

# Decision Theory I

## Problem Set 3

1. Consider a finite set of prizes  $X$  and probabilities  $P$  on them. Suppose that an individual's preferences  $\succ$  on  $P$  have an expected utility representation with utility function  $u : X \rightarrow \mathfrak{R}$ . Show that  $\succ$  satisfies the independence axiom.
2. Consider a finite set of prizes  $X$  and probabilities  $P$  on them. Suppose that an expected utility maximizer's preferences  $\succ$  on  $P$  have an expected utility representation with utility function  $u : X \rightarrow \mathfrak{R}$ . Show that if  $a > 0$  then  $v(\cdot) = au(\cdot) + b$  also represents  $\succ$ .
3. GRAD Consider a finite set of prizes  $X$  and probabilities  $P$  on them. Suppose that an expected utility maximizer's preferences  $\succ$  on  $P$  have an expected utility representation with utility function  $u : X \rightarrow \mathfrak{R}$ . Show that if  $v : X \rightarrow \mathfrak{R}$  also represents  $\succ$  then there exist real numbers  $a > 0$  and  $b$  such that  $v(\cdot) = au(\cdot) + b$ .
4. In the proof of the von Neumann-Morgenstern Expected utility theorem we made the following claim. For any  $\alpha, \beta \in [0, 1]$ ,  $\beta b + (1 - \beta)w \succ \alpha b + (1 - \alpha)w$  if and only if  $\beta > \alpha$ . We proved the "if" part. Prove the "only if" part.
5. Recall that  $f \succ_A g$  iff there is an act  $h$  such that  $f_A h \succ g_A h$ . Prove that  $\succ_A$  is a preference order.
6. Show that if  $A$  and  $B$  are null then  $A \cup B$  is null. (Hint: first prove the result if  $A$  and  $B$  are disjoint.) You may use any facts proved in Prof. Blume's online lecture notes (but not the facts that are stated but not proved.)
7. GRAD: Prove that there are best and worst outcomes; that is, there are outcomes  $x$  and  $y$  such that, for all acts  $f$ , we have  $x \succeq f \succeq y$ . (Recall that an outcome  $x$  is identified with the constant act  $f_x$  that gives  $x$  in all states.)
8. GRAD: Prof. Blume's notes prove the Sure Thing Principle for partitions of size  $n = 2$ . (See Proposition 3.) Complete the proof of the Sure Thing Principle.