

Decision Theory I

Problem Set 1

1. Show that if \succ is negatively transitive and asymmetric then \succ is transitive.
2. Suppose that \succ is a partial, asymmetric and transitive relation on a finite set X . Let $c(\cdot, \succ)$ be the choice function induced by \succ . Does this choice function necessarily satisfy Sen's β ?
3. Suppose $X = \{x, y, z\}$. Consider a choice function $C : P(X) \rightarrow P(X)$ such that $C(\{x, y\}) = \{x\}$, $C(\{x, z\}) = \{z\}$ and $C(\{y, z\}) = \{y\}$. Does this choice function satisfy Sen's α and β ?
4. The set of alternatives is $X = \{a, b, c\}$ and \succ is a binary order on X reflecting strict preference. Suppose that for $x \in \{b, c\}$, $x \not\succeq a$ and $a \not\succeq x$. Suppose also that $b \succ c$. Can this relation be a strict preference relation? Explain.

If we want to include the possibility that there is an alternative a that is not comparable to either b or c in our analysis then we would want the condition above on a to be satisfied. What does this example say about non-comparability?

5. Let \succ be a binary relation on a finite set X . Define \succeq by: $x \succeq y$ if $y \not\succeq x$. Show
 - (a) If \succeq is complete then \succ is asymmetric.
 - (b) If \succeq is transitive then \succ is negatively transitive.
6. **GRAD:** A binary relation that is reflexive, symmetric and transitive is called an equivalence relation. An equivalence relation partitions a set into equivalence classes. Suppose that \succ is a strict preference relation on a finite set X . Then by Proposition 2.4 of Kreps we know that \sim is an equivalence relation on X . For each $x \in X$ define its equivalence class by $I(x) = \{y \in X | y \sim x\}$. Show:
 - (a) The sets $I(x)$ partition X . (A collection of sets $\{A_1, \dots, A_N\}$ partitions X if each $x \in X$ is in at least one A_i and $A_i \cap A_j = \emptyset$ for all $i \neq j$.)

(b) The sets $I(x)$ are strictly ranked. (The equivalence classes are strictly ranked if, for all $x, y \in X$: (1) if $I(x) \neq I(y)$, then either $x \succ y$ or $y \succ x$, and (2) if $x \succ y$ then $x' \succ y'$ for all $x' \in I(x)$ and $y' \in I(y)$.)

7. **GRAD:** In the statement of Sen's α and β we allow the sets A and B to be any subsets of X . So when we proved that these axioms imply that the revealed preference relation is asymmetric and negatively transitive we allowed ourselves to use information about choices from arbitrary subsets of X . We want to know whether there is a smaller class of subsets of X such that the claim in the revealed preference theorem is true if α and β are satisfied on this smaller class of sets. More precisely, find the smallest set S of non-trivial (not just single element sets), non-empty subsets of X such that the following claim is true: If a choice function satisfies Sen's α and β on S then there is a preference order \succ defined on X such that $c(A, \succ) = c(A)$ for all $A \in S$.

8. **GRAD:** In class in the proof of the revealed preference theorem we defined strict revealed preference. Weak revealed preference is defined as follows: $x \succeq y$ if $x \in C(\{x, y\})$. Define induced strict revealed preference \succ^* from revealed preference \succeq by: $x \succ^* y$ if $x \succeq y$ and $y \not\succeq x$. Are strict revealed preference and induced strict revealed preference the same relation?