“The market has no tolerance for uncertainty.”

Lawrence Fink , Blackrock

What is Going On?

- A curious feature of the current credit crisis is the lack of trading volume in many affected markets
  - Currently, the ABS market is "at a standstill" (WSJ Nov. 6, 2008). This includes securitized auto loans, student loans, credit card debt,…
- Many markets appear to be "frozen" as buyers and sellers are unwilling to trade.

Two questions

- What is causing such seemingly aberrant behavior?
- What are these financial assets worth given that there is no actual trading?

Uncertainty 101

- Knight [1921] – Knightian Uncertainty
  - traders care about both risk and uncertainty

- Savage [1952] – the axioms of decision making allow only for risk.
  - Expected utility framework allows no role for uncertainty

- Ellsberg [1961] – the Ellsberg Paradox

Is it really only risk that matters?

Uncertainty 102

- Gilboa and Schmeidler [1989] – weaken the independence axiom of Savage, which leads to a decision-making framework in which a trader has a set of beliefs, rather than the standard single prior.
  - Uncertainty can induce non-participation
    - Extensive literature
  - But it will also lead to lots of trade as investors flee the markets

Is this really consistent with what we are seeing in the credit crisis?

Bewley's model of Knightian Uncertainty

- Bewley [2002] changes the assumption of complete preferences in Savage
  - One portfolio is preferred to another if and only if its expected utility is greater for every belief in the set of beliefs that represents a traders' preferences.
  - A trader moves away from his current position (the status quo) if and only if the move is expected utility improving for every belief in the set of beliefs that represent the trader's preferences.
    - The "inertia assumption"

An Example

- An investor is holding a CLO. The dealer gives a "mark" of 57.
  - This seems way too low to the investor – is the dealer just over reacting? No way she wants to sell – this asset could be worth 75!
- Would she want to buy more?
  - Problem is what if this is a CLO meltdown? No way she wants to buy – the CLO could be worth 30!

She neither buys nor sells

A Simple Model of Trade

- Trade takes place at two dates, t= 0,1.
- At time 0, traders trade a risky asset and a risk-free asset
- At time 1, an unanticipated shock to traders beliefs about the future value of the asset occurs, and traders can re-trade.
- After period 1 ends, asset payoffs are realized

- The risk-free asset has a constant value of 1.
- The risky asset has a price of p_t per unit at date and an uncertain future value, to be realized at the end of period 1, denoted $\tilde{v}$.
  - Think of the asset as a CDO or MBS
- There are I traders indexed by i=1,….I. Traders have heterogeneous beliefs about the future value.
  - They agree the asset is normally distributed with variance σ^2 but they disagree about the mean
  - Trader i's mean is $\bar{v}_i$ and we assume that for at least two traders i and j, $\bar{v}_j \neq \bar{v}_i$

Traders

- Trader i's endowment of the risky asset is $\bar{x}_i$.
- Per capita endowment of the risky asset is $\bar{x} = I^{-1}\sum_{i=1}^{I}\bar{x}_i$
- All traders have constant absolute risk aversion utility of wealth, w, at the end of period one with risk aversion coefficient 1.
- At time 0, each trader maximizes expected utility of wealth w given beliefs and the time zero price, p_0, of the risky asset.
- At time 1, the unanticipated shock occurs.

Shocks

- First scenario: trader i's expected future value of the risky asset declines to $\bar{v}_{i1} = \alpha \bar{v}_i$, where $\alpha < 1$.
- Second scenario: the set of possible declines in the expected value of the risky asset is described by $1 - \alpha$ where $\alpha \in [\underline{\alpha}, \bar{\alpha}]$ with $\bar{\alpha} < 1$.

Period 0 Equilibrium

- Trader i's period 0 demand for the risky asset is

$$x_i^* = \frac{\bar{v}_i - p_0}{\sigma^2}.$$

- The average belief about the future mean value of the risky asset is $\hat{v} = I^{-1}\sum_{i=1}^{I}\bar{v}_i$.
- Equilibrium:

$$p_0^* = \hat{v} - \sigma^2 \bar{x}$$

$$x_{i0}^* = \left(\frac{\bar{v}_i - \hat{v}}{\sigma^2}\right) + \bar{x}.$$

Period 1 Equilibrium

Let $\hat{v}_1 = (1/I)\sum_{i=1}^{I} \bar{v}_{i1} = \alpha\hat{v}$ be the mean belief at time one.

- Scenario one equilibrium:

$$p_1^* = v_1 - \sigma^2\bar{x} = \alpha\hat{v} - \sigma^2\bar{x}$$

$$x_{i1}^* = \left(\frac{\bar{v}_{i1} - \hat{v}_1}{\sigma^2}\right) + \bar{x} = \alpha\left(\frac{\bar{v}_i - \hat{v}}{\sigma^2}\right) + \bar{x}.$$

- The optimal trade by trader i is

$$t_i^* = x_{i1}^* - x_{i0}^* = \frac{(\alpha - 1)(\bar{v}_i - v)}{\sigma^2}.$$

- Aggregate volume is

$$(1/2)\sum_{i=1}^{I}\left|\frac{(\alpha - 1)(\bar{v}_i - v)}{\sigma^2}\right| > 0.$$

Scenario Two Equilibrium

- Trader i chooses a non-zero trade only if: (i) it is better than the status quo point for all distributions of returns and, (ii) it is not dominated by another trade.
- So his trade t_i must solve the following inequalities:

$$\text{(i)}\quad \bar{v}_{i1}x^*_{i0} + t_i(\bar{v}_{i1} - p_1) + m^*_{i0} - (1/2)\sigma^2(x^*_{i0} + t_i)^2 \geq \bar{v}_{i1}x^*_{i0} + m^*_{i0} - (1/2)\sigma^2(x^*_{i0})^2$$

$$\text{for all } \bar{v}_{i1} \in [\underline{\alpha}\bar{v}_i, \bar{\alpha}\bar{v}_i], \text{ and}$$

(ii) there does not exist a trade t' such that

$$\bar{v}_{i1}x^*_{i0} + t'(\bar{v}_{i1} - p_1) + m^*_{i0} - (1/2)\sigma^2(x^*_{i0} + t')^2 > \bar{v}_{i1}x^*_{i0} + t_i(\bar{v}_{i1} - p_1) + m^*_{i0} - (1/2)\sigma^2(x^*_{i0} + t_i)^2$$

$$\text{for all } \bar{v}_{i1} \in [\underline{\alpha}\bar{v}_i, \bar{\alpha}\bar{v}_i].$$

Conditions for a No Trade Equilibrium

- No trade by i if $\overline{\alpha}\overline{v}_i - \sigma^2 x^*_{i0} \geq p_1 \geq \underline{\alpha}\overline{v}_i - \sigma^2 x^*_{i0}$.

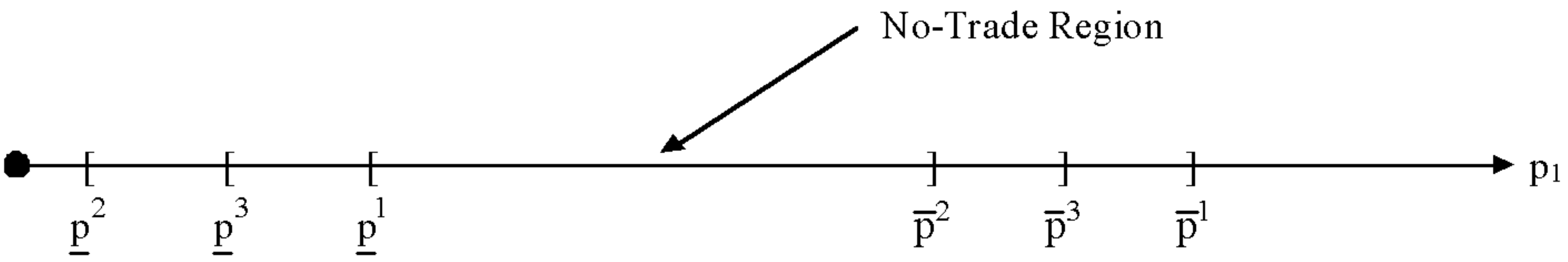
- Equilibrium with no-trade if the intersection of the trader's no-trade regions is non-empty

$$\bigcap_{i=1}^{I} [\underline{\alpha}\overline{v}_i - \sigma^2 x^*_{i0}, \overline{\alpha}\overline{v}_i - \sigma^2 x^*_{i0}] \neq \phi.$$

- Equivalent to

$$(1-\overline{\alpha})\mathrm{Max}_i\{\overline{v}_i\} < (1-\underline{\alpha})\mathrm{Min}_i\{\overline{v}_i\}.$$

The No-Trade Region of Prices



The interval of prices in which trader i will not trade is given by $\left[\underline{p}^i, \overline{p}^i\right]$. The intersection of these intervals, the no-trade region, is $\left[\underline{p}^1, \overline{p}^2\right]$.

No Trade Equilibrium

- Suppose that $\underline{\alpha}<\bar{\alpha}<1$, then there is an equilibrium with no-trade if

$$\frac{\mathrm{Max}_i\{\bar{v}_i\}}{\mathrm{Min}_i\{\bar{v}_i\}} < \frac{1-\underline{\alpha}}{1-\bar{\alpha}}.$$

- LHS is the ratio of the most optimistic trader’s mean to the least optimistic trader’s mean.
- RHS measures the ambiguity about the percentage decline in future mean values.

Example

Suppose there are three traders with prior means

$$\bar{v}_{10} = 1, \bar{v}_{20} = 2, \text{ and } \bar{v}_{30} = 3.$$

- Now consider the "ambiguity case"

  Suppose the most optimistic view is $\bar{\alpha} = 1$ (i.e., values have not fallen)

  Then by Theorem 1, if $\underline{\alpha} < 1$, there will be no trade.

- Consider the no-ambiguity case

  Suppose traders agree price has fallen by 25%. Then

  Trader one buys $(4\sigma^2)^{-1}$ shares

  Trader three sells $(4\sigma^2)^{-1}$ shares

  Trader two does not trade

Prices in the No-Trade Equilibrium

- There is a range of prices at which there is no-trade.
- The maximum price in this range is the lowest price at which some trader is willing to sell the risky asset-----Ask price.
- The minimum price in the no-trade range is the highest price at which some trader is willing to buy the asset----Bid price.
- The difference between the bid and the ask is an “Ambiguity Spread”.

Bid and Ask Prices

- Ask price

$$ask = \mathrm{Min}_i [\bar{\alpha}\bar{v}_i - \sigma^2 x^*_{i0}]$$
$$= \mathrm{Min}_i [\hat{v} - \sigma^2 \bar{x} - (1-\bar{\alpha})\bar{v}_{i0}]$$
$$= \hat{v} - \sigma^2 \bar{x} - \mathrm{Max}_i [(1-\bar{\alpha})\bar{v}_{i0}]$$

- Bid price

$$bid = \mathrm{Max}_i [\underline{\alpha}\bar{v}_i - \sigma^2 x^*_{i0}]$$
$$= \mathrm{Max}_i [\hat{v} - \sigma^2 \bar{x} - (1-\underline{\alpha})\bar{v}_{i0}]$$
$$= \hat{v} - \sigma^2 \bar{x} - \mathrm{Min}_i [(1-\underline{\alpha})\bar{v}_{i0}].$$

- Ambiguity Spread

$$ask - bid = (1-\underline{\alpha})\mathrm{Min}_i\{\bar{v}_i\} - (1-\bar{\alpha})\mathrm{Max}_i\{\bar{v}_i\}.$$

Valuation in an Uncertain World

- In this uncertain world, equilibrium is characterized by a range of prices, and no trade actually occurs at any of them.
  - Markets are illiquid, with no trade occurring and potentially wide spreads
- How then to value an asset in this uncertain world?
  - A huge problem in the current credit crisis, particularly for financial firms holding portfolios of MBS, CDOs, etc,
  - Recent accounting rule changes are important

Fair Value 101

- Actually, FAS 157 “Fair Value Measurements”
  - Defines fair value for financial reporting
  - Establishes a framework for measuring fair value
  - Expands disclosures about fair value measurement
- FAS 159 “The Fair Value Option for Financial Assets and Financial Liabilities”

FAS 157 is required for firms for fiscal years beginning after November 15, 2007

How to define a Fair Value?

- Fair Value: “the price that would be received to sell an asset or transfer a liability in an orderly transaction between market participants”.
  - An exit price, not an entry price

- But what if there are no “orderly transactions”?
  - FAS 157 suggests using “the assumptions that market participants would use in pricing the asset or liability”

What price makes sense?

- A natural candidate to consider is the highest prices at which the asset could be sold – the bid price.

- Two problems
  - No one is actually trading at any price
  - No one would be willing to sell the asset at this bid price. Instead, the lowest price at which anyone would sell the asset is the typically higher ask price.

Prices?

- The Bid price is set by the most optimistic trader about the worst possible outcome---one trader's view of the "the best of a bad situation".
- The Ask price is set by trader who is the least optimistic about the best possible outcome---one trader's view of the "worst of a good situation".

Is there a better price?

- FAS 157 suggests that if an market exists for an identical asset (or a correlated asset) then the fair value is the price from that market.
  - However, in the current credit crisis, for certain classes of assets virtually all markets are frozen.

Midpoint of the Spread

- The principle of insufficient reason suggests

$$(\overline{\alpha} + \underline{\alpha}) / 2.$$

- If there is no dispersion in prior beliefs, then the midpoint is equal to the average notional price of the asset.