CS 5860 Practice Specifications

Fall 2011

Which of these first-order specifications is programmable?
Write the programs when the spec is programmable.
Revise the spec to an "equivalent" one (in the sense of Boolean logic) which is programmable. Ponder the question of whether such revisions are always possible. (They are!) For $\Leftrightarrow$ show $\Rightarrow$ and $\Leftarrow$ separately.

1. $\forall x.(A(x) \Rightarrow B(x)) \Rightarrow (\forall x.\, A(x) \Rightarrow \exists x\, B(x))$

2. $\forall x.\,(A(x) \Rightarrow B(x)) \Rightarrow \forall x.\, A(x) \Rightarrow \forall x.\, B(x)$

3. $\forall x.(A(x) \Rightarrow B(x)) \Rightarrow \exists x.\, A(x) \Rightarrow \exists x.\, B(x)$

4. $\exists y \forall x.(\, F(x,y) \Leftrightarrow F(x,x)) \Rightarrow \sim \forall x.\, \exists y.\, \forall z.(F(z,y) \Leftrightarrow \sim F(z,x))$

5. $\sim \exists y \forall x\, F(x,y) \Leftrightarrow \sim \exists z.(F(x,z)\, \&\, F(z,x))$

6. $\sim \exists y.\, \forall x.(F(x,y) \Leftrightarrow \sim F(x,x))$

7. $\forall x\, \exists z.(\, F(x) \Rightarrow \exists y.(G(y) \Rightarrow H(z))) \Leftrightarrow$
$(\exists x.\, F(x)\, \&\, \forall x.(G(x) \Rightarrow \exists x.\, H(x)))$

8. $\forall x\, \exists y.(\, F(x) \Rightarrow G(y)) \Leftrightarrow \exists y.\, \forall x.(\, F(x) \Rightarrow G(y))$

9. $\forall z\, \exists y\, \forall x.(\, F(x,y) \Leftrightarrow F(x,z)\, \&\, \sim F(x,x)) \Rightarrow \sim \exists z\, \forall x.\, F(x,z)$

10. $(\forall x.\, A(x) \Rightarrow \exists x.\, B(x)) \Leftrightarrow \exists x.(A(x) \Rightarrow B(x))$

11. $(\exists x.\, A(x) \Rightarrow \forall x\, B(x)) \Rightarrow \forall x.(A(x) \Rightarrow B(x))$

12. $(\exists x\, A(x) \vee \exists x\, B(x)) \Leftrightarrow \exists x.(A(x) \vee B(x))$

13. $\forall x.\, A(x)\, \&\, \forall x\, B(x) \Leftrightarrow \forall x\,(A(x)\, \&\, B(x))$

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14. Resolution $(C \vee P) \& (D \vee \neg P) \Leftrightarrow (C \vee P) \& (D \vee \neg P) \& (C \vee D)$

    (also with C or D missing)

15. $(P \Rightarrow (Q \& (R \vee S)) \& (\neg Q \vee \neg R) \Rightarrow (P \Rightarrow S)$

16. $\forall x_1, x_2. \exists y. (P(x_1) \Rightarrow Q(x_2,y)) \Rightarrow \forall z.((Q(x_2,y) \Rightarrow R(z)) \Rightarrow (P(x_1) \Rightarrow R(z)))$

17. $\exists x \exists y.(P(x,y) \& \forall r. \neg Q(x,r)) \vee \forall x,y. \exists z.(\neg P(x,y) \vee Q(y,x) \vee$
    $(Q(x,z) \& \neg P(y,z)))$

18. $\forall x,y. F(x,y) \Rightarrow \exists x,y. F(x,y)$

19. $\forall x \exists z.(F(x) \Rightarrow \exists y.(G(y) \Rightarrow H(z))) \Leftrightarrow \exists x(F(x) \& \forall x\, G(x) \Rightarrow \exists x. H(x))$

20. $\forall x (\exists y\, F(x,y) \Rightarrow \exists y\, G(x,y)) \Leftrightarrow \forall x \forall y. \exists z.(F(x,y) \Rightarrow G(x,z))$

21. $\exists y (\exists x\, F(x,y) \Leftrightarrow G(y)) \Leftrightarrow$

    $\exists y \forall x \exists z.((F(x,y) \Rightarrow G(y)) \& (G(y) \Rightarrow F(z,y)))$

22. $\forall x,y. \exists z.(F(x) \& G(y) \Rightarrow H(z)) \Leftrightarrow \forall y \exists z \forall x.(F(x) \& G(y) \Rightarrow H(z))$

23. $\sim \exists y \forall x (F(x,y) \Leftrightarrow \sim F(x,x))$

24. $\forall z \exists y \forall x.(F(x,y) \Leftrightarrow (F(x,z) \& \sim F(x,x))) \Rightarrow \sim \exists z \forall x. F(x,z)$

25. $\sim \exists y \forall x.(F(x,y) \Leftrightarrow \sim \exists z.(F(x,z) \& F(z,x)))$

26. $\exists y \forall x.(F(x,y) \Leftrightarrow F(x,x)) \Rightarrow \sim \forall x \exists y \forall z.(F(z,y) \Leftrightarrow \sim F(z,x))$

27. $(P \Rightarrow (Q \Rightarrow R)) \Leftrightarrow (Q \Rightarrow (P \Rightarrow R))$