

## Allais Paradox

The set of prizes is  $X = \{\$0, \$1,000,000, \$5,000,000\}$ .

- Which probability do you prefer:  
 $p_1 = (0.00, 1.00, 0.00)$  or  $p_2 = (0.01, 0.89, 0.10)$ ?
- Which probability do you prefer:  
 $p_3 = (0.90, 0.00, 0.10)$  or  $p_4 = (0.89, 0.11, 0.00)$ ?

40% of the class reported  $p_1 \succ p_2$  and  $p_3 \succ p_4$

## Inconsistent with EUT

Suppose  $(u_0, u_1, u_5)$  represents  $\succ$ .

Then  $p_1 \succ p_2$  implies

$$u_1 > .01u_0 + .89u_1 + .1u_5$$

$$.11u_1 - .01u_0 > .1u_5$$

$$.11u_1 + .89u_0 > .1u_5 + .9u_0.$$

So  $p_4 \succ p_3$ .

What axiom is violated?

Independence

## Ellsberg Paradox

There are two urns filled with Red and Black balls.

- **Urn 1** 100 balls, a mix of Red and Black balls.
- **Urn 2** 50 Red balls and 50 black balls.

A ball will be drawn at random from one of the urns.

- Which do you prefer:
  1. A ball is drawn from Urn 1, you are paid \$100 if it is Red and nothing if it is Black.
  2. A ball is drawn from Urn 2, you are paid \$100 if it is Red and nothing if it is Black.
- Which do you prefer:
  1. A ball is drawn from Urn 1, you are paid \$100 if it is Black and nothing if it is Red.
  2. A ball is drawn from Urn 2, you are paid \$100 if it is Black and nothing if it is Red.

### **Inconsistent with SEU**

73% of the class chose option 2 both times.

If there are subjective probabilities then the first choice implies that the probability of a red ball in Urn 1 is less than .5 and the second choice implies that it is greater than .5.

Which axiom is violated?

Independence.

## Multiple Priors

Suppose that the decision maker has multiple priors for the urn with an unknown mix of Red and Black balls and chooses using the most pessimistic belief. Could this explain the outcomes in the Ellsberg Paradox?

Let  $p$  be a probability for the decision maker that the ball to be drawn from Urn 1 is Red. The decision maker has a set  $P \subset [0, 1]$  of probabilities.

In any choice situation the decision maker chooses using a maximin rule: For each action evaluate expected utility using the probability in  $P$  that minimizes expected utility. Select the action that maximizes over these minimized values. (See Professor Halpern's lecture on decision rules.)

If  $P$  contains all probabilities then, for any utility function with  $u(100) > u(0)$ , chose Urn 2 in each case.