Problem Set 4

Problems 1, 3 and 8 are required only for graduate students. Others may feel free, and are encouraged, to take them on.

1. For grads only. Let S denote a set of states, O a set of outcomes, L the set of all Savage acts from S to O, and $\succ$ a preference order on L. Suppose that $\succ$ has an expected utility representation with payoff function u and probability distribution p. Show that if A is not a null event, then $f \succ_A g$ iff the conditional expected utility of f given A exceeds that of g.

2. Exercises on null sets:

   (a) Show that $\emptyset$ is null.

   (b) Show that if $B \subset A$ and A is null, then B is null

   (c) Show that if A and B are disjoint and null, then $A \cup B$ is null.

3. More exercises on null sets, for Grads only.

   (a) Show that if A and B are null, then $A \cup B$ is null.

   (b) show that A is null iff $A \approx \emptyset$.

4. We will reconsider the insurance problem from class but allow for a more general distribution of losses. An individual has wealth $\$w$. There are two possible losses that the individual can insure against: loss 1 of $\$L_1$ and loss 2 of $\$L_2$. The losses are independent events and the individual believes that they occur with probability p_1 and p_2, respectively. For each loss i, insurance is available that costs q_i per dollar to be paid to the decision-maker in the event of loss i. Let $\$\pi_i$ be the amount that the decision-maker chooses to be paid in the event of loss i; then $q_i\pi_i$ is the cost of the insurance. Suppose that the decison-maker is an expected utility maximizer with payoff function $u(\cdot)$ on wealth. Suppose that u is twice continuously differentiable with a strictly positive first derivative and a stricly negative second derivative.

   (a) Suppose that for each i, $p_i = q_i$. Show that full insurance is optimal, i.e. $\pi_i = L_i$.

(b) Suppose that for each i, $\alpha p_i = q_i$ for some $\alpha > 1$. Does optimal insurance require a common deductible, i.e. $L_i - \pi_i = x$ for all i?

5. Consider a decision problem with 2 states and 3 actions. The payoffs $u(\omega, d)$ are given in the following table:

	d_1	d_2	d_3
ω_1	0	-10	-4
ω_2	-8	0	-3

The decisionmaker is an expected utility maximizer, and beliefs are represented by a probability distribution $p = \big(p(\omega_1), p(\omega_2)\big)$.

(a) The DM does not know p. Her prior beliefs on p are that $p(\omega_1)$ is is equally likely to be 1/4, or 3/4. What d_i will she choose?

(b) Before she chooses she is told that the last time a state was drawn from the probability distribution p, the outcome was ω_1. Draws are independent and her prior beliefs are as in part (c). What will she choose?

(c) Suppose she is told that the outcome of the last draw was ω_2. What will she choose?

(d) What is the value to her of being told the last draw.

6. Consider the same payoffs, but with the maximin criterion. What is the value of each of the following pieces of information:

(a) The last draw?

(b) The true probability distribution p?

(c) The true state?

7. Recall the following decision problem, like that discussed in class. Two coins are to be flipped. Two bets are available, bet A and bet B. The dollar payoffs of each bet depend upon the outcome of the flips, and are described in the table. Assume that the utility function is $u(x) = x$, so this table also describes utility payoffs. Suppose the coin-flipper offers to tell you the outcome of the first flip (again as discussed in class). What is the value of the information in each of the following cases:

	HH	HT	TT	TT
A	10	2	0	-20
B	0	0	1	1

Figure 1: Payoffs for problem 7.

(a) The decisionmaker is an EU maximizer, and believes flips to be independent and from a fair coin.

(b) The decisionmaker has minimax regret preferences.

8. For grads only. Consider the previous problem. For minimax regret preferences, how much would a decisionmaker be willing to pay for the information? Hint: If the decisionmaker wants to use a contingent strategy, he must buy the information at price p, so payoffs must be adjusted accordingly. Alternatively, he can choose an uncontingent act, with no payoff adjustment.