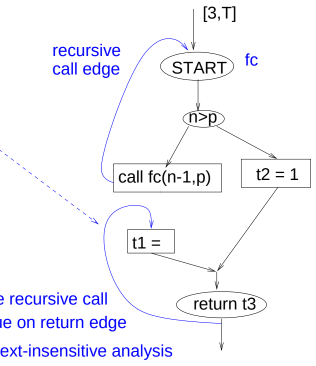
For our example:

fc(n,p) = 1

Computing each element of sequence: make a table of output for each possible input value
Checking convergence: same table is obtained for two successive elements in sequence.
Termination: from monotonicity and fineteness of domain.

2. Solve context-insensitive problem and use result to approximate the effect of recursive calls.
   - value on "return edge" will be an upper bound of possible return values
   - replace recursive call with that value and solve acyclic context-sensitive problem

```
fc(n,p) {
  t1 = t2 = t3 = bottom;
  if (n > p)
     then t1 = fc(n-1,p);
     else t2 = 1;
  t3 = join(t1,t2);
  return t3;
}
```



Interprocedural dataflow problem: computing MOD [Banning]

For any statement s, the set MOD-s is the set of variables visible to s that may be modified directly or indirectly by execution of s.

```
GLOBAL G1, G2, G3
MAIN()
   var p,q
   p = f(G1,3); //S1 => MOD-S1 is {p,G2}
   ...
PROCEDURE f(x,y)
   var z,a
   if (x > 0) G2 := x;     // S2 => MOD-S2 is {G2}
   z = G1;                 // S3 => MOD-S3 is {z}
   ...f(G1,z)...
```

Auxiliary sets: *For any procedure f, GMOD-f is the set of global variables that may be modified directly or indirectly by invoking f.*

In example, GMOD-MAIN = {G2}, and GMOD-f = {G2}

For any statement s, MOD-s is the union of

- set of variables that may be modified directly in statement (IMOD)
- set of globals that may be modified directly or indirectly by procedure invocations in s (GMOD)

So given GMOD sets, MOD sets are easy to compute.


```
GLOBAL G1, G2, G3;
MAIN ()
   ... f(...)
   .... g(...)
   ...f(...)
   ...
f(..)
   G1 := ...
   ...f(...)
   ...g(...)

g(..)
   ...h(...)..
   G1 := ...
h(..)
   G3 := ...
   ...g(...)...
```

GMOD-main = GMOD-f U GMOD-g

GMOD-f = {G1} U GMOD-g U GMOD-f

GMOD-g = {G1} U GMOD-h

GMOD-h = {G3} U GMOD-g

Program

Exploiting structure in inter-procedural dataflow analysis

- Just as in intra-procedural case, inter-procedural problems may have structure that can be exploited to speed up solution.
- Exploiting intra-procedural structure: as before
- Inter-procedural structure: in the call graph
- For many problems, we can exploit strongly connected components in call graph to speed up analysis (eg. MOD computation)

How do we compute GMOD sets?

Write down a set of lattice equations and solve them.

- Lattice: power-set of global variables
- Equations: if procedure f has assignments to globals Gi,Gj,.. and it may invoke procedures g,h,.. equation for GMOD-f is

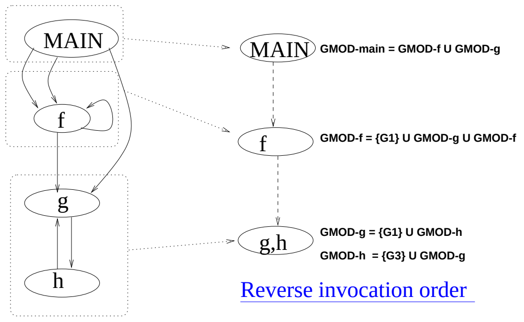
  GMOD-f = {Gi,Gj...} U GMOD-g U GMOD-h...

Observations

- We can use any iterative technique we discussed in intra-procedural case to solve these inter-procedural equations.
- Is there structure that can be exploited to reduce number of iterations? Yes!!
  - In our problem, information flows from invoked procedure to invokee.
  - So consider equations in "reverse invocation order". See next slide.


```
GLOBAL G1, G2, G3;
MAIN ()
   ... f(...)
   .... g(...)
   ...f(...)
   ...
f(..)
   G1 := ...
   ...f(...)
   ...g(...)

g(..)
   ...h(...)..
   G1 := ...

h(..)
   G3 := ...
   ...g(...)...
```

Program



Reverse invocation order

Call Graph

Further simplification: note that GMOD sets for all procedures in a single scc of call graph must be indentical.

So collapse equations for all procedures in a single scc into a single equation!

```
GMOD-g = {G1} U GMOD-h  => GMOD-gh = {G1,G3} U  GMOD-gh
GMOD-h = {G3} U GMOD-g
```

Solving single equation for least solution: trivial! Just drop the recursive term.

So in example, `GMOD-g = GMOD-h = {G1,G3}`

Summary: GMOD/MOD computation for call-by-value language

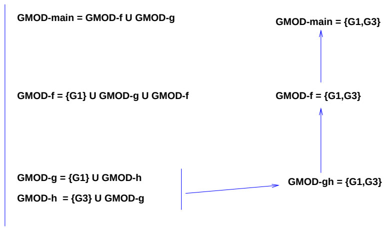
- Write down GMOD equations for the program.
- Partition equations by scc's in call graph.
- Collapse equations in each scc into a single equation.
- In reverse topological order of acyclic condensate of call graph, read off solutions to GMOD equations.
- For each statement, compute MOD set.

Complexity: O(size of program*number of variables)

Note: we can exploit scc's any time we have a set of equations (eg, block triangular systems in linear algebra)

Running example:

```
GLOBAL G1, G2, G3;
MAIN ()
    ... f(...)
    .... g(...)
    ...f(...)
    ...
f(..)
    G1 := ...
    ...f(...)
    ...g(...)
g(..)
    ...h(...)..
    G1 := ...
h(..)
    G3 := ...
    ...g(...)...
```



Program

Call-by-reference

Complications:

1. Effect of a procedure is not just globals it modifies but also what happens to parameters

```
x := f(G1,w)  //to compute MOD, we need to know what happens to G1 and w!
```

2. Aliasing: two program names for same location

```
GLOBAL G1,G2
procedure f(x,y)
  x := 3; //S1
  ....
procedure g(z,w)
  ..f(G1,G2)...f(z,z)...f(w,z)
```

For first call to f, x and y are not aliases
For second call to f, x and y are aliases
For third call to f, x and y are aliases if w and z are aliases!
What is MOD-S1???

Let us handle these problems one at a time.

Aliasing

- MUST-ALIAS: two program names that definitely refer to the same memory location.

```
GLOBAL G1;
procedure f(x) { <---- x and G1 are MUST-ALIASes within f
  x = 7;}
procedure main() {
  f(G1);
  print(G1); }
```

MUST-ALIAS is an equivalence relation on names.

- MAY-ALIAS: two program names that may or may not refer to the same memory location. MAY-ALIASing usually arises from MUST-ALIASes through loss of information such as when we merge information along different program paths.

```
GLOBAL G1,G2;
procedure f(x) { <---- x and G1 are MAY-ALIAS's within f
  x = 7;}
procedure main() {
  f(G1) + f(G2);
  print(G1); }
```

MAY-ALIAS relation is reflexive and symmetric but not necessarily transitive.

Representation of aliasing information: alias pairs

```
GLOBAL G1,G2;
procedure f(x) {  //ALIAS = {<x,x>,<x,G1>,<x,G2>}
  x = 7;}
procedure main() {
  f(G1) + f(G2);
  print(G1); }
```

If <a,b> does not occur in an ALIAS relation, variables a and b are definitely not aliased at the point in program where ALIAS relation holds.

Some people also store may/must flag with each alias pair.

Alias pairs are an example of store-free alias representation.

Store-based alias representation: see when we talk about pointer analysis.

Using ALIAS relations in inter-procedural dataflow analysis:

For our language model, all statements in a procedure have same alias relation (not true when we have pointers as in C).

Modify the dataflow transfer functions of statement with alias information

- Constant propagation:

```
procedure f(x,y) {      function fc(x,y) {
 ....                     ....
 x := e;                  V-out = {let n = Eval(e,V-in);
                                   return V-out where
                                     V-out[i] = V-in[i] if i is not aliased with x
      }                              V-out[i] = join(Vin[i],n) if i is MAY-ALIASed with x
                                     V-out[i] = n if i is MUST-ALIASed with x
                                   }
 .....                    .....
}                       }
```


- MOD computation: close affected variables under aliasing

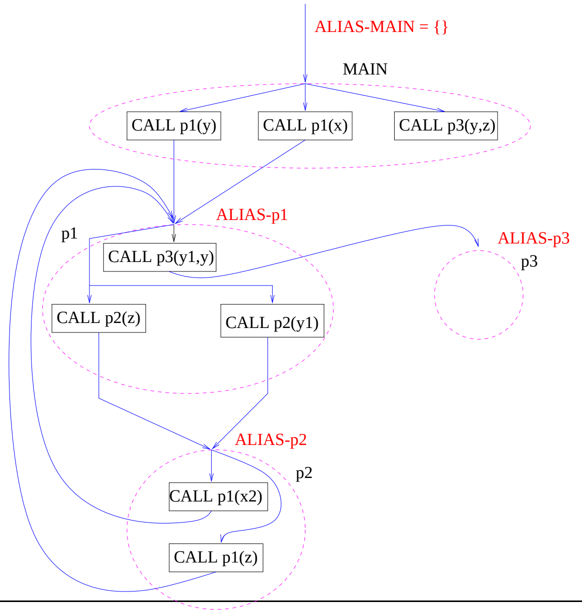
```
GLOBAL G1,G2
procedure f(x,y)
  x := 3; // <--- MOD = {x,G1,y}
  ....
procedure g(z,w)
  ..f(G1,G2)...f(z,z)...f(w,z)
```


Computing alias relation for our program model: treat as a dataflow problem...

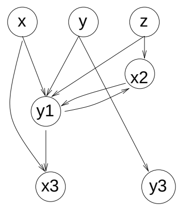
- Compute one alias set for each procedure.
- Each call-site has an associated transfer function that generates output alias set from alias set of caller (see next slide).
- Alias set of procedure = union of alias sets generated at its call sites.
- Transfer functions are monotonic and lattice is finite.

Example: [adapted from Banning]

```
GLOBAL x,y,z;
 procedure MAIN()         ALIAS-MAIN = {}
    p1(y);                ALIAS-C1   = {<y,y1>}
    p1(x);                ALIAS-C2   = {<x,y1>}
    p3(y,z);              ALIAS-C3   = {<y,x3>,<z,y3>}

 procedure p3(x3,y3)      ALIAS-p3   = ALIAS-C3 U ALIAS-C6
    x3 := ...;
    y3 := ...;
    x  := ...;

 procedure p1(y1)         ALIAS-p1   = ALIAS-C1 U ALIAS-C2 U ALIAS-C7 U ALIAS-C8
    p2(z);                ALIAS-C4 =   {<x2,z>}
    p2(y1);               ALIAS-C5 =   Transfer(ALIAS-p1,(y1)->(x2),{x,y,z})
    p3(y1,y);             ALIAS-C6 =   Transfer(ALIAS-p1,(y1,y)->(x3,y3),{x,y,z})

 procedure p2(x2)         ALIAS-p2 =   ALIAS-C4 U ALIAS-C5
    p1(x2);               ALIAS-C7 =   Transfer(ALIAS-p2,(x2)->(y1),{x,y,z})
    p1(z);                ALIAS-C8 =   {<z,y1>}

Solution: ALIAS-p1 = {<y,y1>,<x,y1>,<z,y1>}
ALIAS-p2 = {<y,x2>,<x,x2>,<z,x2>}
ALIAS-p3 = {<y,y3>,<x,x3>,<z,x3>,<y3,x3>,<y,x3>,<z,y3>}
```


```
procedure f(x,y) {
  ...}
procedure g(a,b,c) {  //suppose alias set is A
  ....f(p,q)... //output alias set is B
}

 Transfer(A, list of actuals, list of formals, globals)
      B = { };
      for each actual parameter p in call do {
          if (p is a global variable) then append <p,x> to B;
          else  for each tuple <p,V> in A do
                if V is a global variable then append <x,V> to B;
      }
      for each pair (p,q) of actual parameters in call do {
          if ((<p,q> is in A) or (p and q are same variable))
          then append <x,y> to B where x/y are formals bound to p/q by call.
      return B;
```



Binding graph

Structure of binding graph:

- one node for each reference parameter and global
- if procedure f passes its reference parameter/global r1 to procedure g as reference parameter r2, put an edge from r1 to r2

Finding aliases between globals and formals:

all reference parameters reachable from a global variable in the binding graph may be aliased to that global

In our example:

x may be aliased to y1, x3, x2
y may be aliased to y1, y3, x2,x3
z may be aliased to y1, x2, x3

Example: [adapted from Banning]

```
GLOBAL x,y,z;
 procedure MAIN()            GMOD-MAIN()
    p1(y);                      return GMOD-p1(y) U GMOD-1(x) U GMOD-p3(y,z)
    p1(x);
    p3(y,z);

 procedure p3(x3,y3)         GMOD-p3(m,n)
    x3 := ...;                  return {m,n,x}
    y3 := ...;
    x  := ...;

 procedure p1(y1)            GMOD-p1(m)
    p2(z);                      return GMOD-p2(z) U GMOD-p2(m) U GMOD-p3(m,y)
    p2(y1);
    p3(y1,y);

 procedure p2(x2)            GMOD-p2(m)
    p1(x2);                     return GMOD-p1(m) U GMOD-p1(z)
    p1(z);
```

Need to solve recursive functional equations.

More efficient ways of computing alias sets:

- Determine which globals are aliased with which formals by using binding graph (see next slide).
  - Graph has a node for each global and formal parameter; if global/formal v1 is passed to formal v2, put an edge from v1 to v2.
  - All formals reachable from node for global g are aliased to g.
- Determining which formals are aliased to each other: use pairwise binding graph. Left to reader.

Concern 2:GMOD must tell us what happens to globals AND parameters.

One model: make GMOD into a function from variables to variables

```
 w := f(G4,s);  //S1

procedure f(x,y)       =>    GMOD-f(v1,v2)
  var a,b;                      return {G1,v1}
  G1 := 5;
  if (y>x) x:= 7;
  ...
```

Intuition: (assuming s,w have no non-trivial aliases)

```
MOD-S1 = {w} U GMOD-f(G4,s) = {w} U {G1,G4} = {w,G1,G4}
```

Without recursion, we can use "interpret equations as program" trick.

Another approach: since our lattice is finite, we can always use tabular approach.

```
GMOD-p3(m,n)
   return {m,n,x}
GMOD-p1(m)
   return GMOD-p2(z) U GMOD-p2(m) U GMOD-p3(m,y)
GMOD-p2(m)
   return GMOD-p1(m) U GMOD-p1(z)
```

Iterations:

	1	2	3	... final
GMOD-p3(m,n):	{}	{m,n,x}	{m,n,x}	... {m,n,x}
GMOD-p2(m):	{}	{}	{}	... {m,y,x,z}
GMOD-p1(m):	{}	{}	{m,y,x}	... {m,y,x,z}

Exploiting structure for computing GMOD

- call-by-value: we found GMOD sets without iteration
  - find scc's in call graph
  - for each scc, compute union of side-effects to globals by procedures in scc
  - propagate GMOD sets in reverse invocation order
- can we use this trick for call-by-reference as well?
- Problem: GMOD sets contain both globals and parameters, so GMOD sets of mutually recursive procedures will be different in general.
- Idea: separate computation of side-effects to parameters from side-effects to globals

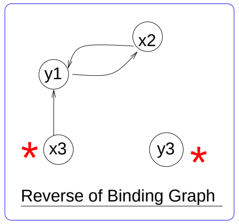
Side-effects to parameters:

`procedure f(a,b) ...`

RMODf-a = true if execution of f modifies a directly or indirectly

```
GLOBAL x,y,z;
 procedure MAIN()
    p1(y);
    p1(x);
    p3(y,z);

 procedure p3(x3,y3)     RMODp3-x3 = true
    x3 := ...;           RMODp3-y3 = true
    y3 := ...;
    x  := ...;

 procedure p1(y1)        RMODp1-y1 = RMODp2-x2 V RMODp3-x3
    p2(z);
    p2(y1);
    p3(y1,y);

 procedure p2(x2)        RMODp2-x2 = RMODp1-y1
    p1(x2);
    p1(z);

Solution: RMODp3-x3 = RMODp3-y3 = RMODp1-y1 = RMODp2-x2 = true
```


Graphical way of solving RMOD equations: marker propagation



A reference parameter of a procedure f may be modified by execution of f if

- f may write to parameter directly
- f passes it to procedure g as a reference parameter, and g may modify the parameter

RMOD computation

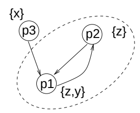
- build reverse of binding graph (ignore globals)
- if f modifies reference parameter r1 directly, put a mark on r1
- propagate marks along reverse binding graph edges
- at the end of propagation, any variable that is marked corresponds to a reference parameter that may be modified

From RMOD and GLOBALS, we can read off GMOD sets:

```
GMODp1(m) = {x,m}
GMODp2(m) = {x,m}
GMODp3(m,n) = {x,m,n}
```

Oops...

What went wrong?



Algorithm for side-effects to globals: GLOBAL

- Find acyclic condensate of the call graph.
- For each procedure, determine set of globals either assigned to directly in procedure or passed by reference as a parameter to a procedure that modifies that parameter (use RMOD information for this).
- Union these sets for all procedures in an scc.
- Propagate these global sets in reverse invocation order in the acylic condensate.

Side-effects to globals: similar to call-by-value

```
GLOBAL x,y,z;
 procedure MAIN()
    p1(y);
    p1(x);
    p3(y,z);

 procedure p3(x3,y3)    GLOBALp3 = {x}
    x3 := ...;
    y3 := ...;
    x  := ...;

 procedure p1(y1)       GLOBALp1 = GLOBALp2 U GLOBALp3
    p2(z);
    p2(y1);
    p3(y1,y);

 procedure p2(x2)       GLOBALp2 = GLOBALp1
    p1(x2);
    p1(z);

Solution: GLOBALp1 = GLOBALp2 = GLOBALp3 = {x}
```


We did not take into account side-effects to globals that were passed as parameters![Kennedy and Cooper]

Correct equations for GLOBAL sets: use RMOD information for globals passed as parameters:

```
GLOBAL x,y,z;
 procedure MAIN()
    p1(y);
    p1(x);
    p3(y,z);

 procedure p3(x3,y3)    GLOBALp3 = {x}
    x3 := ...;
    y3 := ...;
    x  := ...;

 procedure p1(y1)       GLOBALp1 = {z} U GLOBALp2 U GLOBALp3 U {y}
    p2(z);
    p2(y1);
    p3(y1,y);

 procedure p2(x2)       GLOBALp2 = {z} U GLOBALp1
    p1(x2);
    p1(z);
```

This gives the correct sets.

Putting it all together: GMOD computation

- Compute RMOD information
  - build binding graph
  - mark every node that represents a reference parameter modified directly by its procedure
  - propagate marks in biding graph: efficient approach would compute scc's and propagate in acyclic condensate
  - for each procedure f, read off RMOD-f = set of reference parameters of f that are marked.
- Compute GLOBALs information
  - build call graph
  - for each procedure, find all globals that are either modified directly by procedure or passed as a reference parameter to another procedure that modifies that parameter (use

RMOD for this)
  - find scc's of call graph and propagate sets in reverse invocation order.
- From RMOD and GLOBALs sets, read off GMOD function for each procedure f.

Summary

- Inter-procedural dataflow analysis: unknowns are functions on dataflow values
- Need to solve recursive functional equations
- Important special cases for which exact solution is possible
  - finite lattice: use tabular method
  - acyclic call graph: interpret equations as program
- Approximate solution of functional equations: approximate effect of backedges in call-graph
- Call-by-reference: need to take aliases into account
- Some interprocedural dataflow analysis problems can be reduced to marker propagation by formulating in the right graph. Key structure to exploit: strongly connected components in call graph.