Decision Theory I
Problem Set 1

Handed out: Feb. 17, 2021. Due: Mar. 3, 2021 at 9:30 AM. (If you hand it in late, but before Friday, Mar. 5, at 3 PM, you will get a 15% penalty.) If you are enrolled in CS5846 or ECON6760, you have to do all the questions. If you are in ECON3810, you do not have to do the two questions labeled "GRAD".

1. Show that a choice function C satisfies axioms α and β iff it satisfies WARP (the Weak Axiom of Revealed Preference). (See the class notes for the relevant definitions.)

2. Suppose $X = \{x, y, z\}$. Consider a choice function $C : P(X) \to P(X)$ such that $C(\{x, y\}) = \{x\}$, $C(\{x, z\}) = \{z\}$ and $C(\{y, z\}) = \{y\}$. Does this choice function satisfy Sen's α? Does it satisfy Sen's β?

3. Let $\succ$ be a binary relation on a finite set X. Define $\succeq$ by: $x \succeq y$ if and only if $y \not\succ x$. Show

   (a) If $\succeq$ is complete then $\succ$ is asymmetric.

   (b) If $\succeq$ is transitive then $\succ$ is negatively transitive.

4. Suppose that $\succ$ is transitive (but not necessarily negatively transitive), and define $c(\cdot, \succ)$ as in class. Show that Sen's axiom α holds, but show by example that Sen's β may fail to hold.

5. A binary relation that is reflexive, symmetric, and transitive is called an *equivalence relation*. Suppose that $\succ$ is a strict preference relation on a finite set X. Then by Proposition 2.4 of Kreps we know that $\sim$ is an equivalence relation on X. For each $x \in X$, define $I(x) = \{y \in X : y \sim x\}$; $I(x)$ is called the *equivalence class of* x. Show:

   (a) The sets $I(x)$ partition X. (A collection of sets $\{A_1, \ldots, A_N\}$ partitions X if each $x \in X$ is in at least one A_i and $A_i \cap A_j = \phi$ for all $i \neq j$.)

(b) The sets $I(x)$ are strictly ranked. (The equivalence classes are strictly ranked if, for all $x, y \in X$: (1) if $I(x) \neq I(y)$, then either $x \succ y$ or $y \succ x$, and (2) if $x \succ y$ then $x' \succ y'$ for all $x' \in I(x)$ and $y' \in I(y)$.)

6. **GRAD:** In the statement of Sen's α and β we allow the sets A and B to be any subsets of X. So when we proved that these axioms imply that the revealed preference relation is asymmetric and negatively transitive we allowed ourselves to use information about choices from arbitrary subsets of X. We want to know whether there is a smaller class of subsets of X such that the claim in the revealed preference theorem is true if α and β are satisfied on this smaller class of sets. Suppose that the cardinality of X is N and for each integer $n \leq N$ let S_n be the collection of all non-empty subsets of X of cardinality less than or equal to n. Find the smallest $n > 1$ such that the following claim is true: If a choice function satisfies Sen's α and β on S_n then there is a preference order $\succ$ defined on X such that $c(A, \succ) = c(A)$ for all $A \in S_n$.

7. **GRAD:** In class in the proof of the revealed preference theorem we defined strict revealed preference. Weak revealed preference is defined as follows: $x \succeq y$ if $x \in C(\{x, y\})$. Define induced strict revealed preference $\succ^*$ from revealed preference $\succeq$ by: $x \succ^* y$ if $x \succeq y$ and $y \not\succeq x$. Are strict revealed preference and induced strict revealed preference the same relation?