Corrections to Problem 2

Problem 2 in the homework asked you to show that A1 and A4$'$ together imply A4. That's going to be awfully hard to do, since it's false. It's quite easy to show that A1 and A4$'$ together imply that the following weaker version of A4:

A4^w. If p and q are probabilities on prizes such that $f(S - \{s\})p \succ f(S - \{s\})q$, then $f(S - \{s'\})p \succeq f(S - \{s'\})q$ for all nonnull $s' \in S$.

Note the $\succeq$ in the conclusion here, rather than the $\succ$ in the original statement of A4. This is the best we can do. Here is a modified version of the problem that I hope clarifies the issues:

(a) Prove that A1 and A4$'$ imply A4^w.

(b) Observe that MMEU (Maxmin Expected Utility) satisfies A1 and A4$'$ (you don't have to prove this), but show by example that MMEU does *not* satisfy A4. It follows that A1 and A4$'$ do not imply A4.

(c) I haven't had a chance to check yet whether A1–3 and A4$'$ imply A4, although I suspect not. Consider the following axiom:

 A4^s. If $f(s) \succeq g(s)$ for all $s \in S$ and $f(s') \succ g(s')$ for some nonnull s', then $f \succ g$.

 Show that A1, A4$'$, and A4^s together imply A4.

It follows from part (b) that MMEU does not satisfy A4^s. It is possible that A1 and A4^s already imply A4$'$ (although I suspect not), in which case A4$'$ is unnecessary in part (c). I haven't had a chance to check this yet either. A minor modification of the argument I sketched for Exercise 3 can be used to show that A1 and A4 imply A4^s as well as A$4'$. Bottom line: A4 is slightly stronger than A$4'$.

Sorry for the confusion ...