

Decision Theory First Exam  
October 10, 2002

I (40 points) Provide a brief answer for each of the following.

1. Suppose  $X = \{a, b, c\}$ . Consider the choice function with  $c(\{a, b\}) = \{a\}$ ,  $c(\{a, c\}) = \{a\}$ ,  $c(\{b, c\}) = \{b\}$ , and  $c(\{a, b, c\}) = \{a, b\}$ . Which of Sen's axioms does this choice function violate?
2. An individual's preference relation on probabilities over  $\{1, 2, 3\}$  satisfies the axioms used in the von Neumann-Morgenstern expected utility theorem. Suppose that 2 for sure is strictly preferred to the probability  $p'$  giving 1 with probability  $1/2$  and 2 with probability  $1/2$ . Show that the probability  $p$  giving 2 with probability  $3/4$  and 3 with probability  $1/4$  is strictly preferred to the probability  $q$  giving 1 with probability  $1/4$ , 2 with probability  $1/2$  and 3 with probability  $1/4$ .
3. Suppose that  $u$  represents the preference relation  $\succ$  on  $X$ . Does  $v(x) = (u(x))^3$  also represent  $\succ$ ? Briefly explain.
4. Take  $X$  to be the set of real numbers and define a relation  $\succeq$  on  $X$  such that
 
$$x \succeq y \quad \text{iff} \quad x \geq y - 1/2$$
  - (a) Is  $\succ$  transitive?
  - (b) Is  $\sim$  transitive?

II (20 points) Let  $\succ$  be a preference relation on a finite set  $X$ . Both of the following statements are true. Provide a proof for one and only one of them.

- For any  $A, B \in P(X)$ ,  $C(A \cup B, \succ) \subset C\left(C(A, \succ) \cup C(B, \succ), \succ\right)$ .
- For any  $A, B \in P(X)$ ,  $C\left(C(A, \succ) \cup C(B, \succ), \succ\right) \subset C(A \cup B, \succ)$ .

III (40 points) Suppose that a retailer has  $\$13N$ . He can use it in November to order some combination of sneakers or winter boots. He can get both sneakers and winter boots from a wholesaler at  $\$13/\text{pair}$ . He can sell

all the sneakers he buys at \$16/pair. Up until March, he can sell winter boots at \$19/pair, but if there are any left over in March, he can sell the rest for only \$11/pair. (Do not worry about integer constraints for the number of sneakers or winter boots purchased by the retailer.)

- (a) Express the retailer's problem as a decision problem, by describing carefully the set of states and acts. Outcomes are profits. You do not need to find an expression for profits or to write a table giving profits.
- (b) What is best act according to the maximin rule?
- (c) What is the optimal act according to the maximax rule?
- (d) What is the optimal act according to the minimax regret?
- (e) What is optimal act according to multiplicative regret (i.e., the act that has the best competitive ratio).

Make sure you show enough of your work to make it clear you understand the issues (and how the various rules work).