

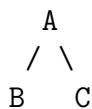
# Decision Theory I

## Problem Set 4

1. GRAD In the proof of the von Neumann-Morgenstern Expected utility theorem we made the following claim. There exist best and worst probabilities  $b, w \in P$ , i.e.  $b, w \in P$  such that  $b \succeq p \succeq w$  for all  $p \in P$ . Prove this claim. [This is Lemma 5.7 in Kreps, whose proof is left as an exercise. You can use any fact proved in Kreps to prove this.]
2. Evaluate the following argument: Max, who is a risk averse expected utility maximizer, is offered an opportunity to buy fair insurance. (The premium equals the expected payoff as in the problem above.) He decides not to buy the insurance. His reasoning is that buying insurance is really gambling; you pay a premium and you may not get a payoff. Fair insurance is a fair gamble, but because he is risk averse he would not accept a fair gamble and thus should not accept fair insurance.
3. GRAD Suppose that an expected utility maximizing individual's utility function,  $u(\cdot)$ , for wealth satisfies  $u'(m) > 0 > u''(m)$  for all  $m > 0$ . The individual's initial wealth is  $w > 0$ . She is offered a bet with probability  $p$  of winning  $t$  and probability  $1 - p$  of losing  $t$ . Assume that  $0 < p < 1$  and  $0 < t < w$ .
  - a. Assume that  $p > \frac{1}{2}$ . Show that if  $t$  is small enough then she will accept the bet.
  - b. Assume that  $p = \frac{1}{2}$ . Is there any value of  $t$  (with  $0 < t < w$ ) for which she will accept the bet?
4. There is a deck with three cards:
  - one is black on both sides,
  - one is white on both sides, and
  - one is black on one side and white on the other.

Alice chooses a card from the deck and puts it on the table with a black side showing.

- (a) What is the probability, according to Bob, that the other side is black? Give at least two answers to the problem, and describe the protocol that generates them.
  - (b) What if Bob doesn't know Alice's protocol (which is probably the case in practice). What would be a good way to model the problem in that case?
5. You're trying to decide whether or not to spend the morning studying for an afternoon test. You don't particularly like studying, but you definitely want to do well on the test. Suppose for simplicity that you get utility 10 if you don't study and do well, 9 if you study and do well, 0 if you don't study and don't do well, and  $-1$  if you study and don't do well. You have previous experience showing that studying is highly correlated with doing well on tests: the probability of doing well given that you study is .9, and the probability that you do well if you don't study is .1. On the other hand, you didn't get much sleep last night, and you know that when you don't sleep well, you neither study (you're too tired) nor do you do well on tests. P
- (a) Describe two causal scenarios: one in which not studying causes poor performance on tests and one in which lack of sleep causes both not studying and poor performance. In both scenarios, define causal probabilities that result in the correlation between studying and doing well given above.
  - (b) Given the probabilities used in part (a), what is the expected utility of studying in each causal model.
  - (c) What information would you need to distinguish the two models?
6. Consider the following Bayesian network containing 3 Boolean random variables (that is, the random variables have two truth values—*true* and *false*):



Suppose the Bayesian network has the following conditional probability tables (where  $X$  and  $\bar{X}$  are abbreviations for  $X = true$  and  $X = false$ ,

respectively):

$$\begin{aligned}\Pr(A) &= .1 \\ \Pr(B \mid A) &= .7 \\ \Pr(B \mid \bar{A}) &= .2 \\ \Pr(C \mid A) &= .4 \\ \Pr(C \mid \bar{A}) &= .6\end{aligned}$$

- (a) What is  $\Pr(\bar{B} \cap C \mid A)$ ?
- (b) What is  $\Pr(A \mid \bar{B} \cap C)$ ?
- (c) Suppose we add a fourth node labeled  $D$  to the network, with edges from both  $B$  and  $C$  to  $D$ . For the new network
  - (i) Is  $A$  conditionally independent of  $D$  given  $B$ ?
  - (ii) Is  $B$  conditionally independent of  $C$  given  $A$ ?In both cases, explain your answer.

7. Consider the following NP(ceteris paribus) network:

A --> B --> C

with the following conditional preference tables:

$a \succ \bar{a}$

$a$	$b \succ \bar{b}$
$\bar{a}$	$\bar{b} \succ b$

$b$	$c \succ \bar{c}$
$\bar{b}$	$\bar{c} \succ c$

- (a) Does it follow from this CPnet that  $abc \succ \bar{a}\bar{b}\bar{c}$ ? (Explain why or why not; a simple yes or no will not get any points.)
- (b) Does it follow from this CPnet that  $\bar{a}\bar{b}\bar{c} \succ \bar{a}bc$ ? (Again, explain why or why not.)