Urn Experiments

Urn A contains 50 Red and 50 Yellow balls

Urn B contains 100 balls each of which is either Red or Yellow.

There are two acts:

1. A ball will be drawn at random from Urn A; win \$100 if the ball is Red and \$0 otherwise.
2. A ball will be drawn at random from Urn B; win \$100 if the ball is Red and \$0 otherwise.

Suppose the decision-maker has utility function u over money and uses the maximizing expected utility rule for known probabilities. Then the expected utility of act a_1 is

$$E_{1/2}(u_{a_1}) = 1/2u(100) + 1/2u(0).$$

How do we evaluate act a_2?

One argument based loosely on the principle of insufficient reason is to treat all probabilities of R and Y as equally likely and use the expected probability which is 1/2.

This gives act a_2 the same expected utility as act a_1.

- Does this make sense?
- Is it reasonable to use the principle of insufficient reason here?
- Is it reasonable to use any particular probability on the set of probabilities of R and Y?

Another alternative is to use a set of probabilities.

If use $\mathcal{P}_S$ and maximin then $\underline{E}_{\mathcal{P}_S}(u_{a_2}) = u(0)$.

Alternatively, could use a restricted set of probabilities $\mathcal{P}$ in which the probability of Red is between $\underline{p}$ and $\overline{p}$.

Then maximin gives

$$\underline{E}_{\mathcal{P}}(u_{a_2}) = \underline{p}u(100) + (1 - \underline{p})u(0),$$

which is less than $E_{1/2}(u_{a_1})$ if $\underline{p} < 1/2$.

We will see more on this idea later in the course.

Portfolio Choice

There are two assets: cash and a risky asset. The risky asset could be stock, Goldman Sachs debt, a mortgage portfolio or perhaps even a bank deposit or money market fund.

Cash always has a value of 1. The value of the risky asset can be either Low ($L < 1$) or High ($H > 1$).

There are two acts:

1. Hold cash
2. Hold the risky asset

Suppose the decision-maker has utility function u over money and uses the maximizing expected utility rule for known probabilities.

Known Probability

The probability of High is p_H.

The expected utility of act a_1 is

$$E_{p_L}(u_{a_1}) = u(1).$$

The expected utility of act a_2 is

$$E_{p_L}(u_{a_1}) = p_H u(H) + (1 - p_H)u(L).$$

Chose the risky asset if

$$p_H[U(H) - u(1)] + (1 - p_H)[u(L) - u(1)] > 0$$

Unknown Probability

What happens if instead the decision-maker's uncertainty can no longer be represented by single probability on payoffs on the risky asset?

Suppose the set of possible probabilities is $\mathcal{P}$ in which the probability of High is between $\underline{p}$ and $\overline{p}$.

Then maximin gives

$$\underline{E}_{\mathcal{P}}(u_{a_2}) = \underline{p}u(H) + (1 - \underline{p})u(L),$$

which is less than $E_{p_H}(u_{a_1})$ if $\underline{p}$ is low enough.

Sell the risky asset!