Professors Halpern and Salcedo | Econ 3810/6760
23 May 2017 | CS 5846

Final Exam

INSTRUCTIONS, SCORING: The points for each question are specified at the beginning of the question. There are 82 points on the exam for undergrads and 97 for grads. Note that students taking CS 5846 or ECON 6760 must do all the questions; students taking ECON 3810 should do all but 3(b) and 5(b). Answer questions 1 to 4 in one exam book, and questions 5 to 8 in in a different exam book. Please show your work, so that we can give you partial credit. Good luck!

1. Consider some orderings on the students in our class. In each case provide either a proof or a counterexample.

   (a) [5 points] Suppose that student a is "larger than" student b if and only if a weighs more than b and is taller than b. Is "larger than" a preference relation?

   (b) [5 points] Suppose that student a is "bigger than" student b if and only if a weighs more than b or is taller than b. Is "bigger than" a preference relation?

2. [10 points] Let S be a state space with at least two states. Let the outcome space be the real numbers. Thus, an act is a function from S to the reals. Define the order $\succ$ by $a \succ b$ iff $\max_{s\in S}(b(s) - a(s)) < \max_{s\in S}(a(s) - b(s))$. This is called pairwise minimax regret. Is $\succ$ a preference relation? If you think so, prove it. If you think it isn't, show by example that if does not satisfy some property of preference relations.

3. In this problem the state space is $S = [0, 1]$ (i.e., $\{x \in \mathbf{R} : 0 \leq x \leq 1\}$), the set of acts is $A = \{0, 1\}$ and the payoff of act a in state s is $u_a(s) = -(a - s)^2$.

   (a) [5 points] Compute the regret of each act and identify which decisions are optimal under the minimax regret criterion.

   (b) [7 points, **GRAD**] Now suppose that randomized decision procedures can be used. The set of rules is now $p \in [0, 1]$, where p is the probability of choosing act 0 (so $1 - p$ is the probability of choosing 1). The payoff of act p in state s is its expected payoff, that is, $u(s, p) = -ps^2 - (1 - p)(1 - s)^2$. Find the randomized acts that are optimal under the minimax regret criterion.

4. An individual with initial wealth w will lose L with probability p, where $0 < p < 1$. With probability $(1 - p)$ there will be no loss. The individual is a von Neumann-Morgenstern expected utility maximizer with utility function u. Assume that $u' > 0$ and $u'' < 0$; that is, the individual likes wealth and dislikes risk. Insurance that pays fraction α of the loss is available. The individual can chose any $\alpha \in [0, 1]$; for any such α he must pay a cost (or premium) of $p\alpha L$; and, he receives αL in the event of a loss and nothing if there is no loss.

(a) [5 points] What is the individual's expected utility as a function of α?

(b) [5 points] What is the optimal choice of α?

5. Suppose a von Neumann-Morgenstern decision maker is choosing over lotteries which have payoffs in the set $X = \{x, y, z\}$, and suppose that he has a payoff function u such that $u(x) > u(y) > u(z)$.

   (a) [7 points] Show that if another payoff function v gives the same expected utility ranking of lotteries, then

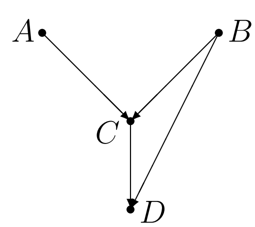
   $$\frac{u(x) - u(z)}{u(y) - u(z)} = \frac{v(x) - v(z)}{v(y) - v(z)}.$$

   [Although you don't need to show it, it follows relatively easily from this that there are numbers $a > 0$ and b such that $v(\cdot) = au(\cdot) + b$. Note that you can't use the latter fact in your proof above though!]

   (b) [8 points, **GRAD**] In part (a), X had three elements. This is critical. Suppose that X contains only the two elements x and y. Suppose u and v are both EU representations of the same preference order on lotteries. Show that there may not be numbers $a > 0$ and b such that $v(\cdot) = au(\cdot) + b$?

6. [10 points] Show that if A and B are two events, and if a decision-maker (DM) believes that $A \cup B$ is more likely than A, then she cannot believe that B is null.

7. Consider the following Bayesian network, where each of A, B, C, and D is a binary random variable:



   (a) [4 points] What (conditional) indepencencies are encoded by this Bayesian network?

   (b) [4 points] There are ten (conditional) probabilities (of the form $\Pr(X = x \mid \vec{Z} = \vec{z})$, where $\vec{Z}$ is a (possibly empty) set of variables) that are needed to convert this qualitative Bayesian network into a quantitative Bayesian network. What are they?

   (c) [7 points] Write an expression for $\Pr(D = 1)$ in terms of the conditional probabilities listed in part (b).

8. [15 points] A population of 1,000 elderly people consisting of 500 men and 500 women were given an option to take an experimental drug. 500 people took the drug. Researchers discovered that

   - Of the people who chose to take the drug, 60% recovered; of the people who didn't choose to take the drug, 40% recovered.
   - The percentage of men who chose to take the drug and recovered is lower than the percentage of men who didn't choose to take the drug and recovered.
   - The percentage of women who chose to take the drug and recovered is lower than the percentage of women who didn't choose to take the drug and recovered.

   Thus, the drug seems efficiacious for the population as a whole, but doesn't seem efficacious for either men or women! (This phenomenon is called *Simpson's paradox*: a treatment seems effective for the population as a whole, but not effective for all subgroups of the population. A number of well-known instances of this phenomenon have ocurred in real life, as you'll see if you Google "Simpson's paradox" or check it out on Wikipedia.)

   (a) [7 points] Provide numbers that justify the conclusions above. [Hint: the fraction of of women choosing to take the drug will be quite different from the fraction of men choosing to take the drug.]

   (b) [5 points] Find a causal model that would explain these numbers. What does your causal model say about who should be proscribed the drug?

   (c) [3 points] Describe a causal model under which Simpson's paradox could not occur. More specifically, describe a causal assumption under which, if the drug were more efficacious for the population as a whole (as in the first bullet above), it could not be less efficacious for both men and women (as in the second and third bullets above).