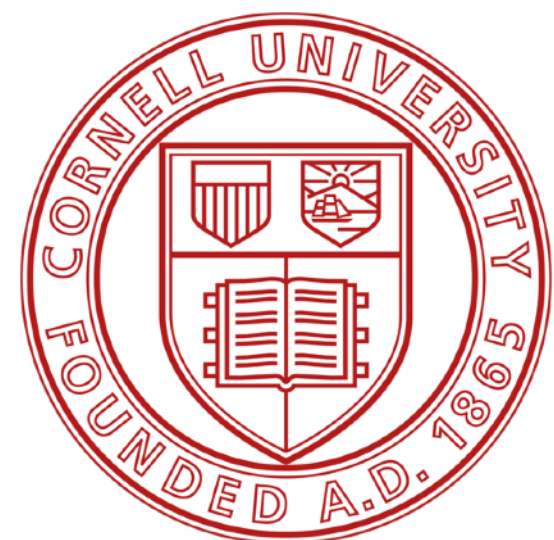


Lecture 14: Diffusion models - Part 1

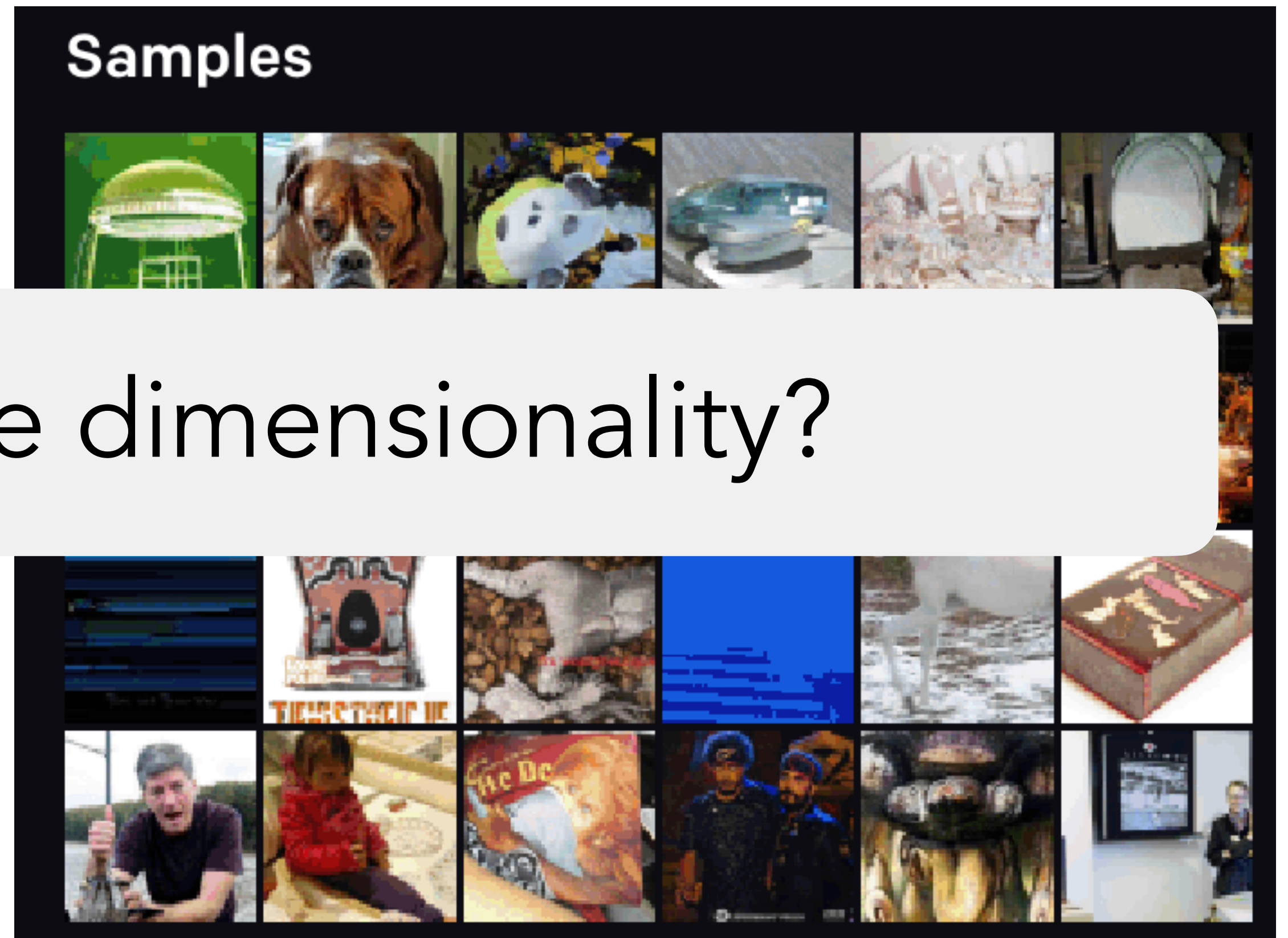
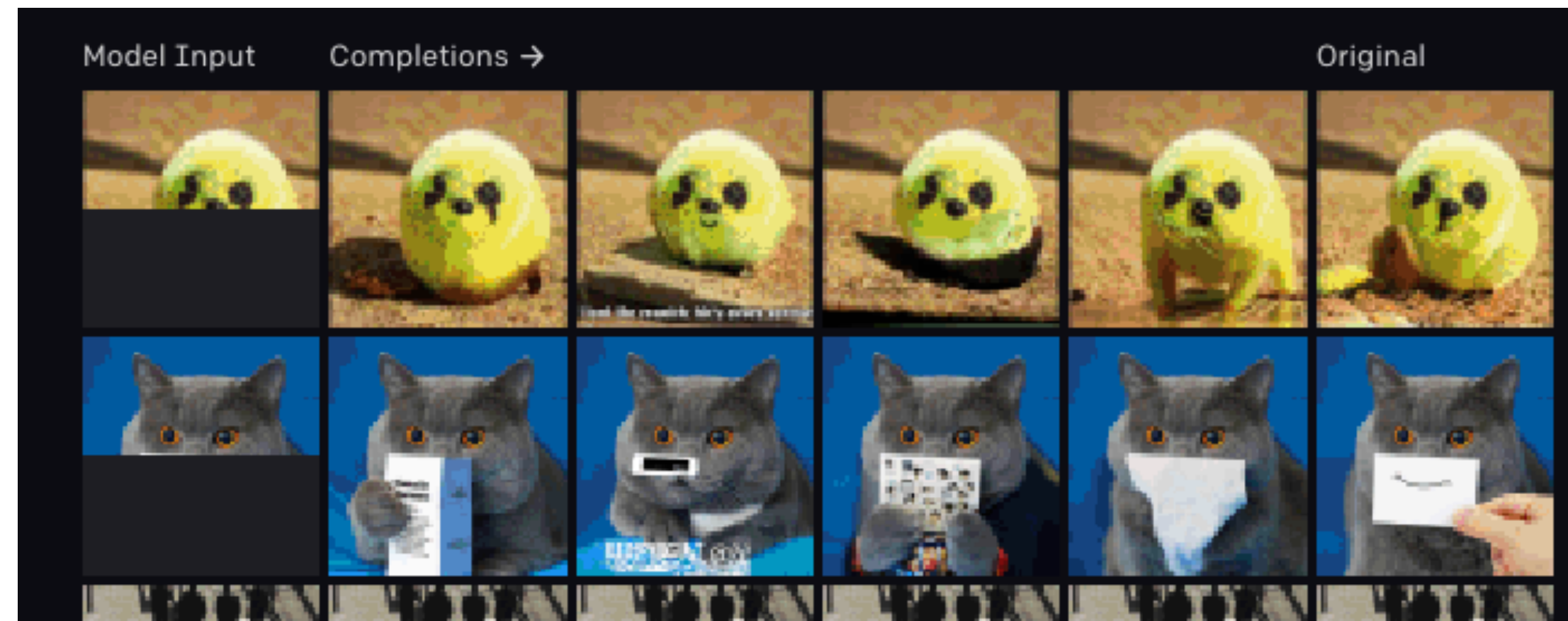
CS 5670: Introduction to Computer Vision



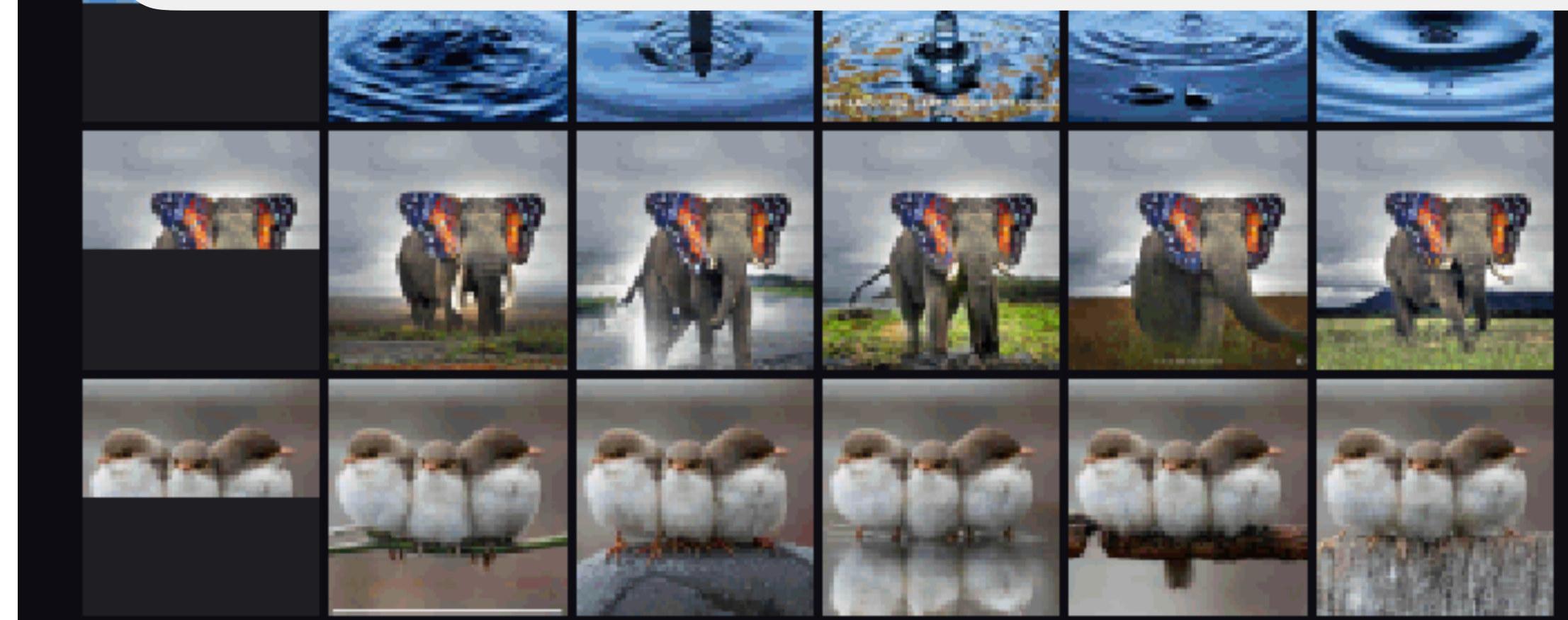
Announcements

- Midterm on Oct. 20

Last class: pixel autoregressive models



Can we lower the dimensionality?

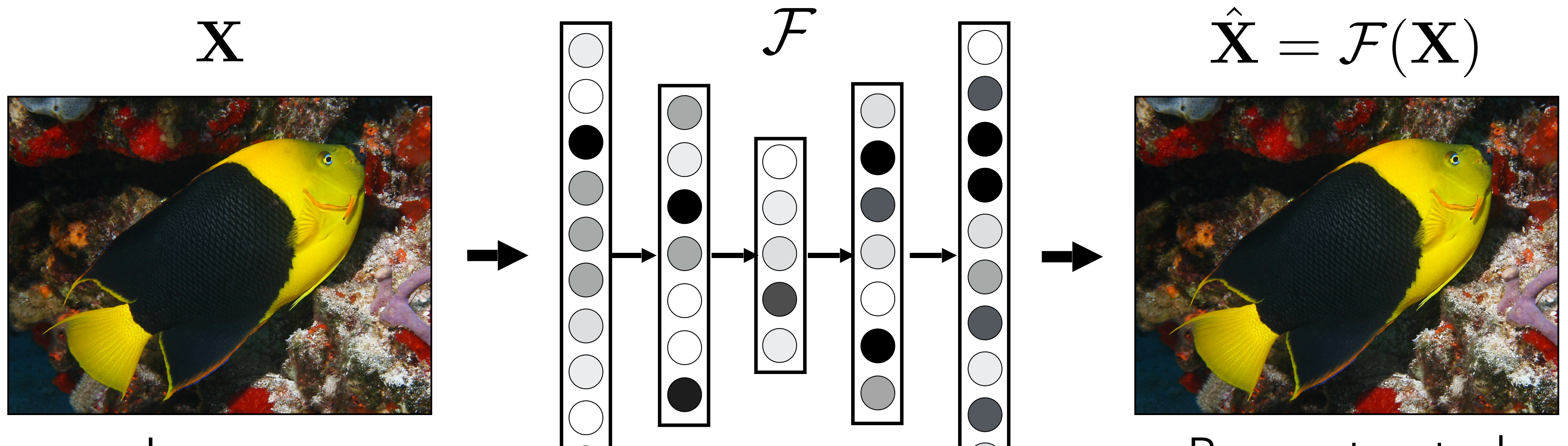


<https://openai.com/blog/image-gpt/>

Today

- Generative models in latent space (continuing from last class)
- Diffusion models

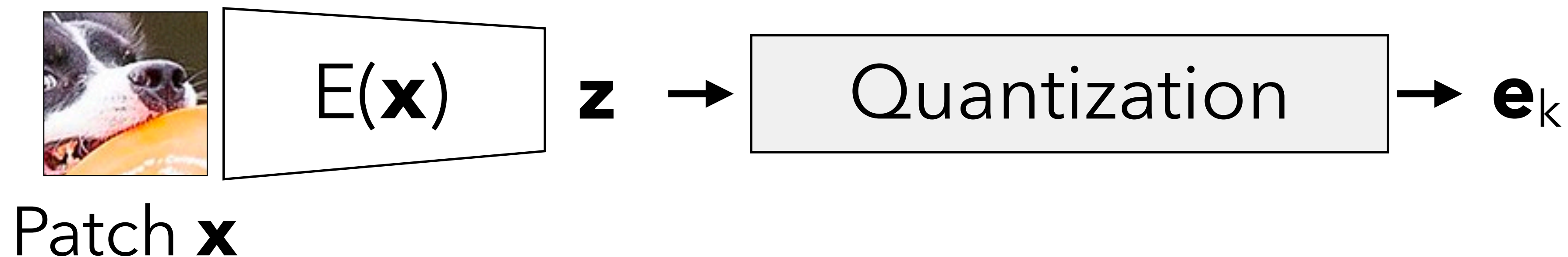
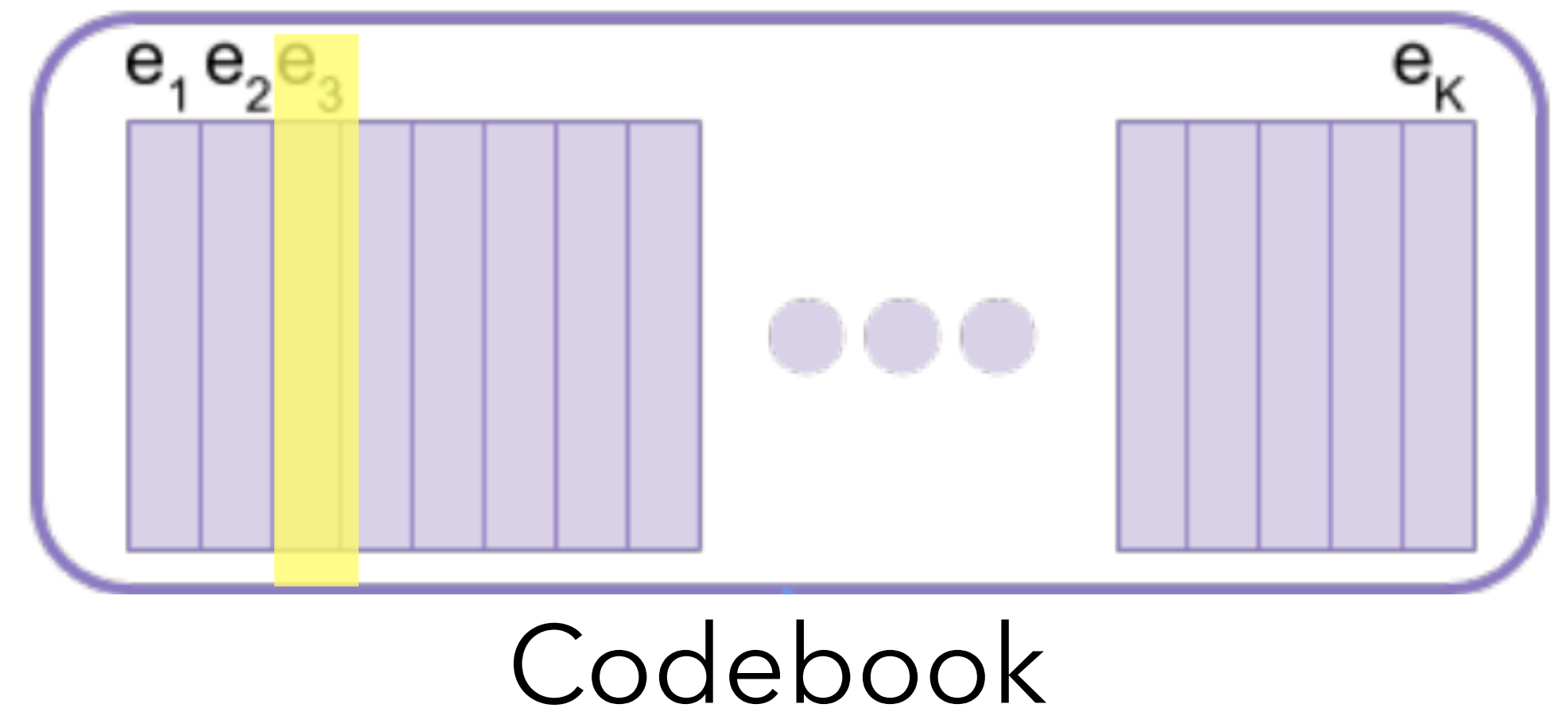
Lower dimensionality using an autoencoder



How can we make the output discrete?

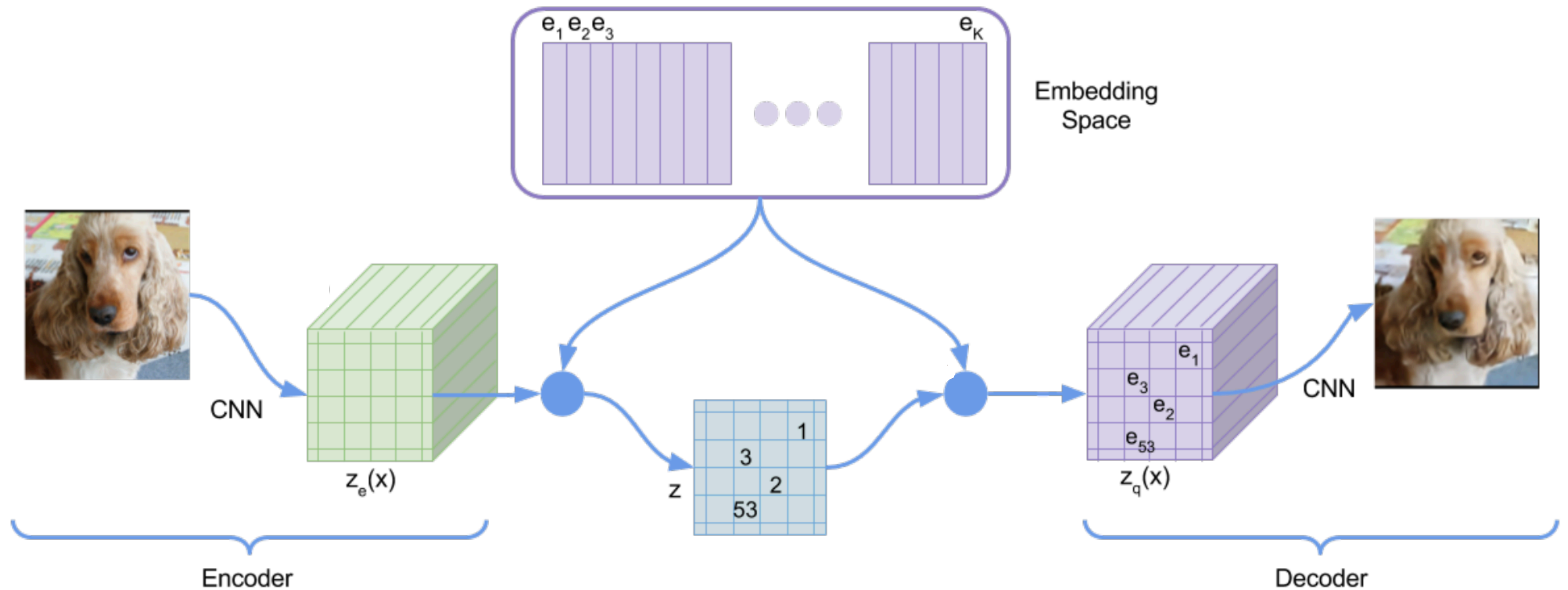
Vector quantization

Predict a real-valued vector, then “snap” it to a nearest neighbor from a *codebook*.

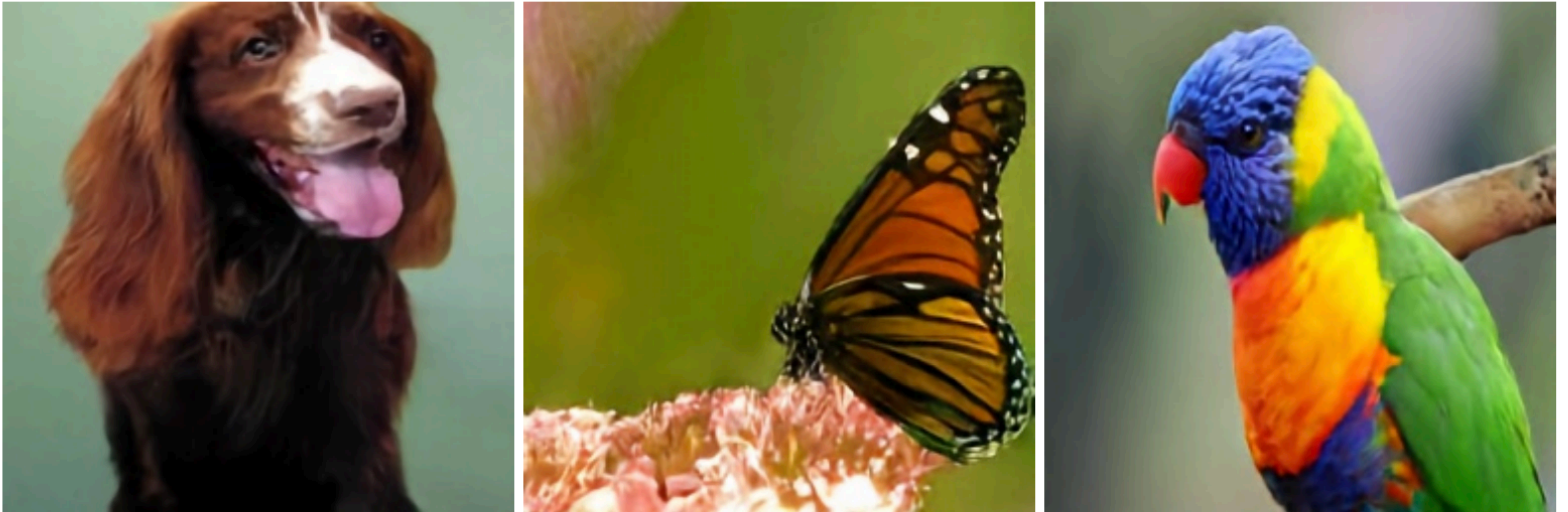


$$\text{Quantize}(E(\mathbf{x})) = \mathbf{e}_k \quad \text{where } k = \arg \min_j ||E(\mathbf{x}) - \mathbf{e}_j||$$

Vector quantized (variational) autoencoder

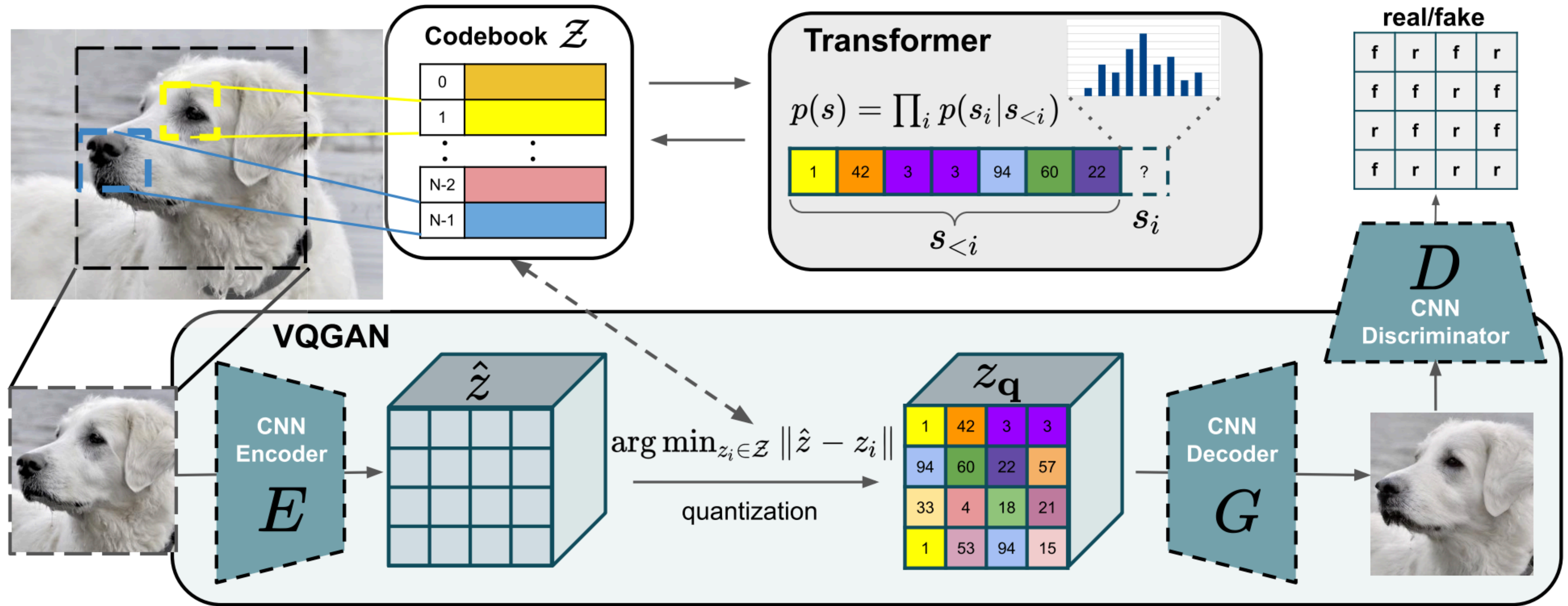


VQ-VAE image generation results



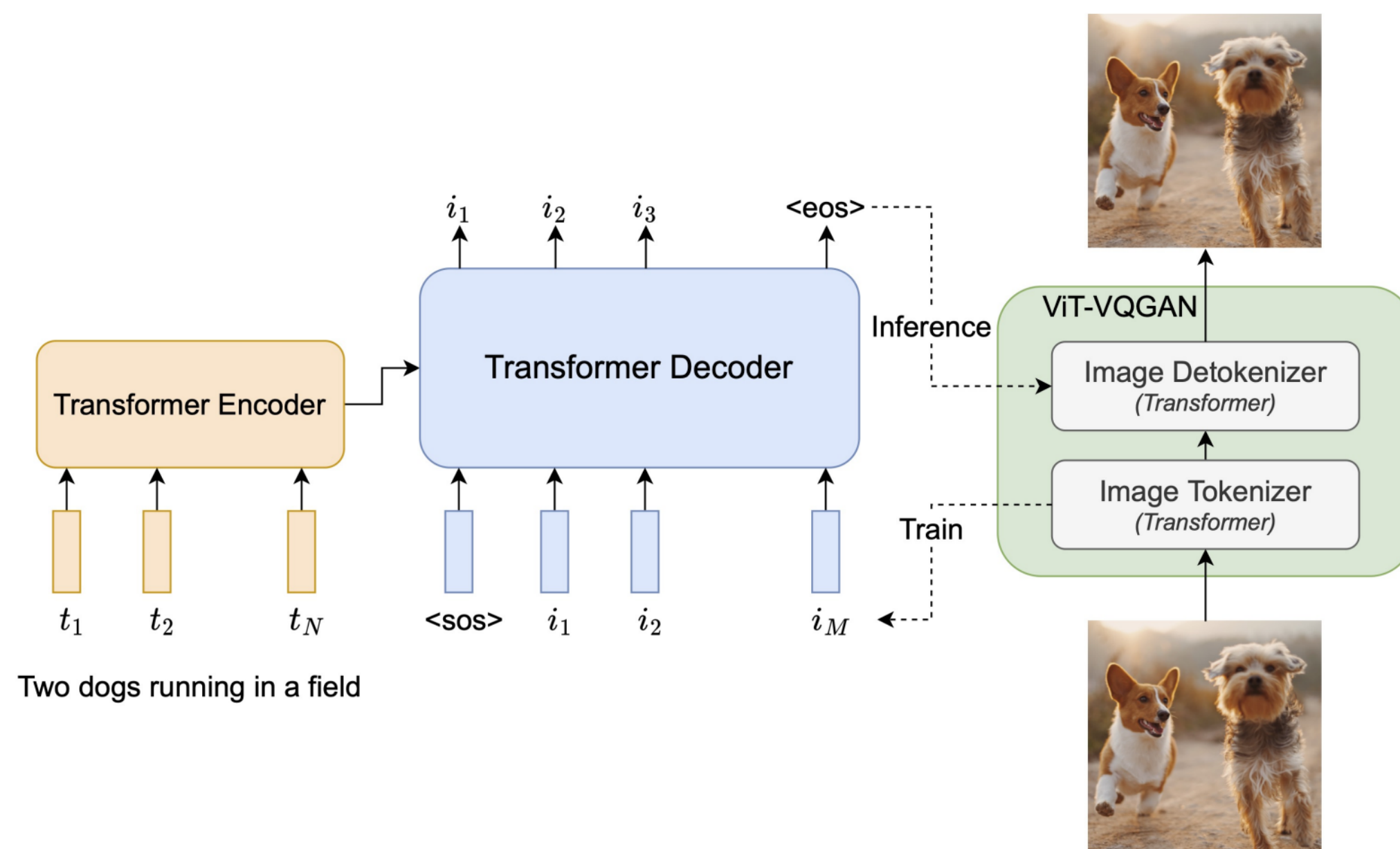
Generated images (256×256)

Vector quantized GAN (VQ-GAN)



Add a GAN loss to get crisper images

Autoregressive text-to-image generation



A. A photo of a frog reading the newspaper named "Toaday" written on it. There is a frog printed on the newspaper too.



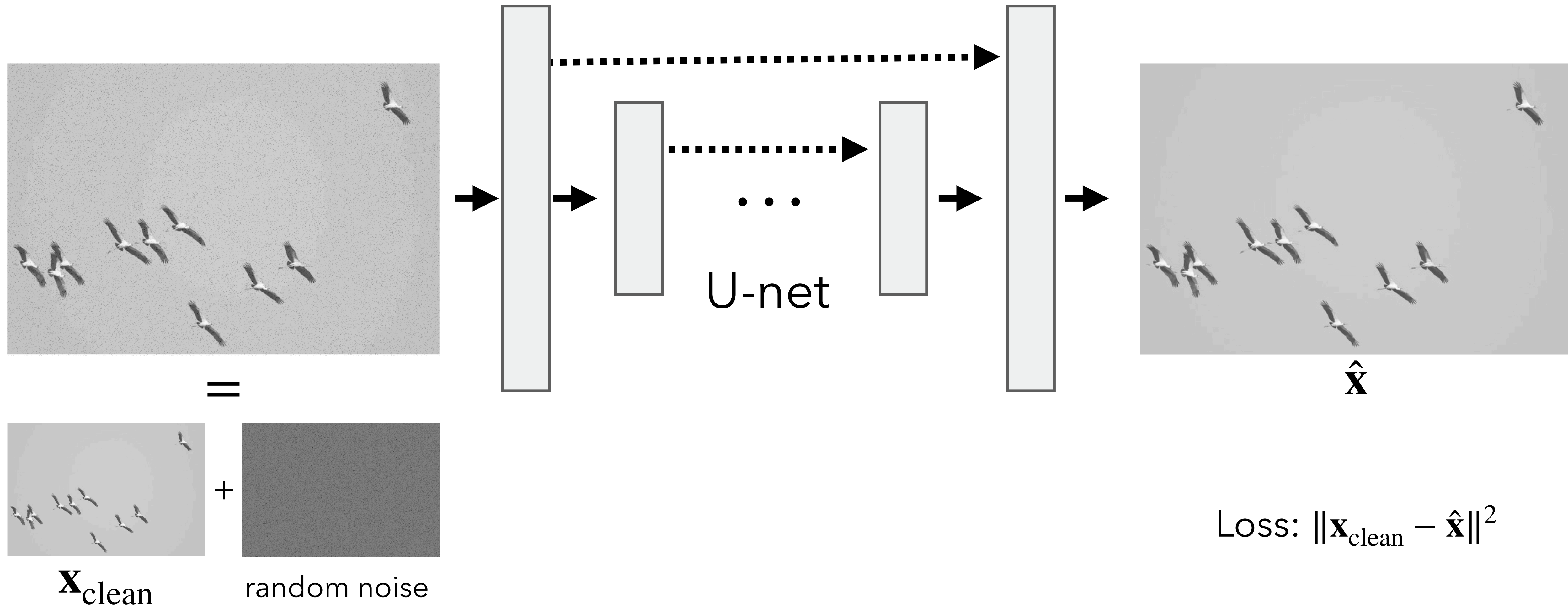
B. A portrait of a statue of the Egyptian god Anubis wearing aviator goggles, white t-shirt and leather jacket. The city of Los Angeles is in the background. Hi-res DSLR photograph.

[Yu et al., "Parti", 2022]

Today

- Generative models in latent space
- **Diffusion models**

Recall: the denoising problem



From noise to an image

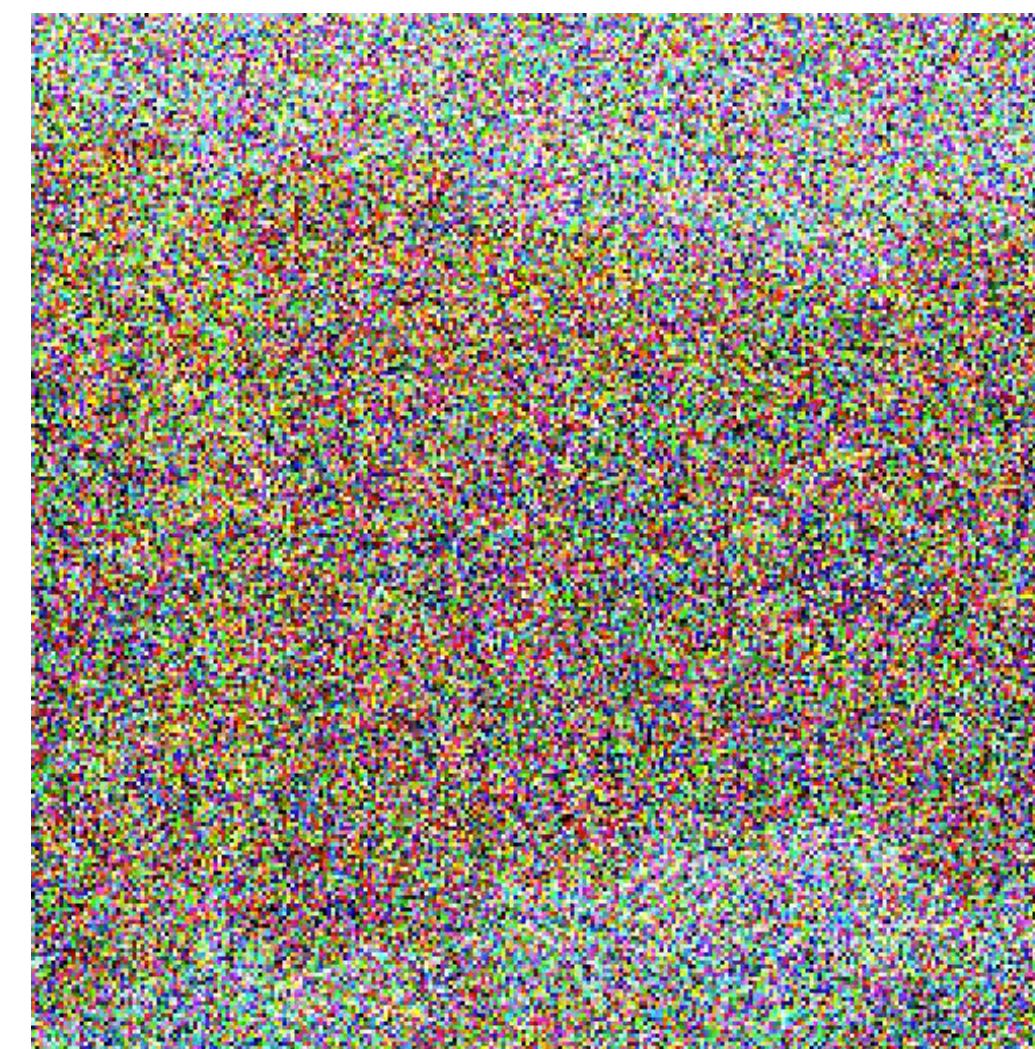


Random noise

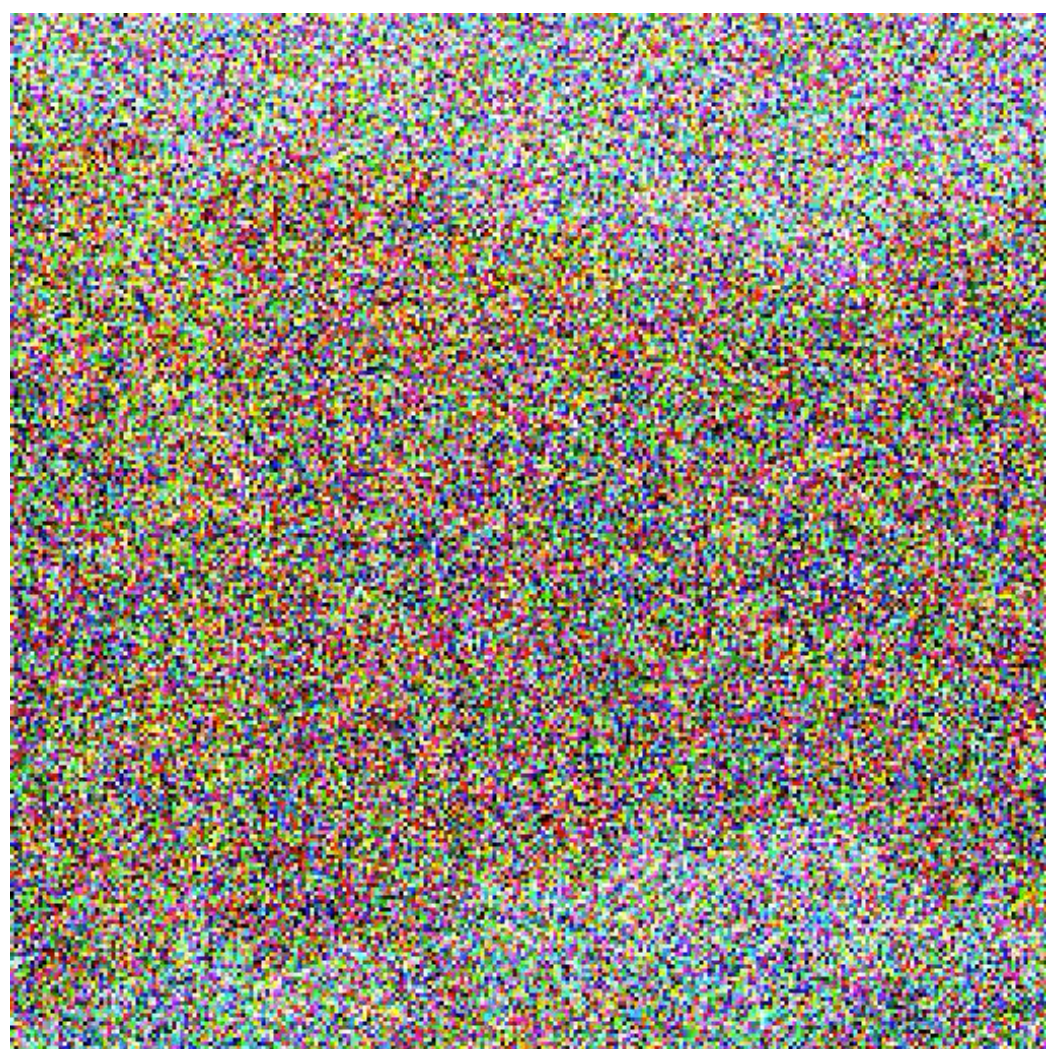
denoise



denoise



From noise to an image



denoise



denoise



From noise to an image



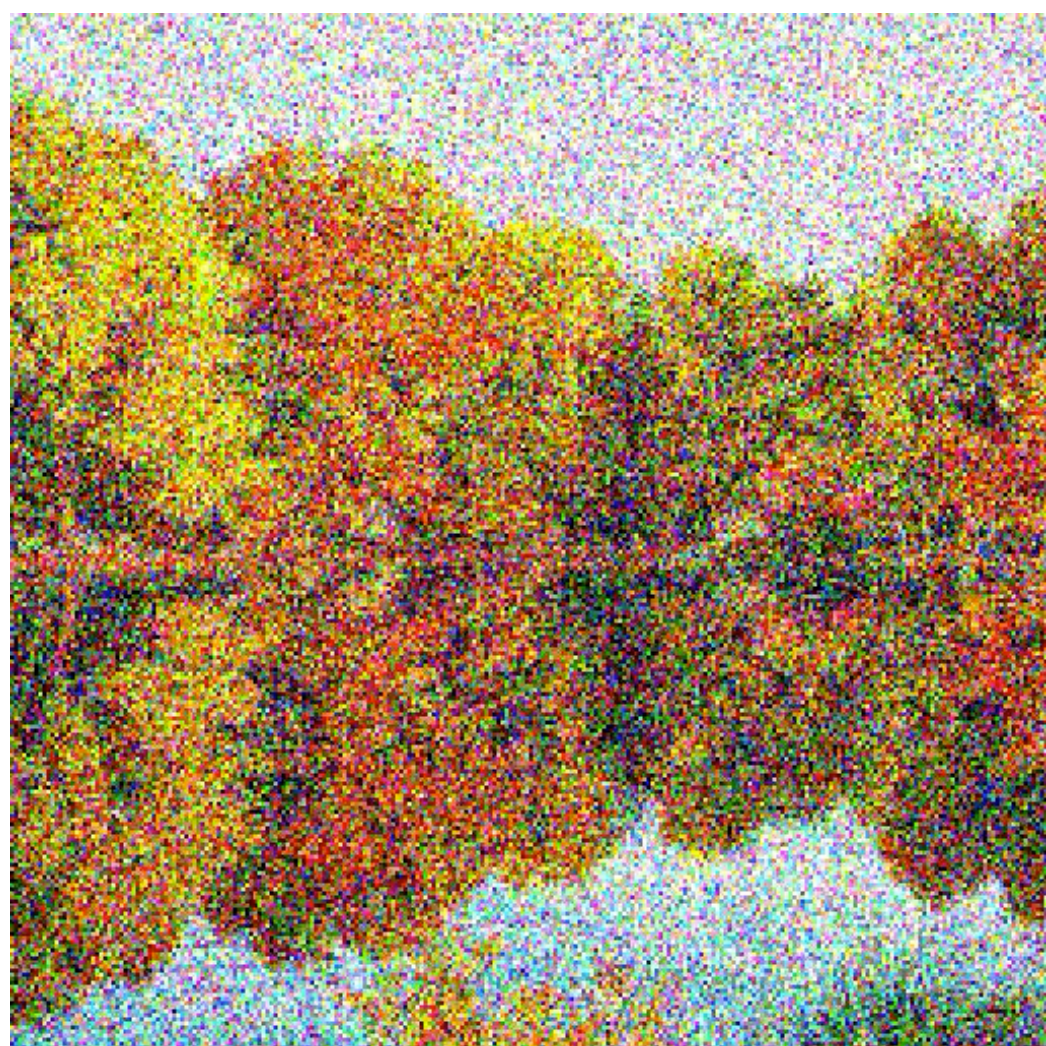
denoise



denoise



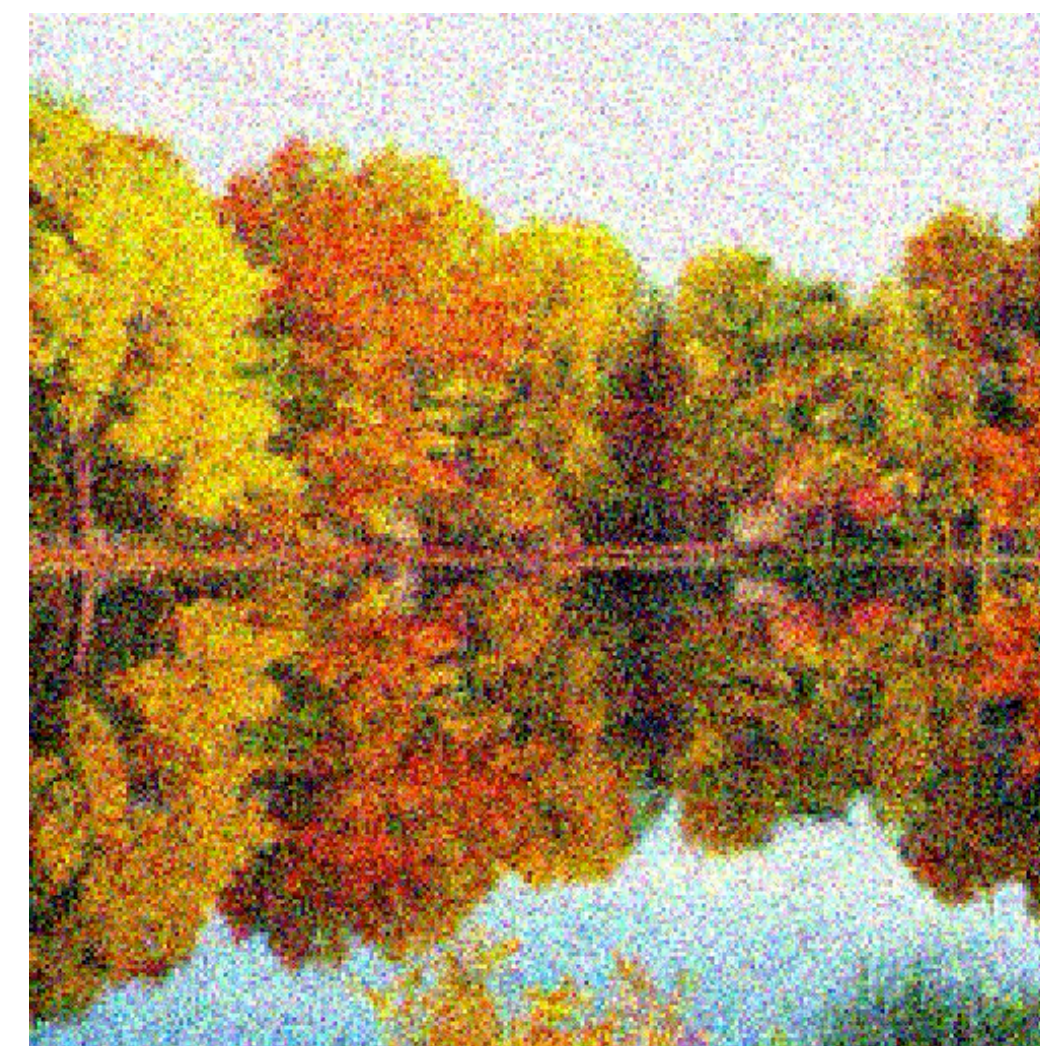
From noise to an image



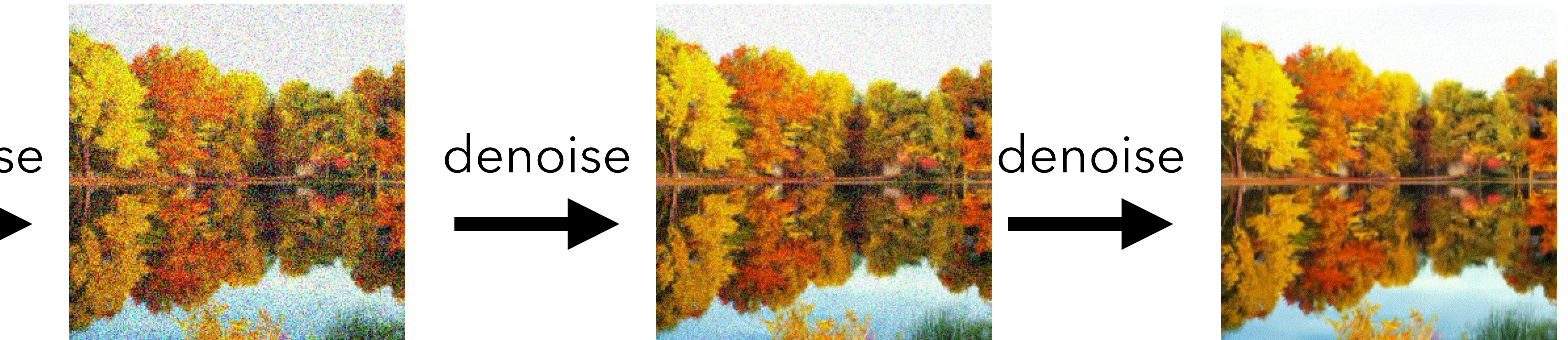
denoise



denoise

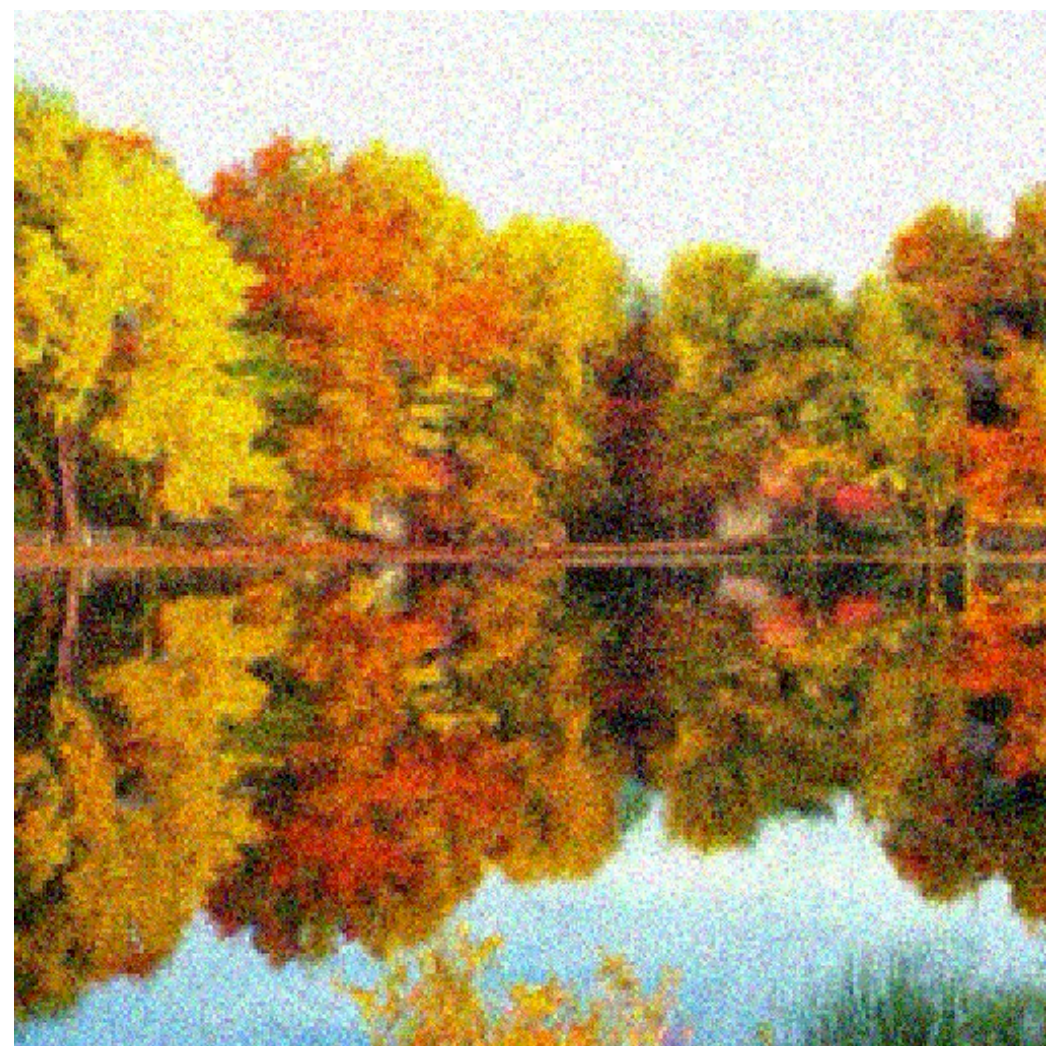
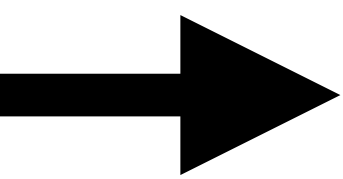


From noise to an image

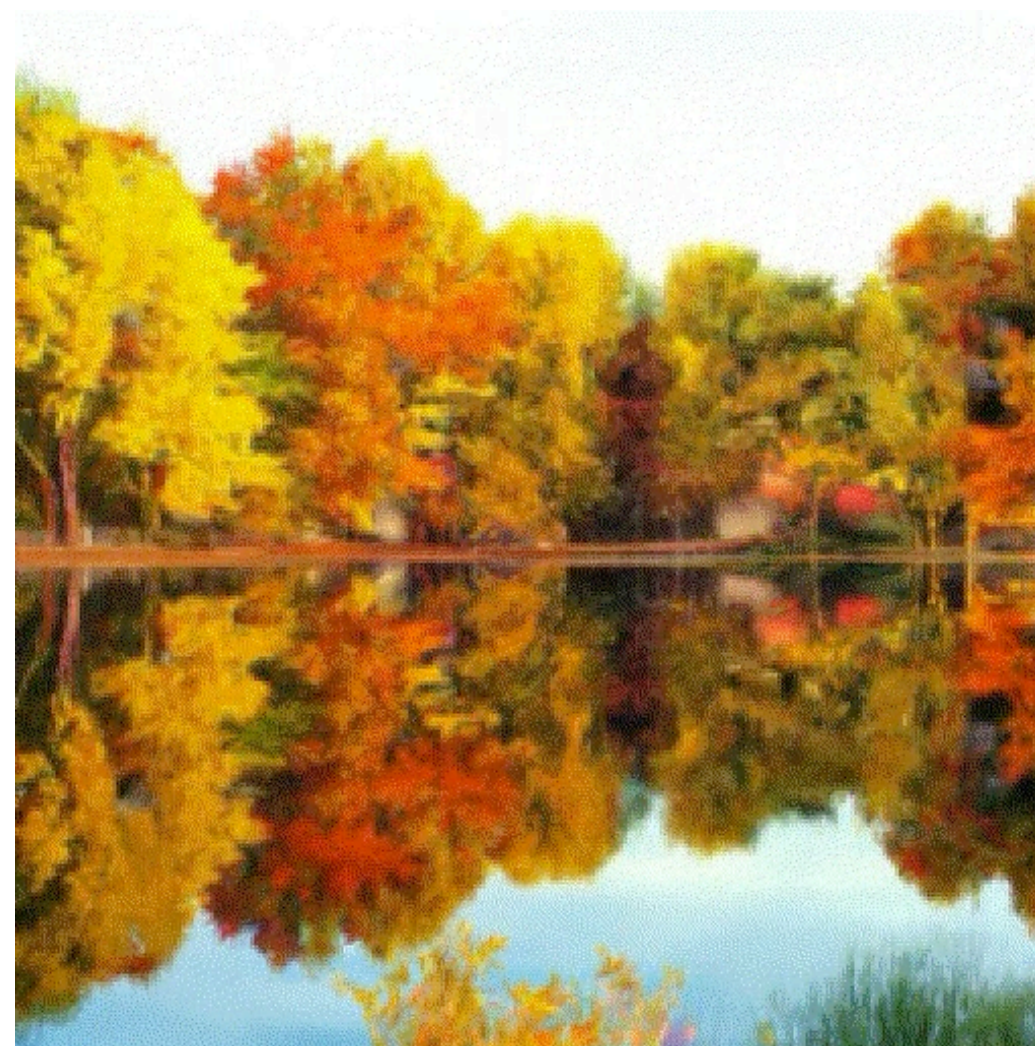


From noise to an image

noise



denoise

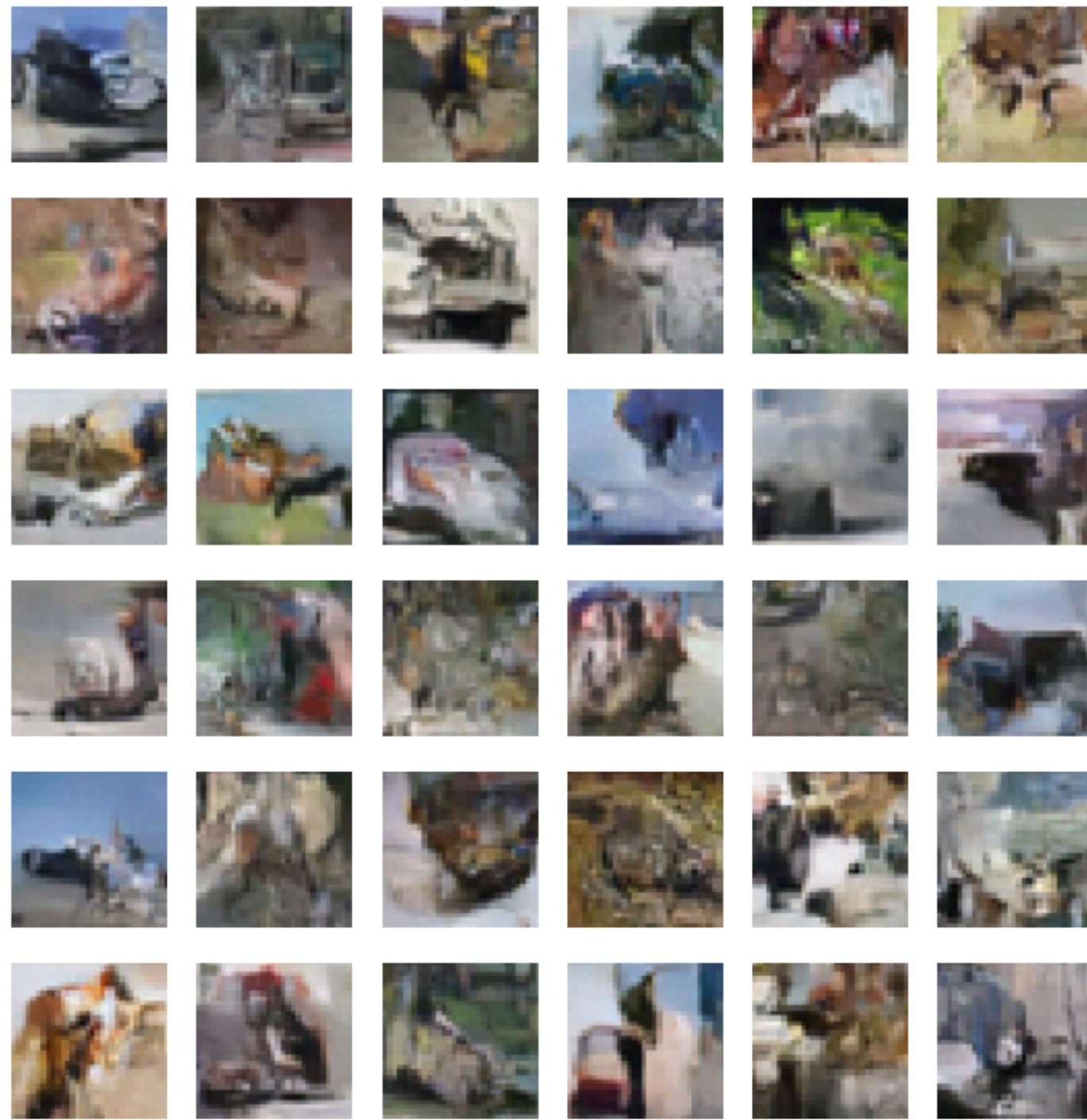


denoise



Generated image!

Diffusion models



[Sohl-Dickstein et al., 2015]

Diffusion models

Deep unsupervised learning using nonequilibrium thermodynamics

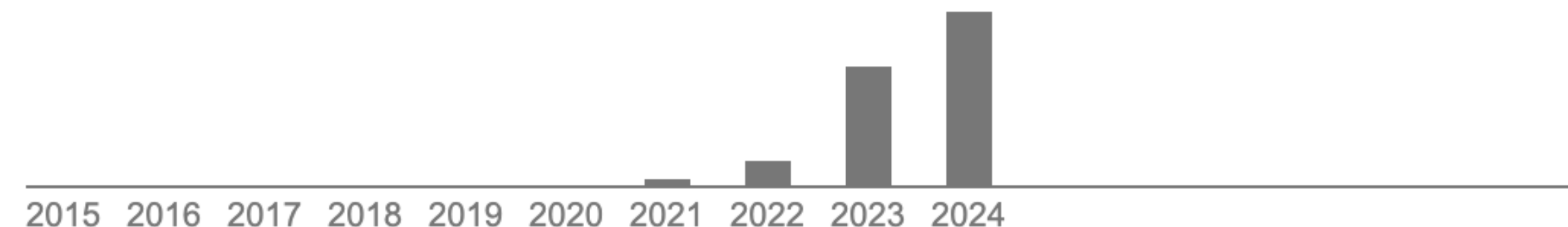
Authors Jascha Sohl-Dickstein, Eric A Weiss, Niru Maheswaranathan, Surya Ganguli

Publication date 2015/3/12

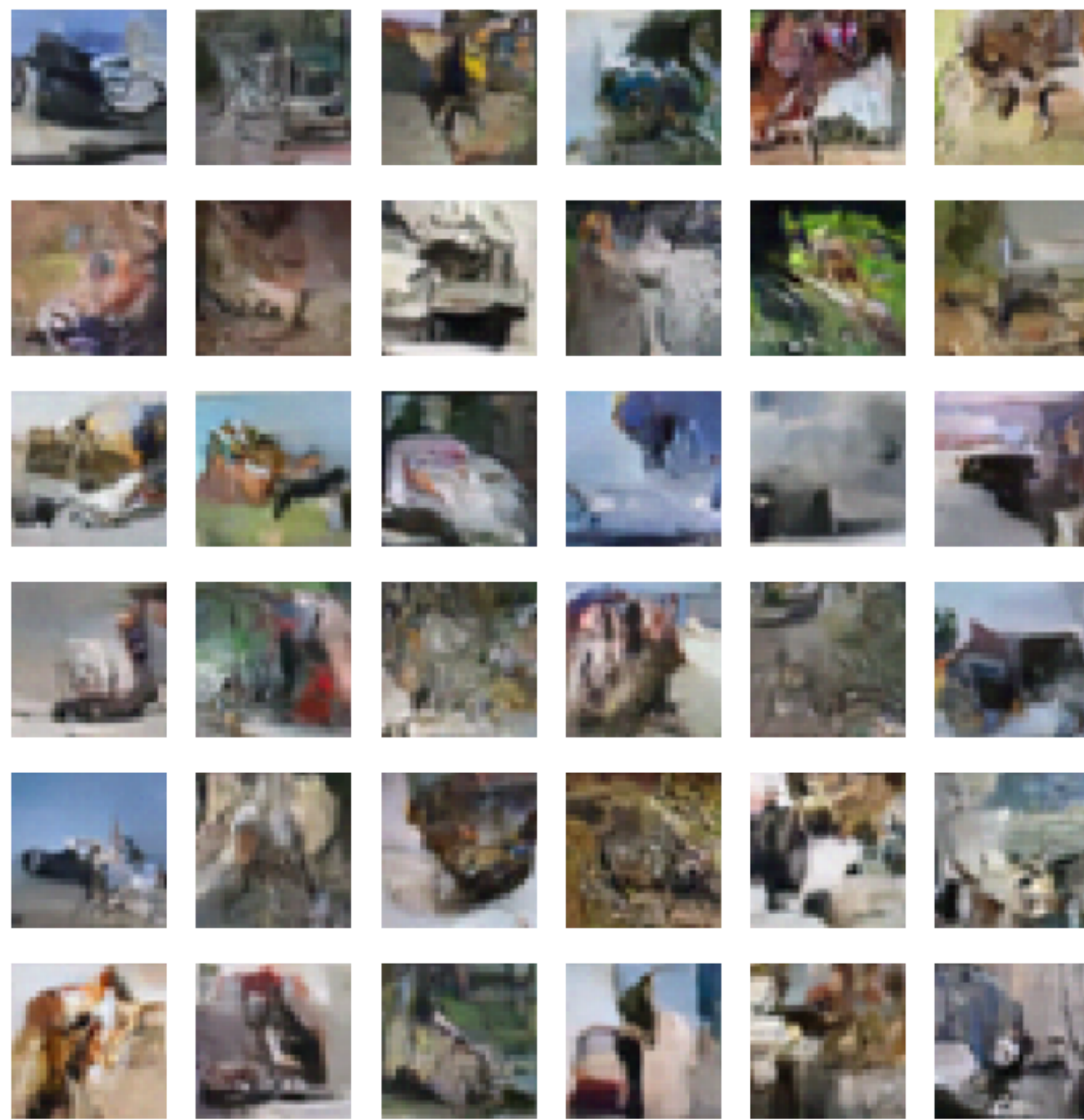
Journal International Conference on Machine Learning

Description A central problem in machine learning involves modeling complex data-sets using highly flexible families of probability distributions in which learning, sampling, inference, and evaluation are still analytically or computationally tractable. Here, we develop an approach that simultaneously achieves both flexibility and tractability. The essential idea, inspired by non-equilibrium statistical physics, is to systematically and slowly destroy structure in a data distribution through an iterative forward diffusion process. We then learn a reverse diffusion process that restores structure in data, yielding a highly flexible and tractable generative model of the data. This approach allows us to rapidly learn, sample from, and evaluate probabilities in deep generative models with thousands of layers or time steps, as well as to compute conditional and posterior probabilities under the learned model. We additionally release an open source reference implementation of the algorithm.

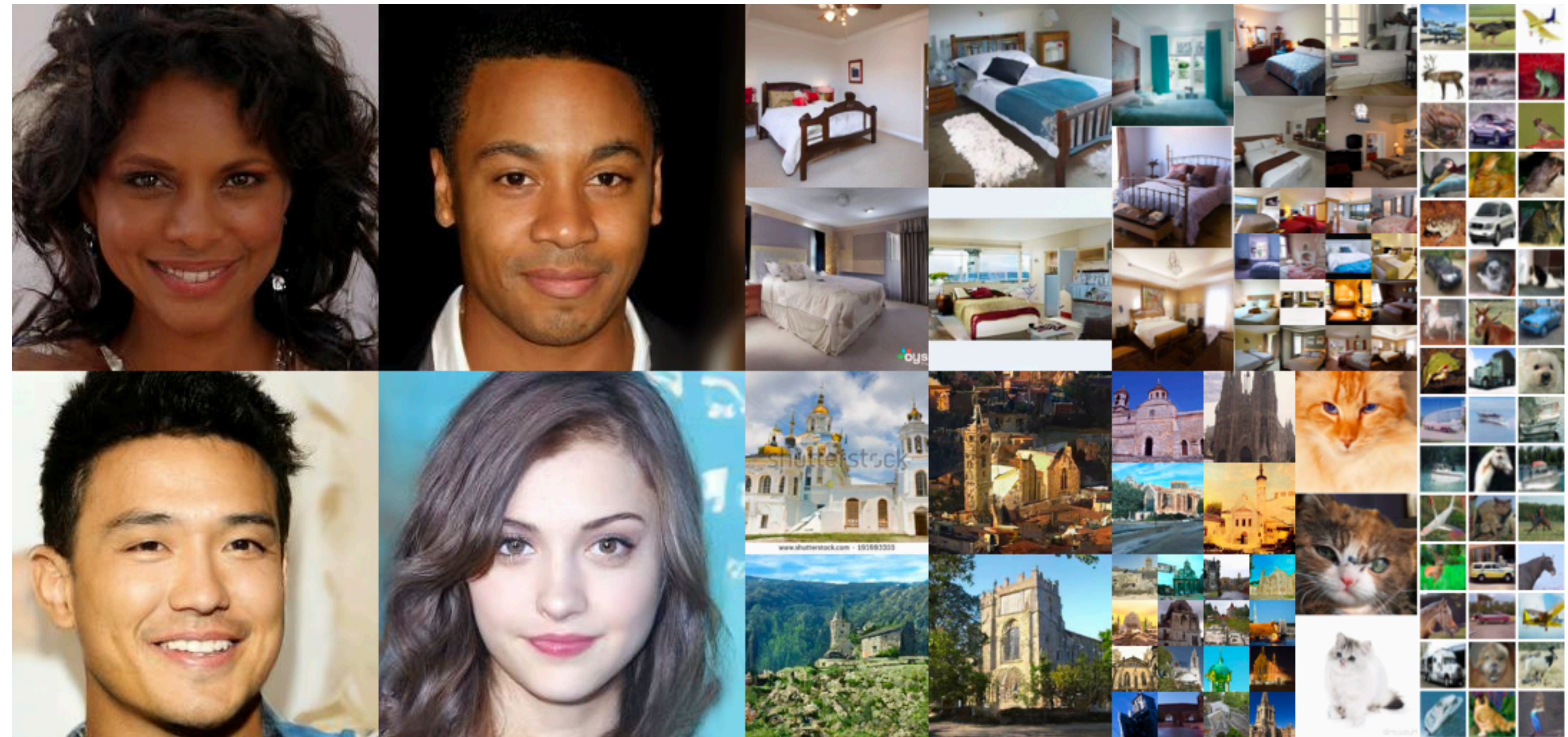
Total citations [Cited by 5829](#)



Diffusion models



[Sohl-Dickstein et al., 2015]



[Ho, et al., "Denoising Diffusion", 2020]



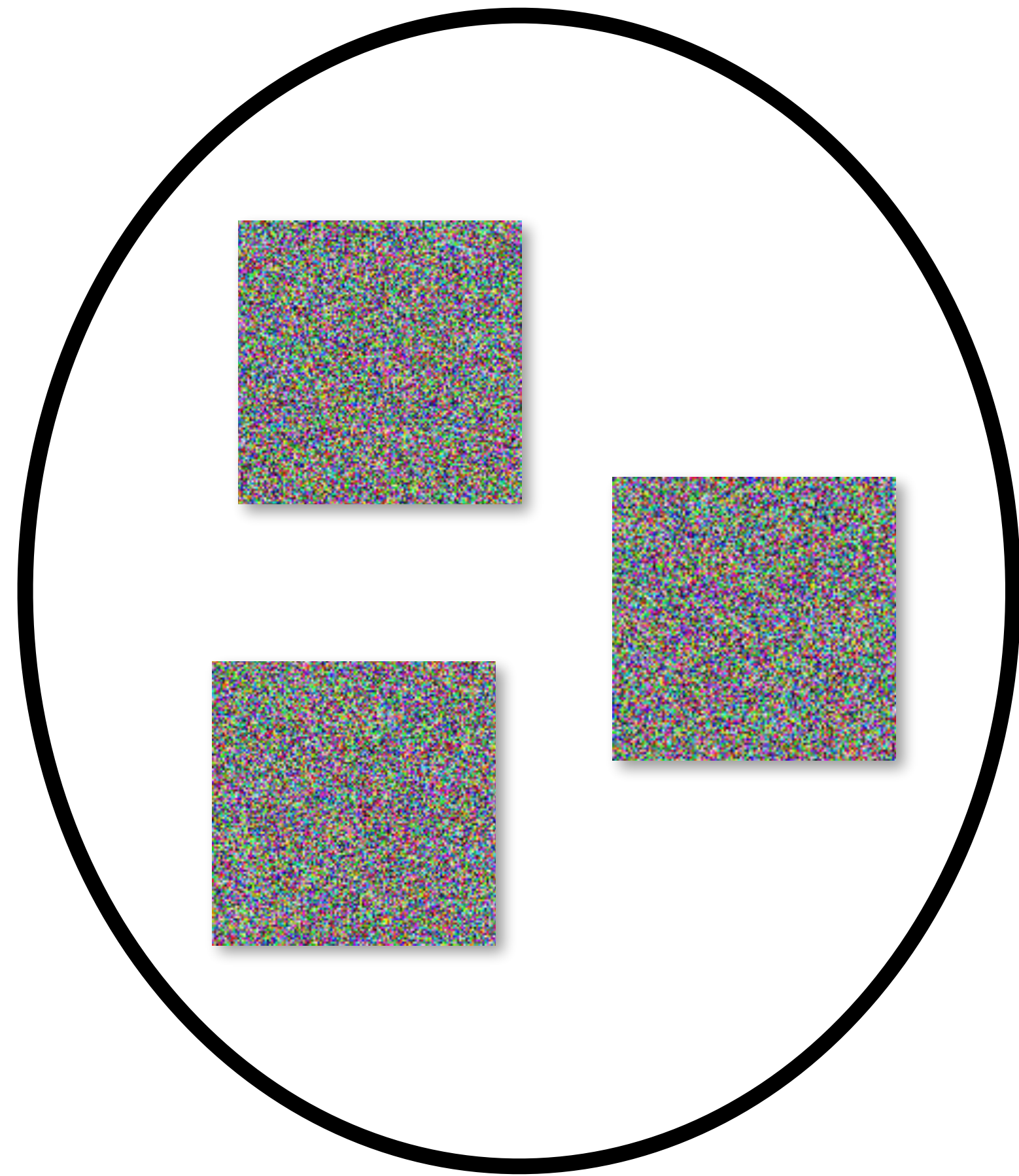
“Sprouts in the shape of text 'Imagen' coming out of a fairytale book.”



“A dragon fruit wearing karate belt in the snow.”

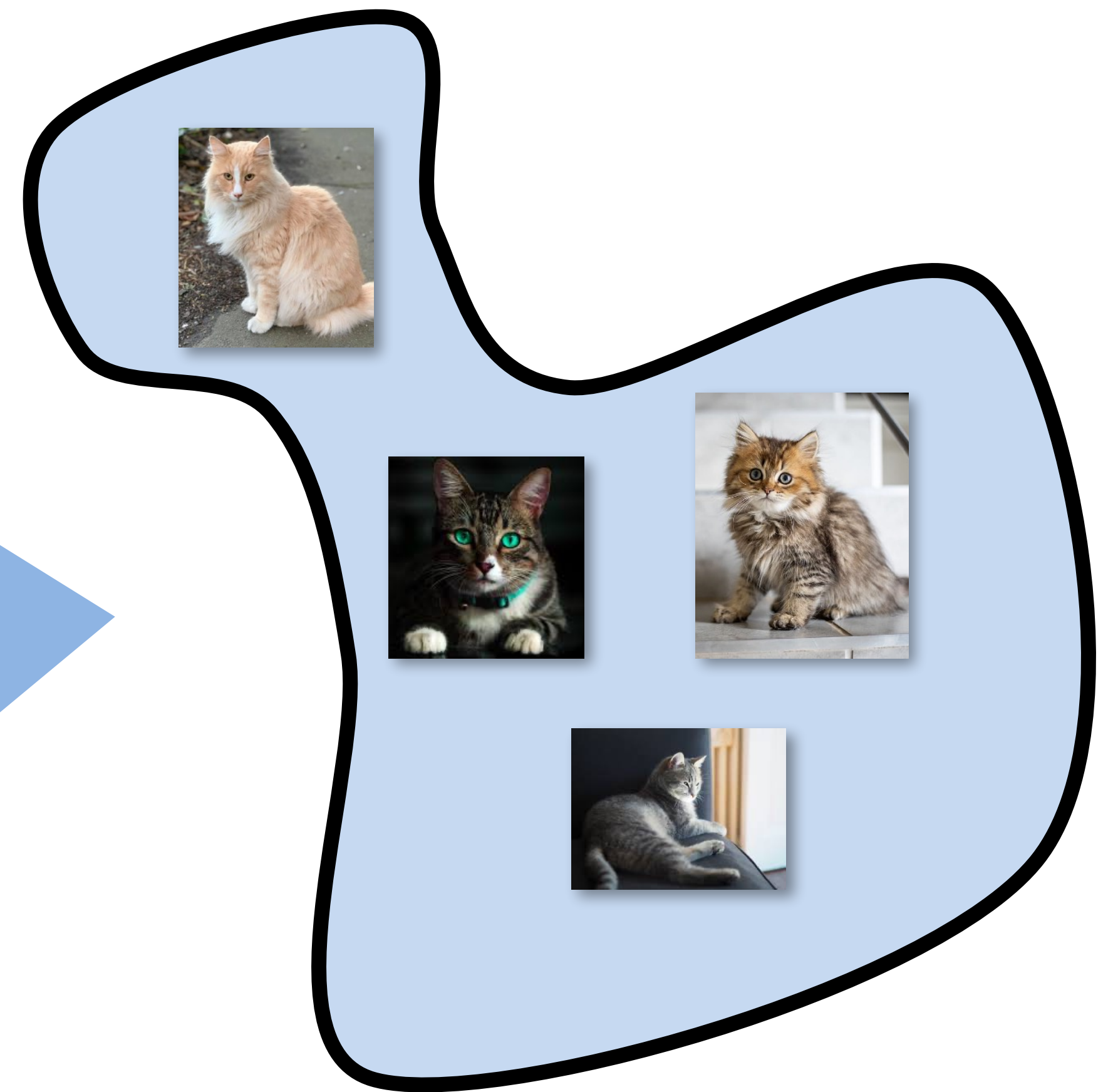


Today: Google Veo 3, 2025

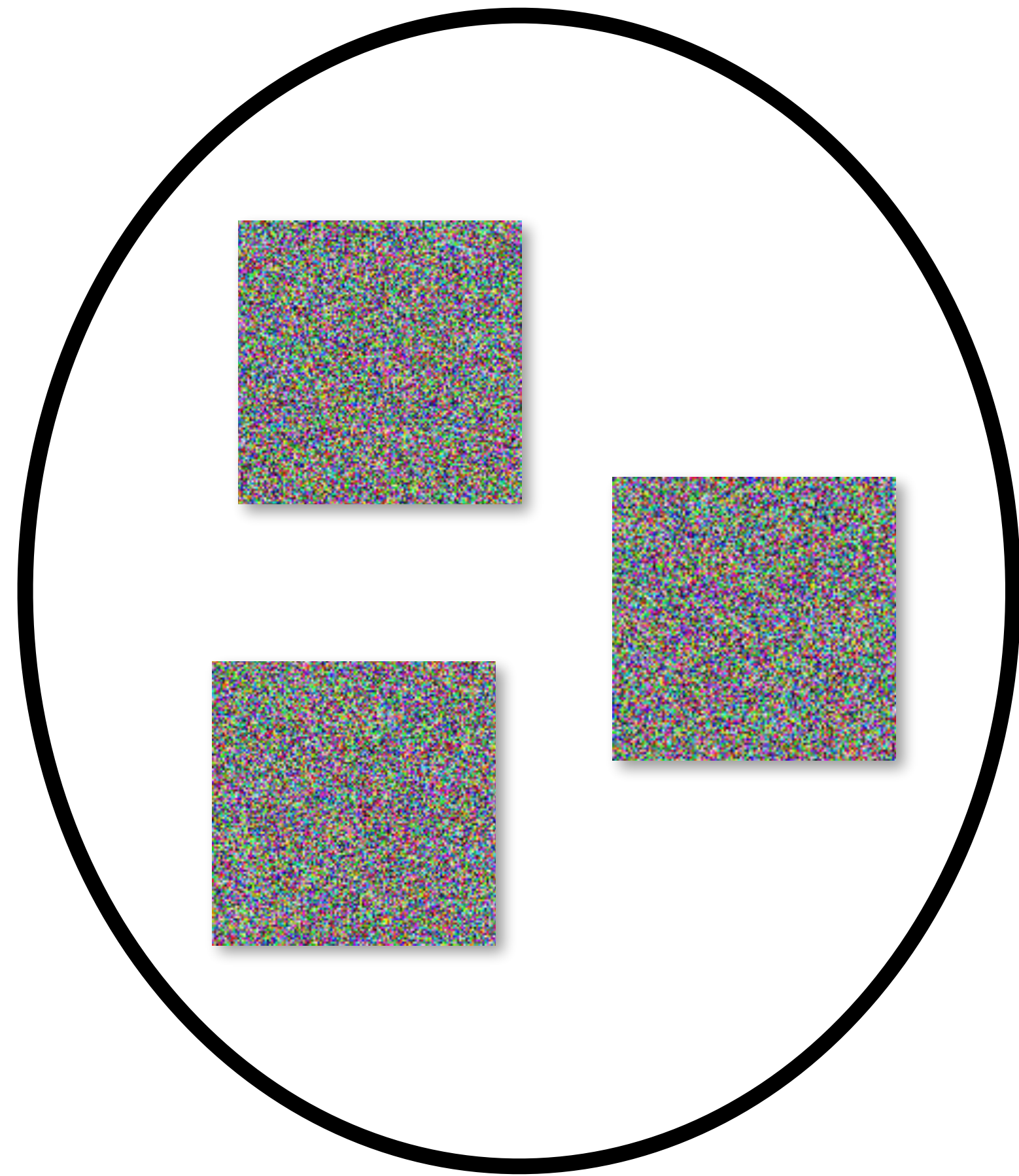


Random images

Diffusion

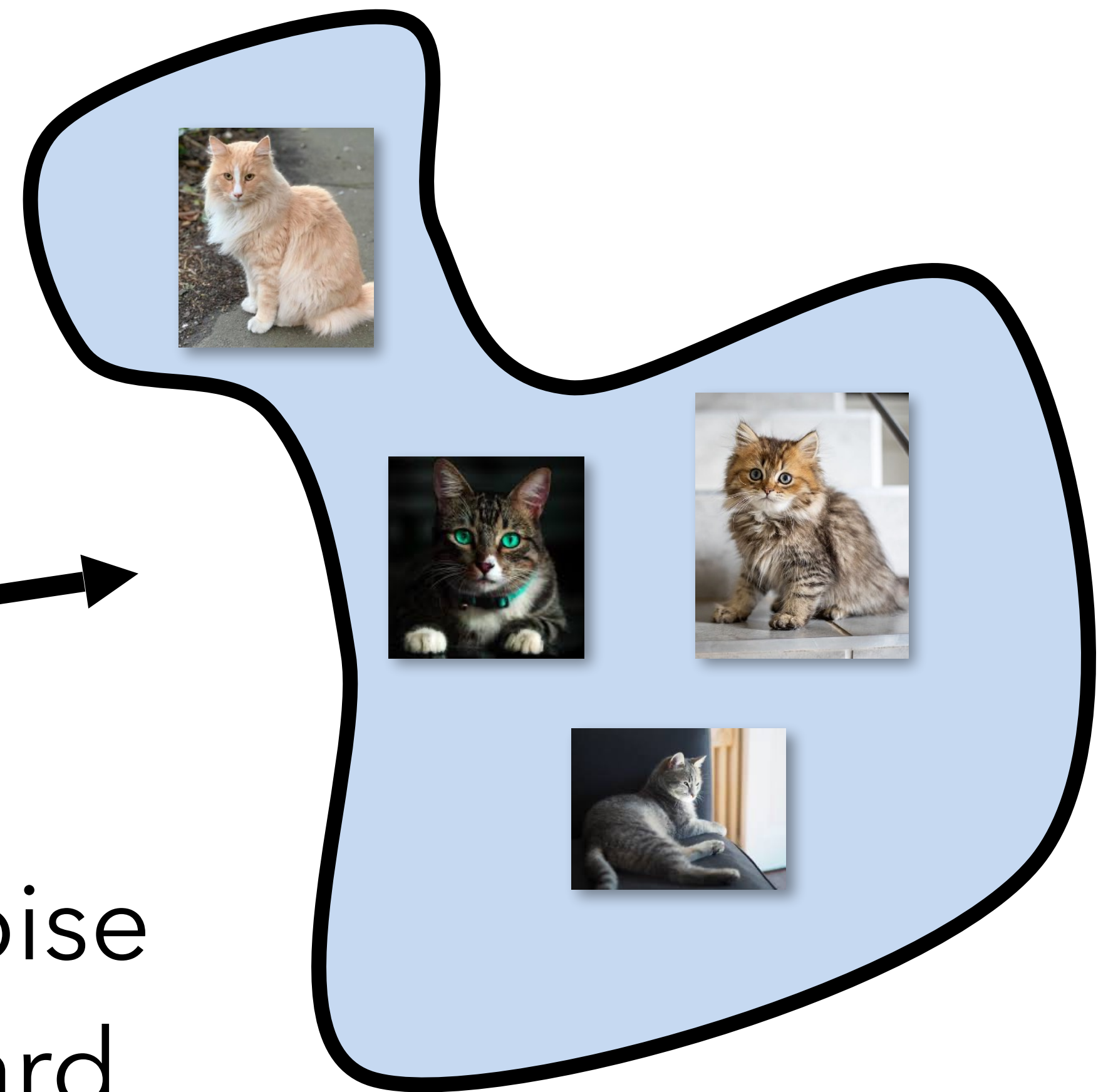


Manifold of cat images

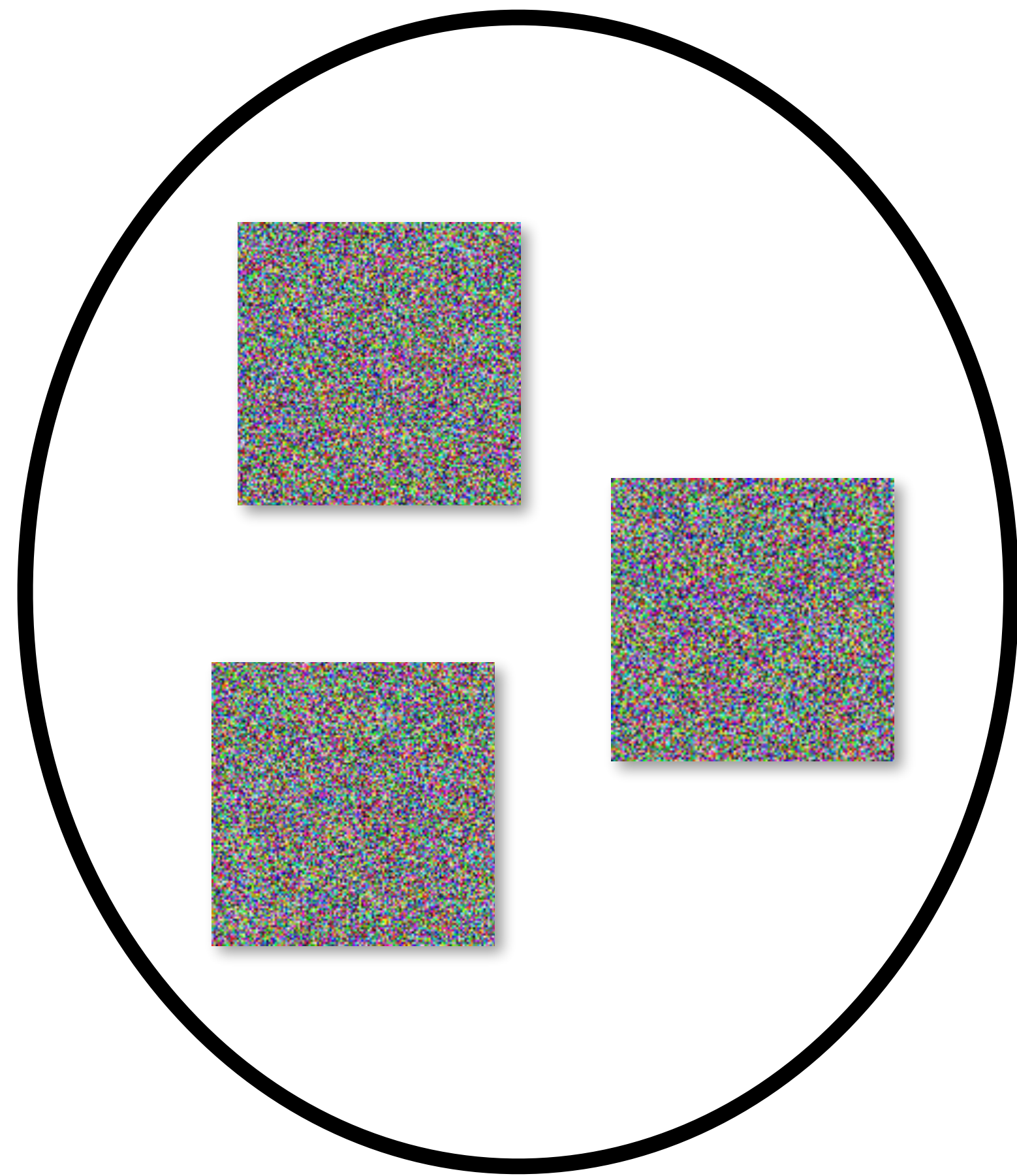


Random images

Forward
mapping (noise
to cats) is hard

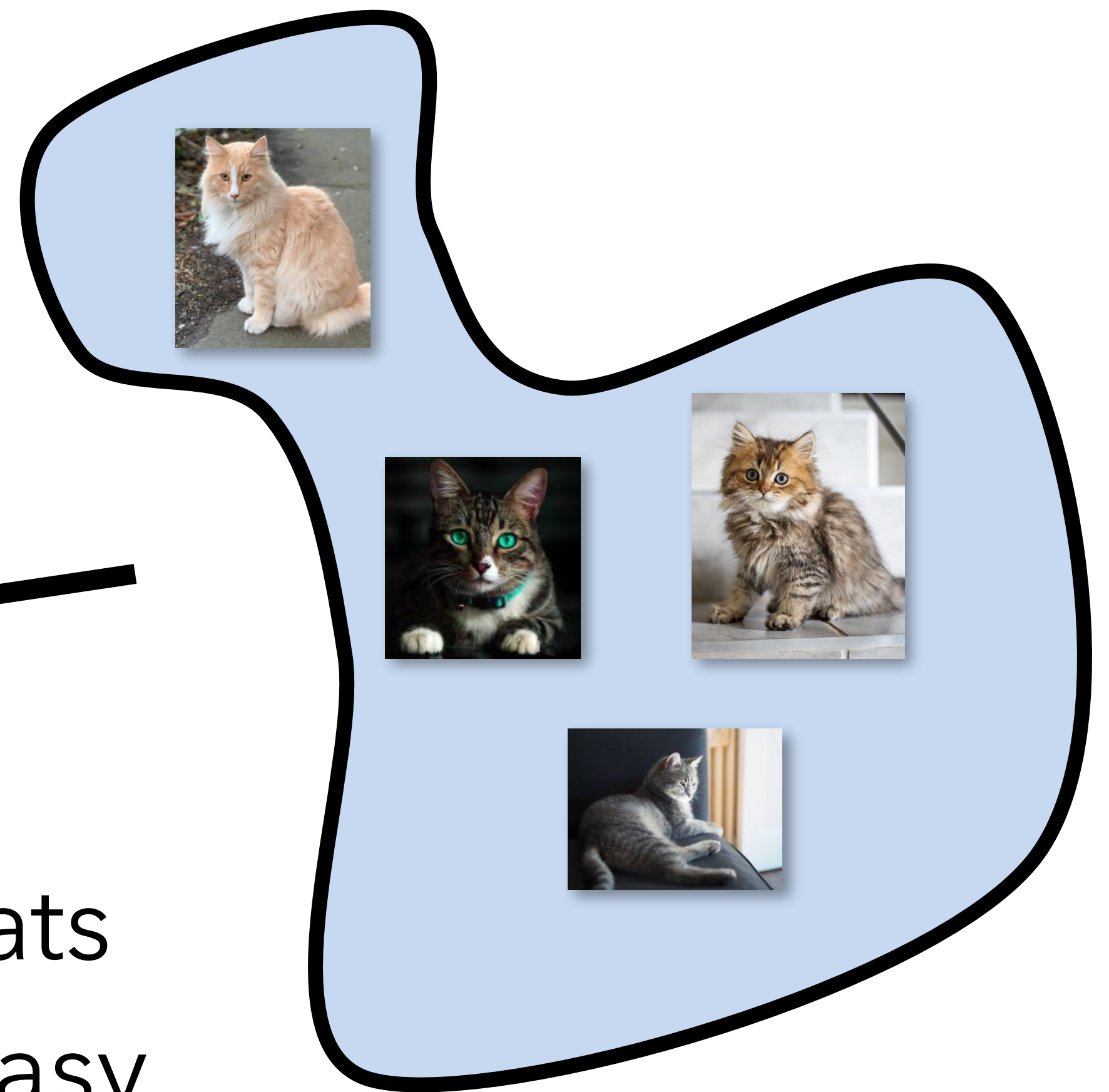


Manifold of cat images

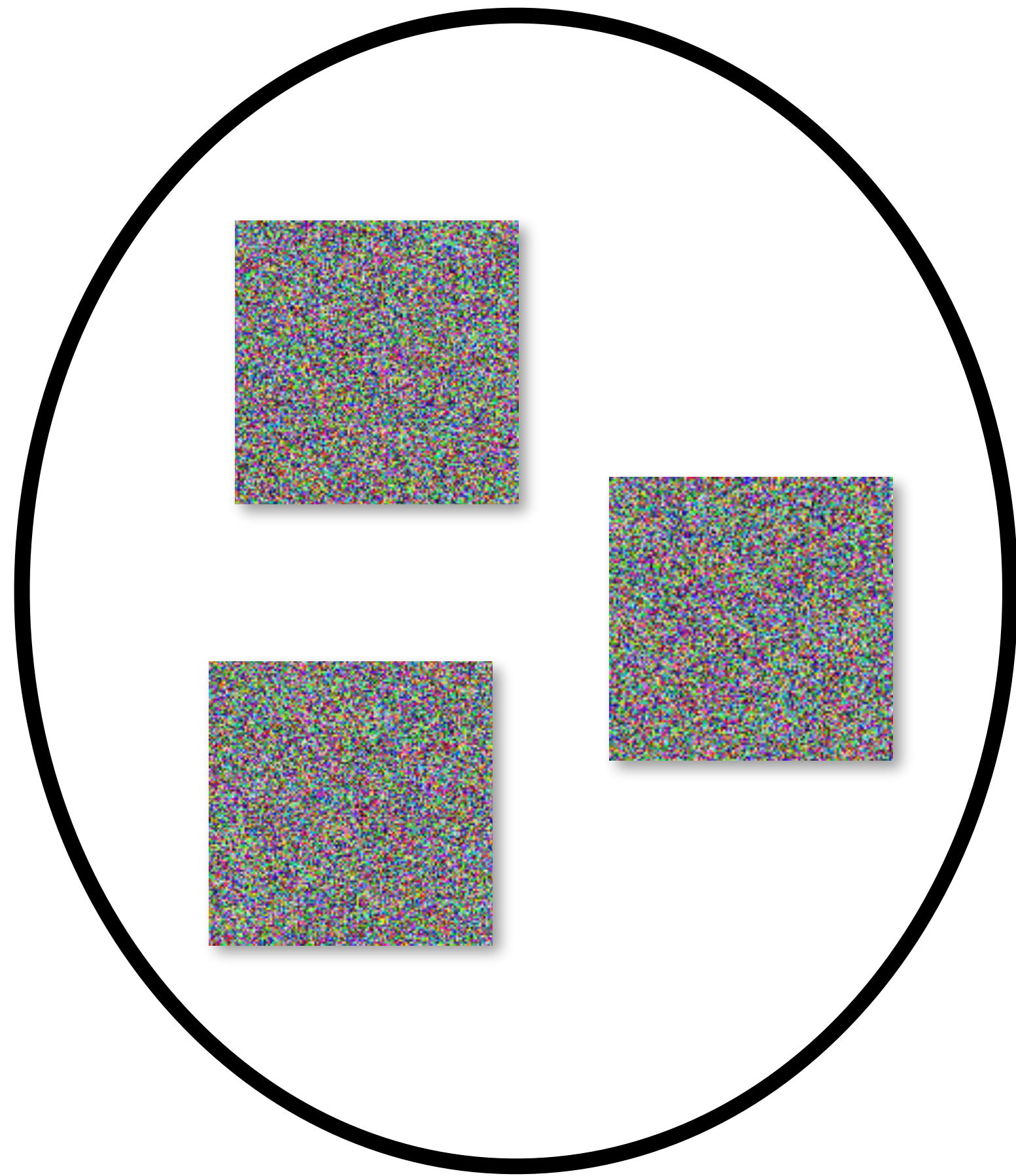


Random images

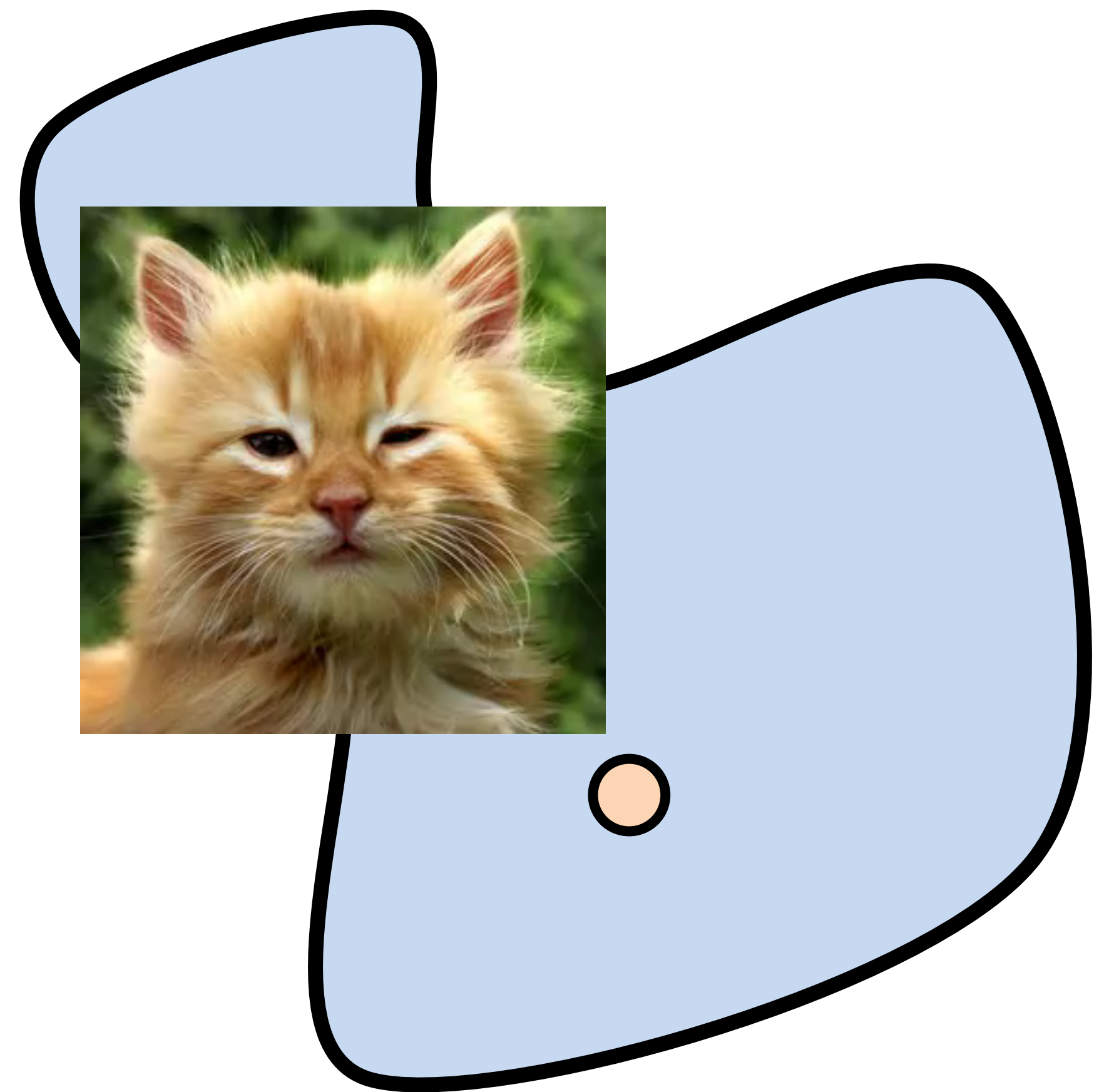
Reverse
mapping (cats
to noise) is easy



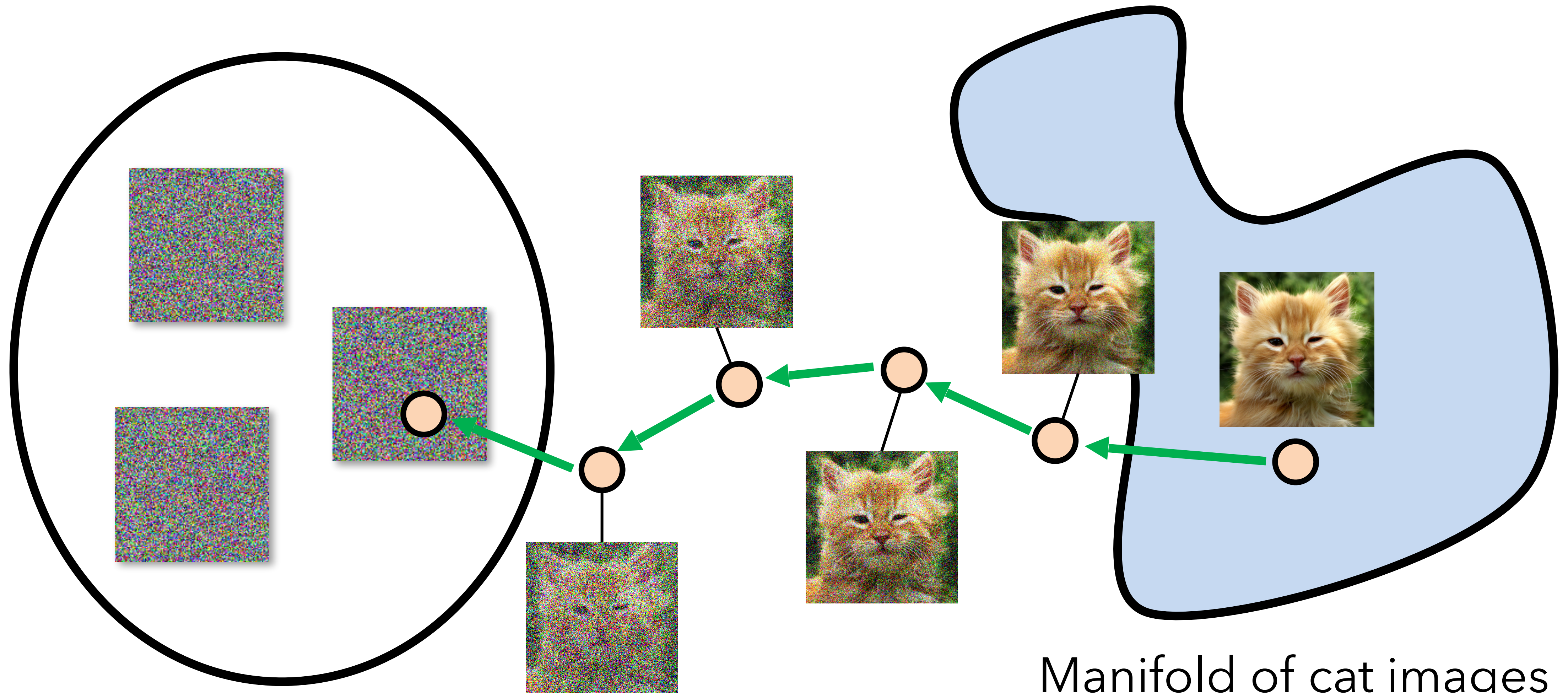
Manifold of cat images



Random images

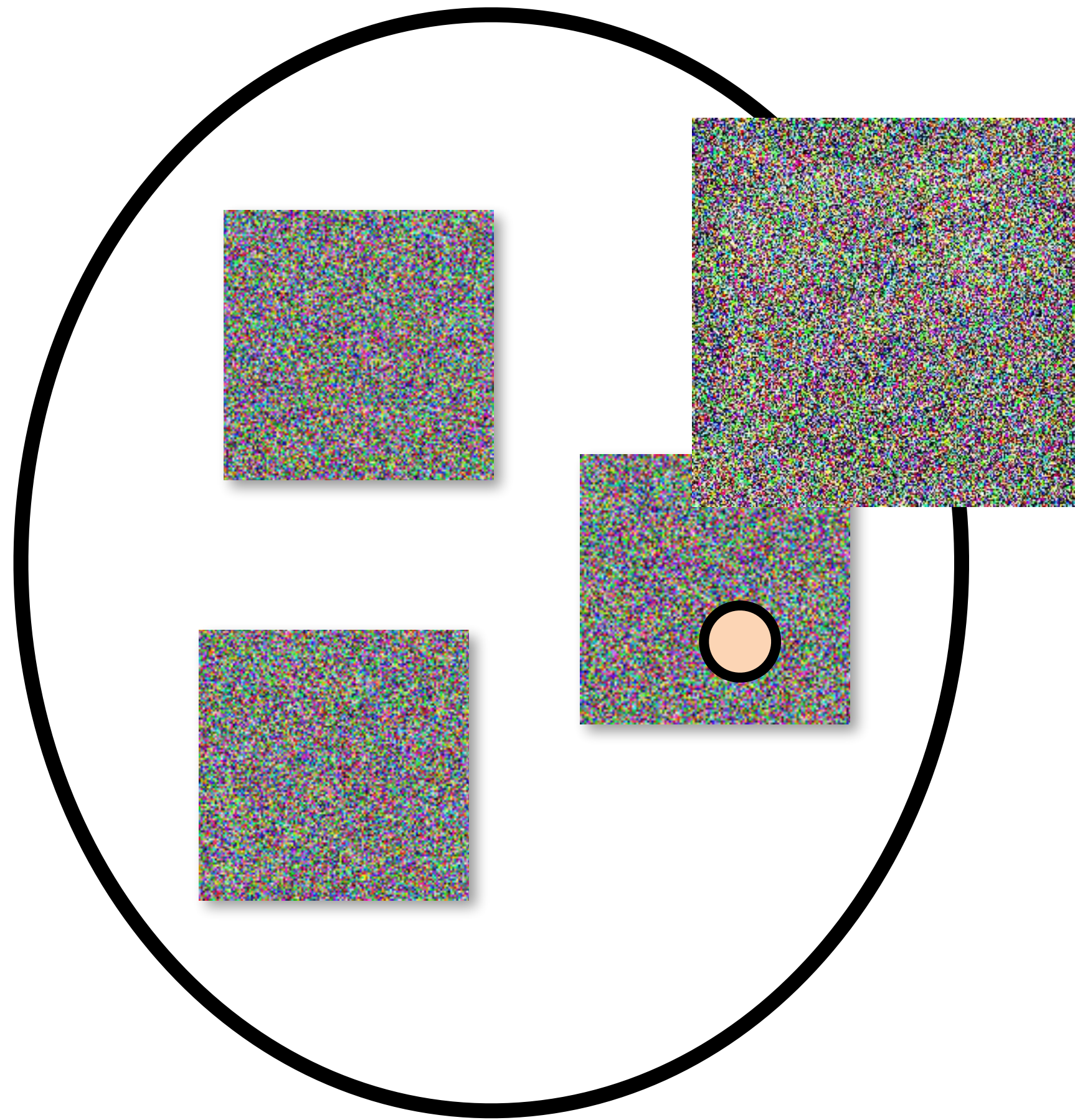


Manifold of cat images

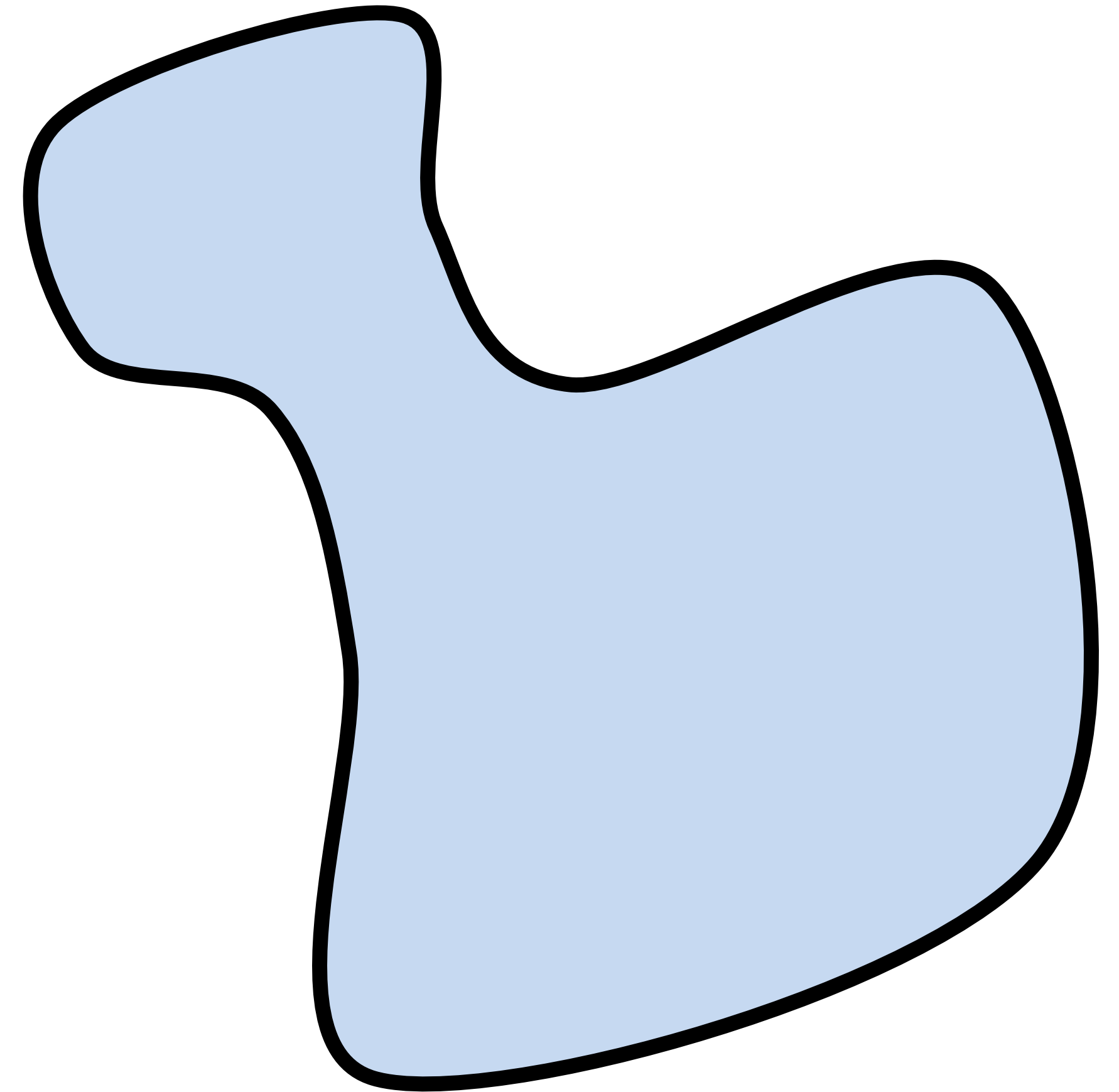


Random images

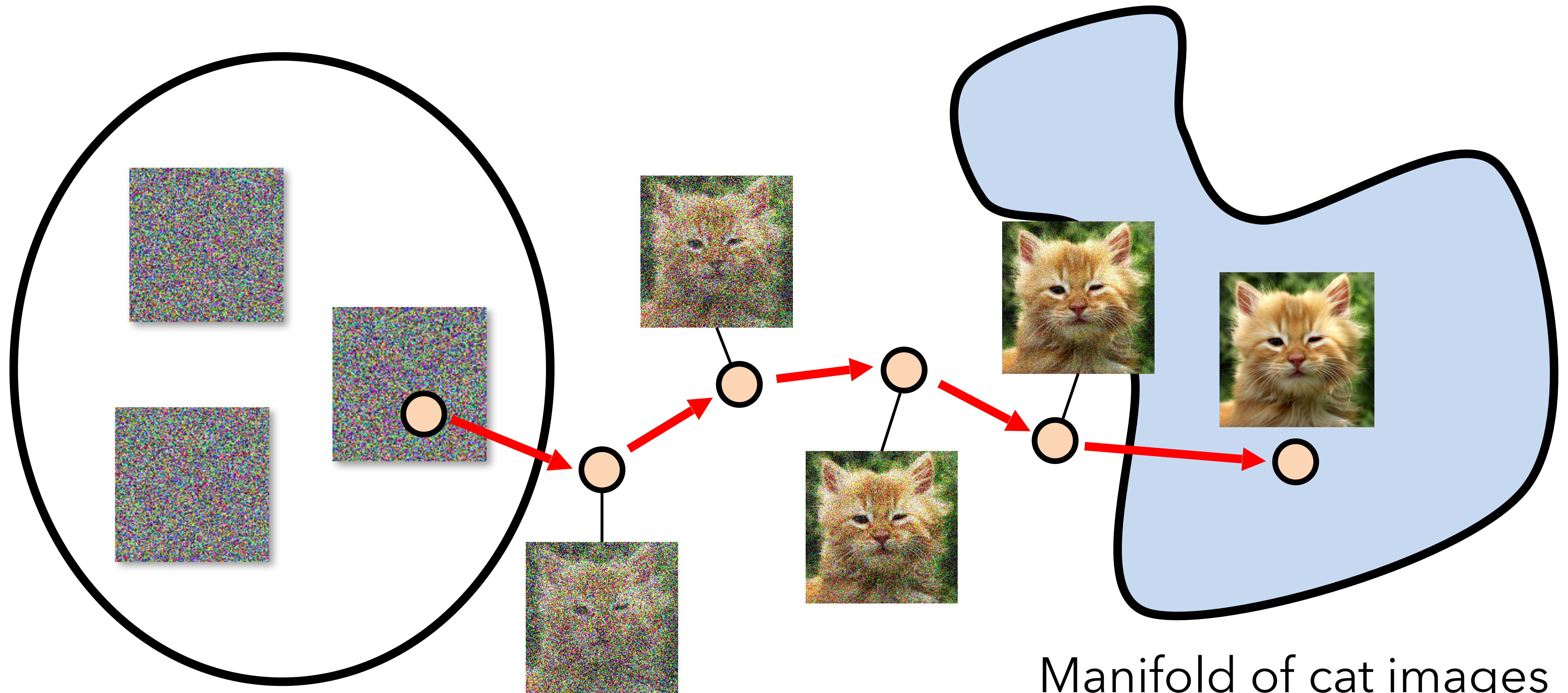
Manifold of cat images



Random images

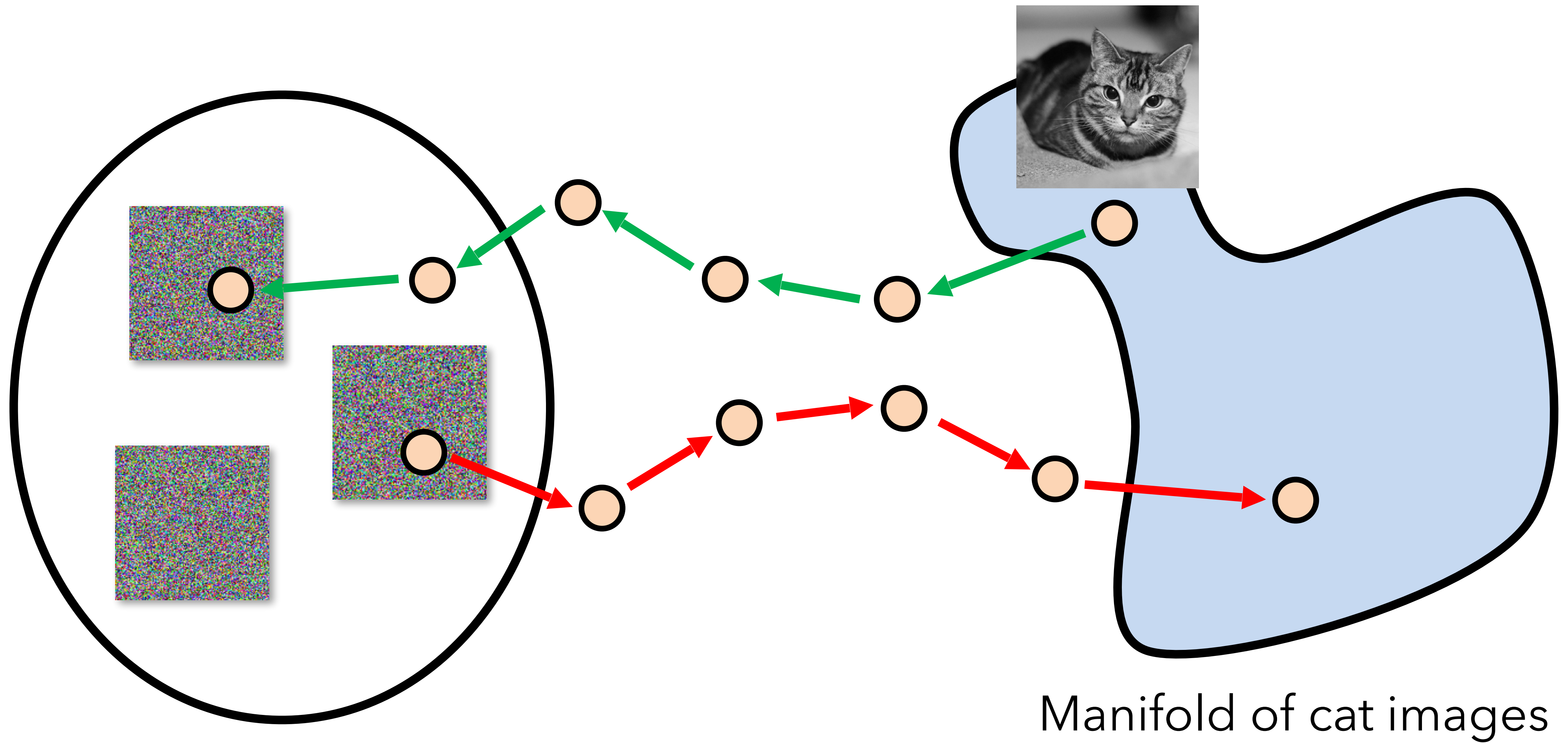


Manifold of cat images



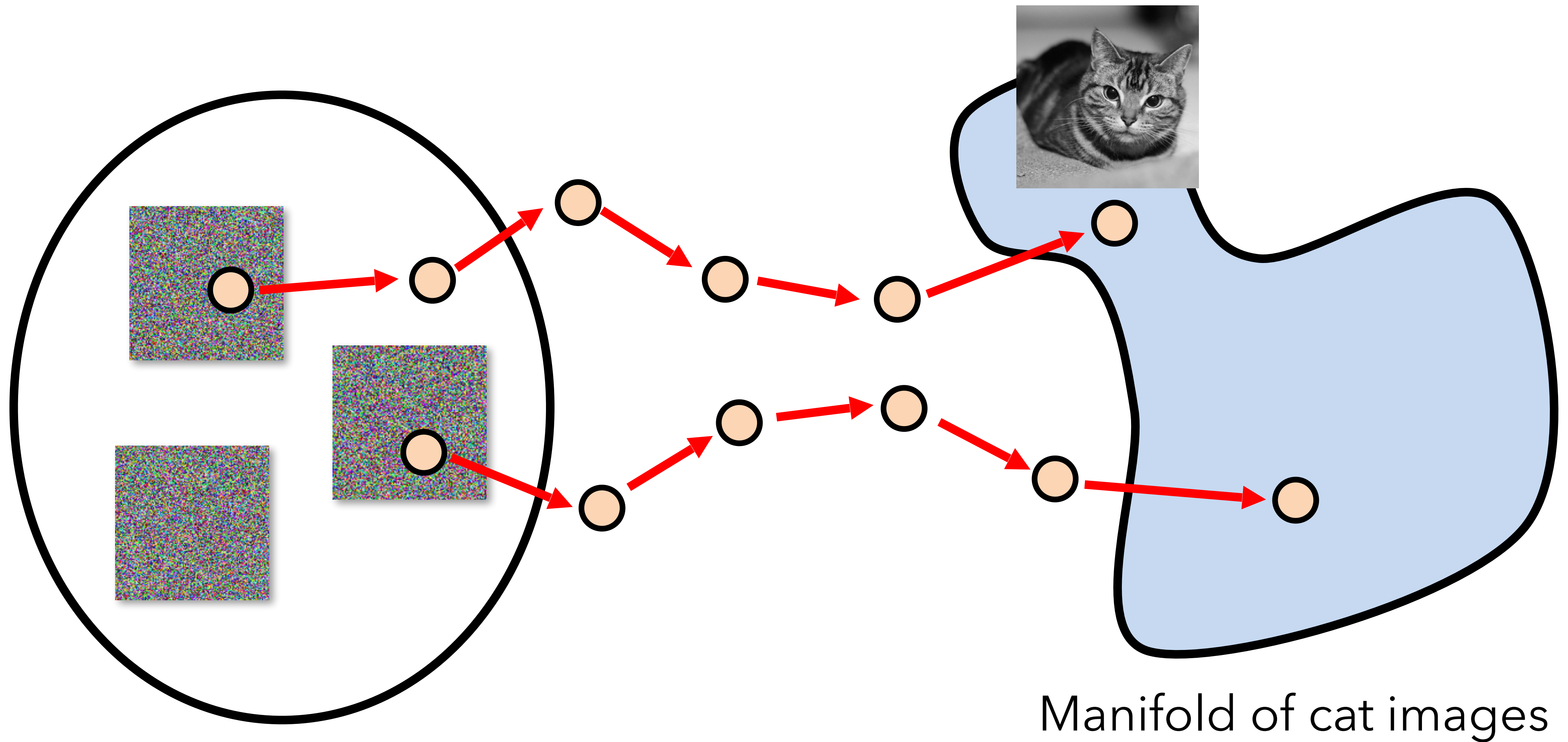
Random images

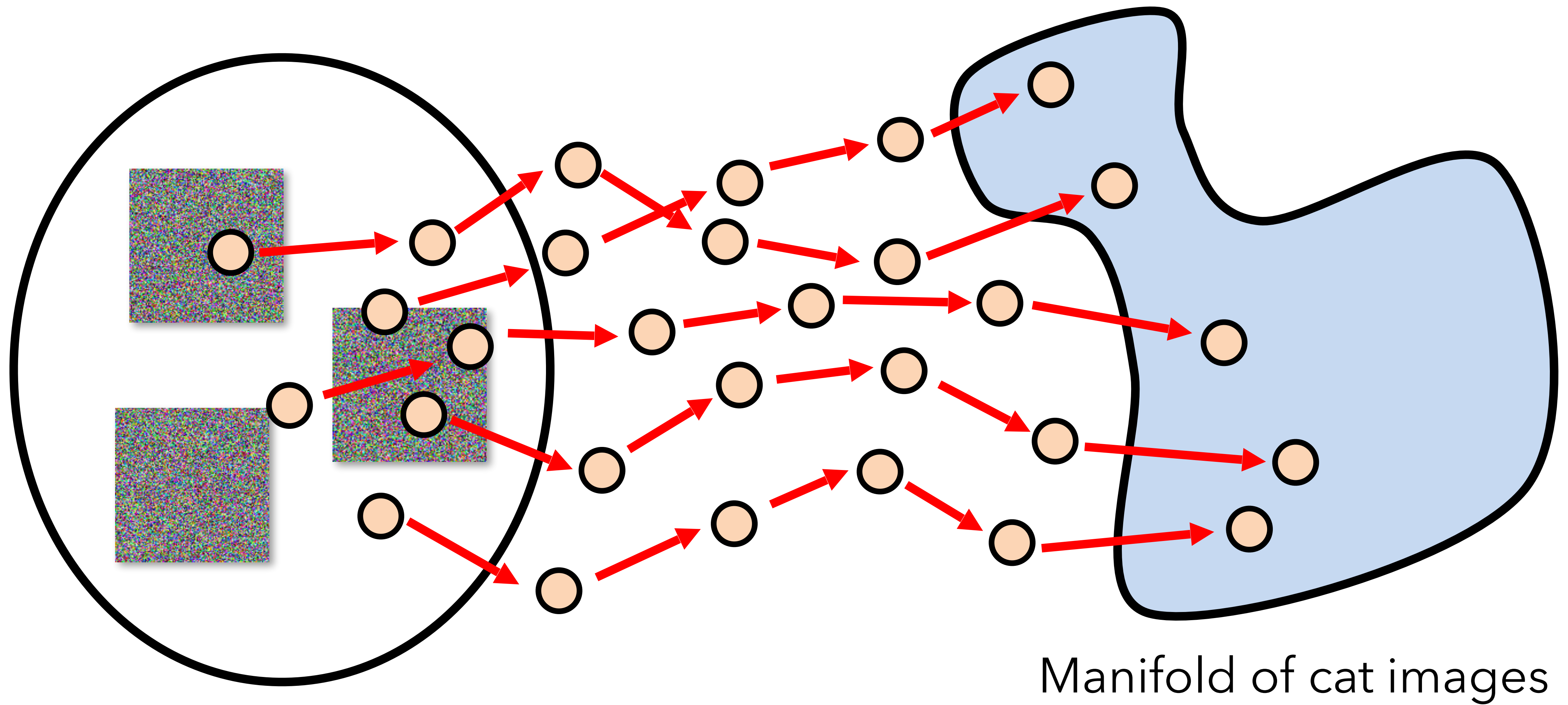
Manifold of cat images



Random images

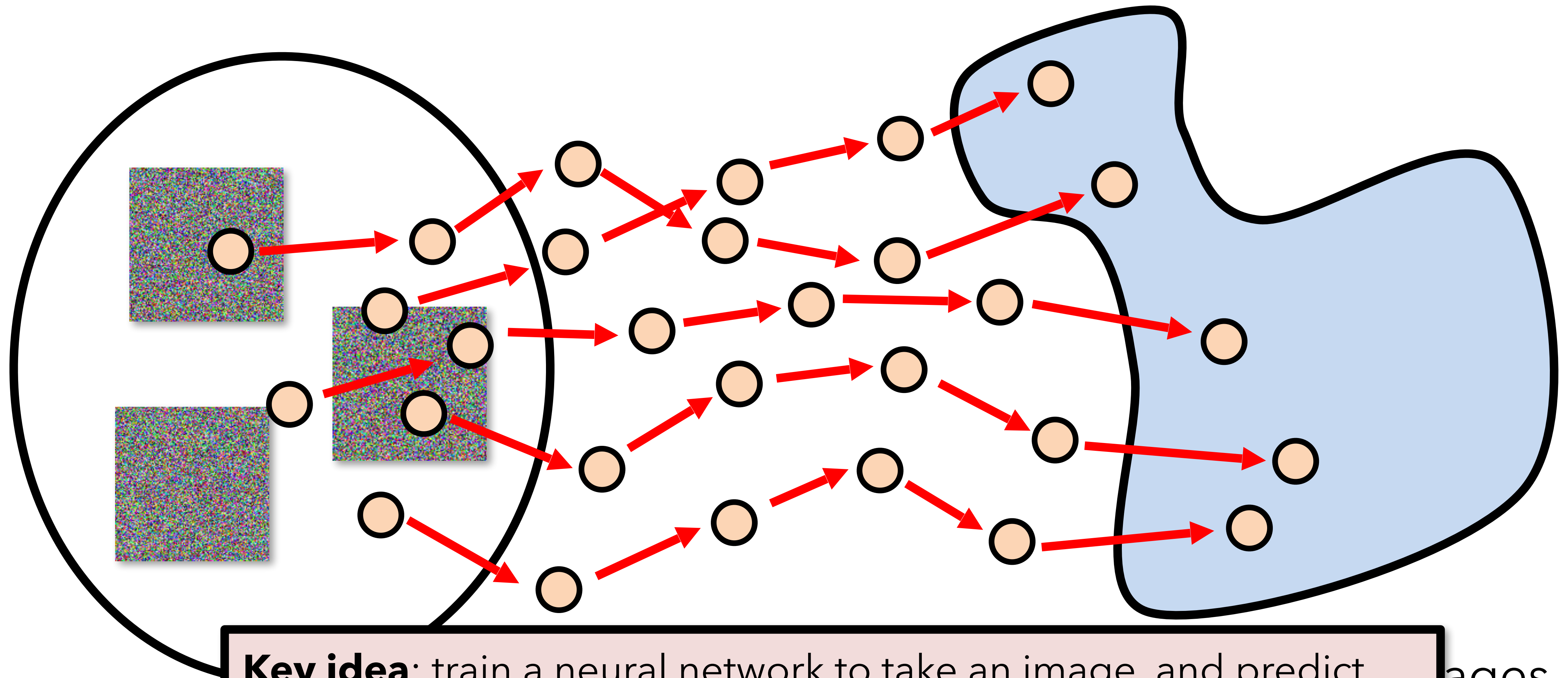
Manifold of cat images





Random images

Manifold of cat images

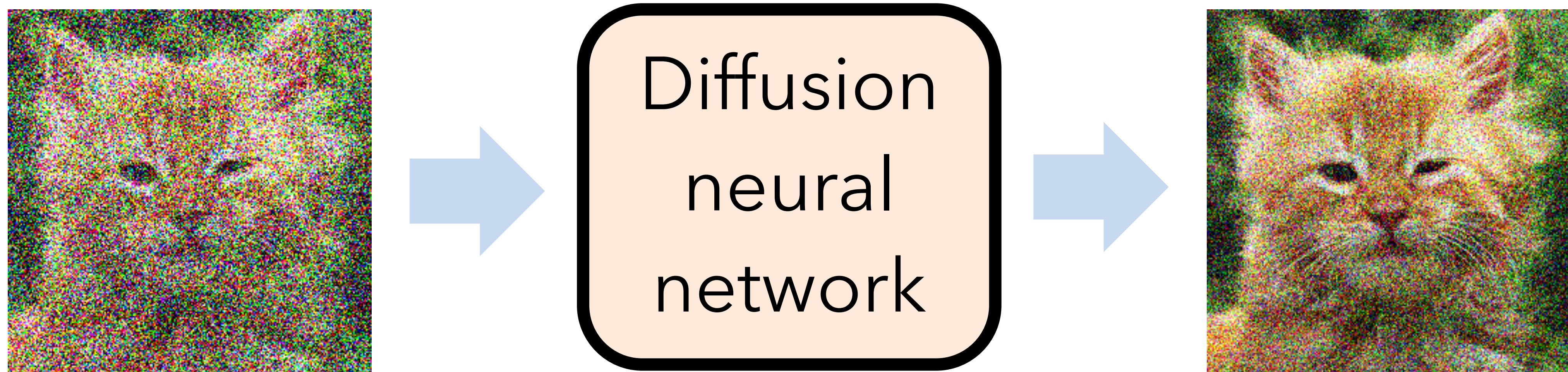


Random

Key idea: train a neural network to take an image, and predict the arrows above; that is, predict to convert a noisy image to a slightly less noisy image that is closer to the desired image manifold, using the example above to train.

images

Denoising diffusion neural network



This network can be a U-Net or other suitable image-to-image network

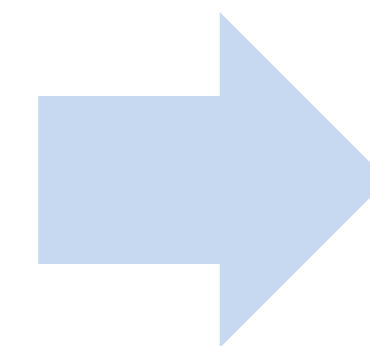
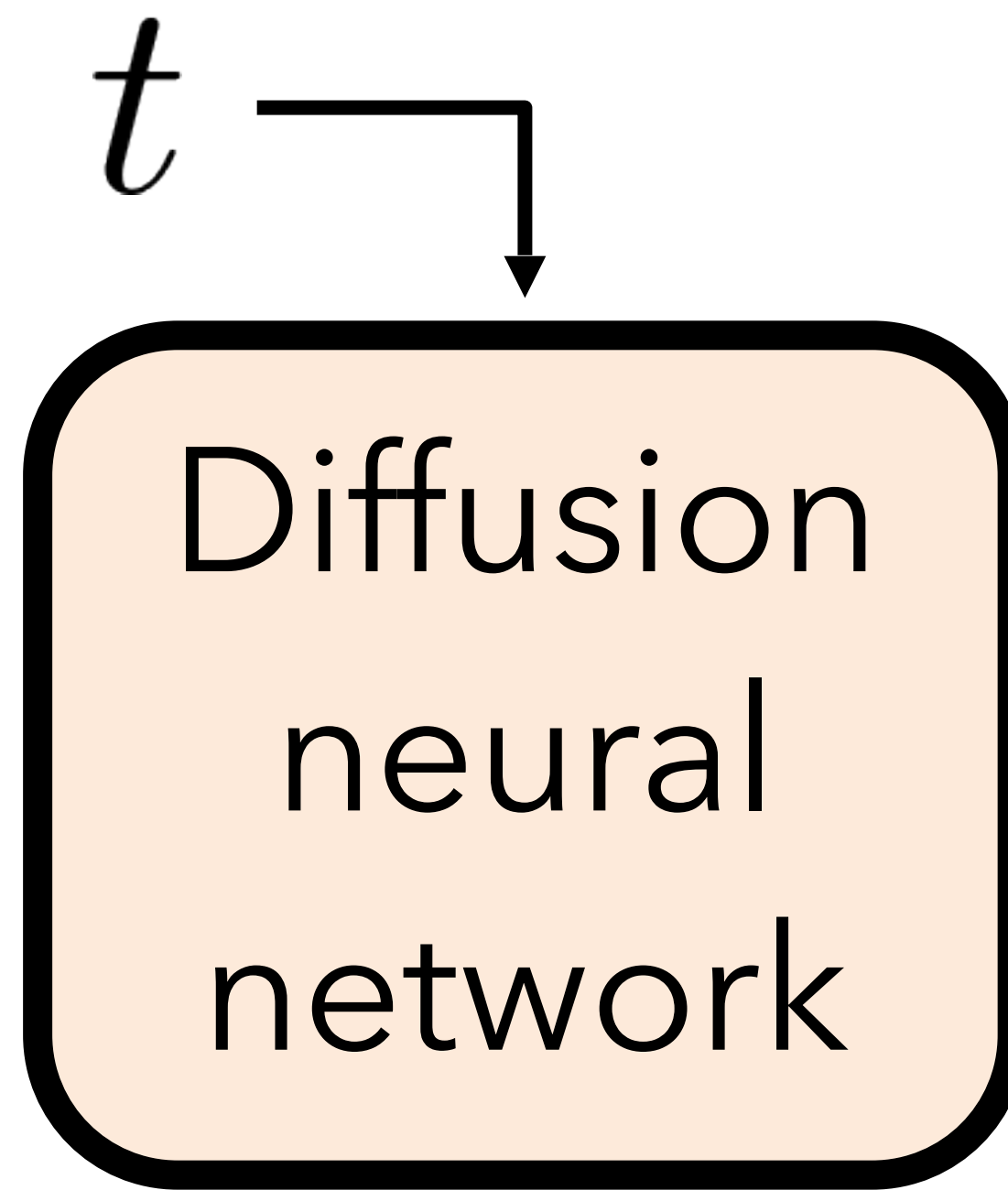
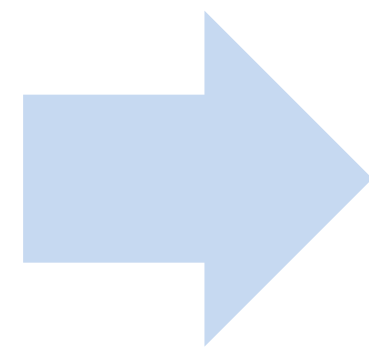
Running a diffusion model for multiple steps



Example source: Aditya Ramesh

Denoising diffusion neural network

Timestep t (from say 0 to 999) is an additional input to the network (positionally encoded)

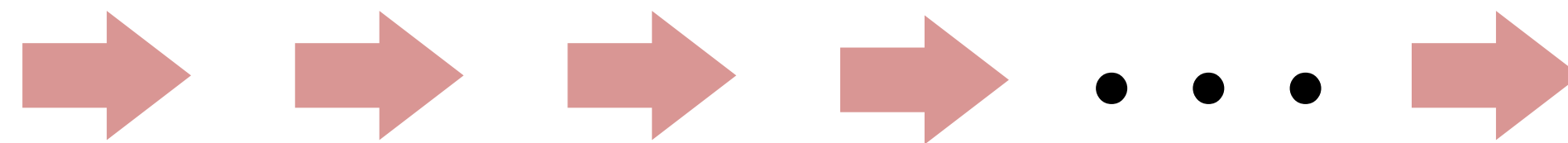


A basic diffusion approach will run this
denoising for many timesteps (e.g., $T = 1000$
steps)

Sampling many outputs



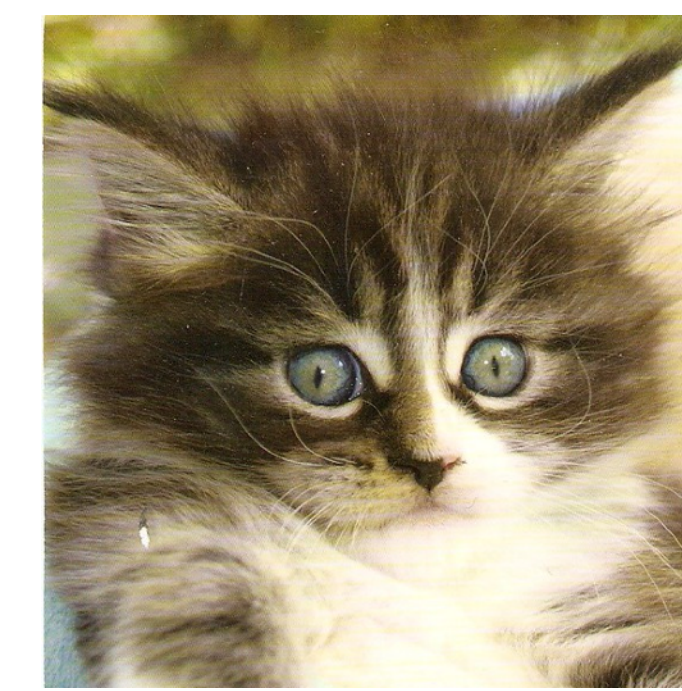
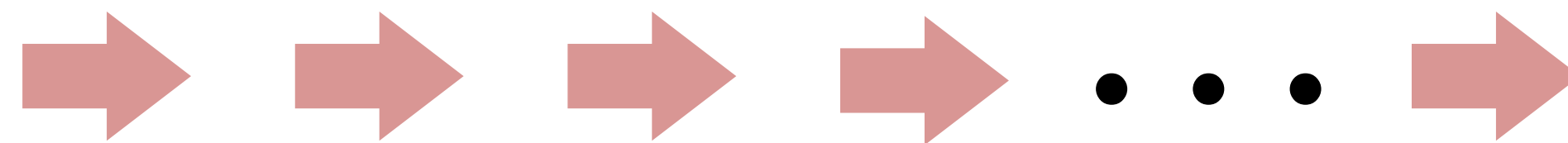
Noise image 1



Output image 1

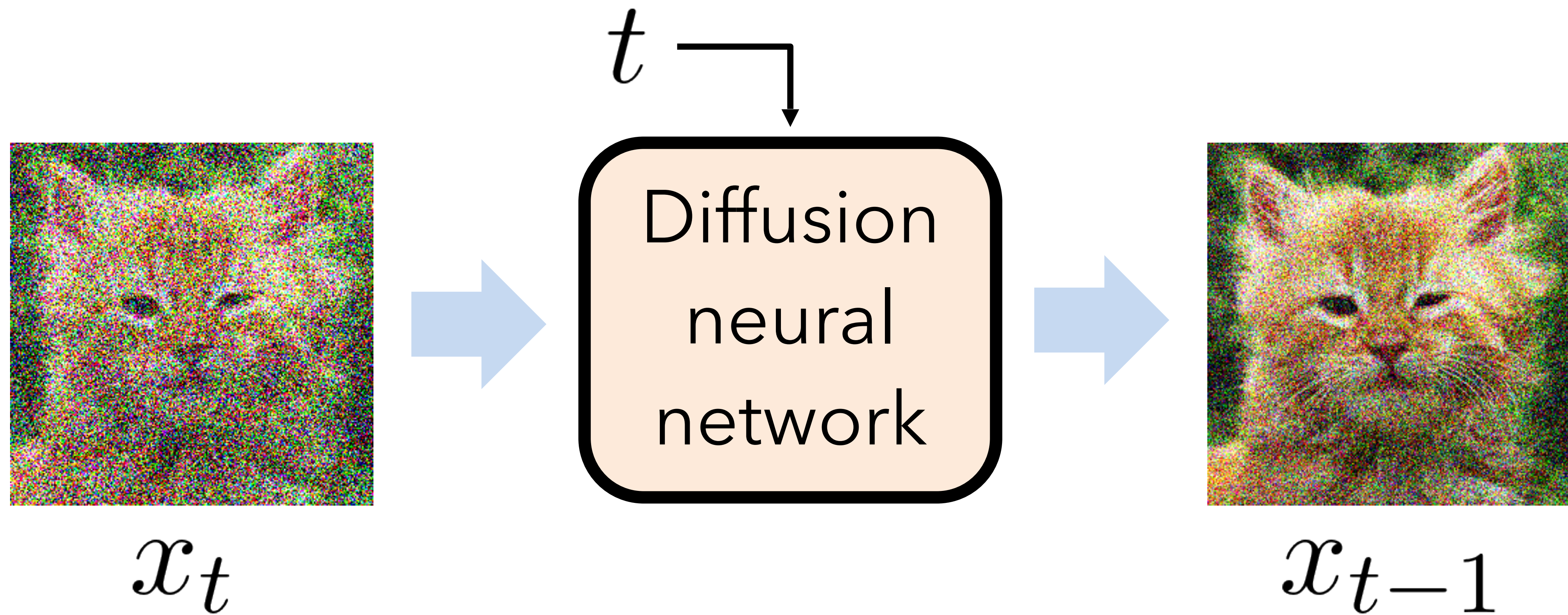


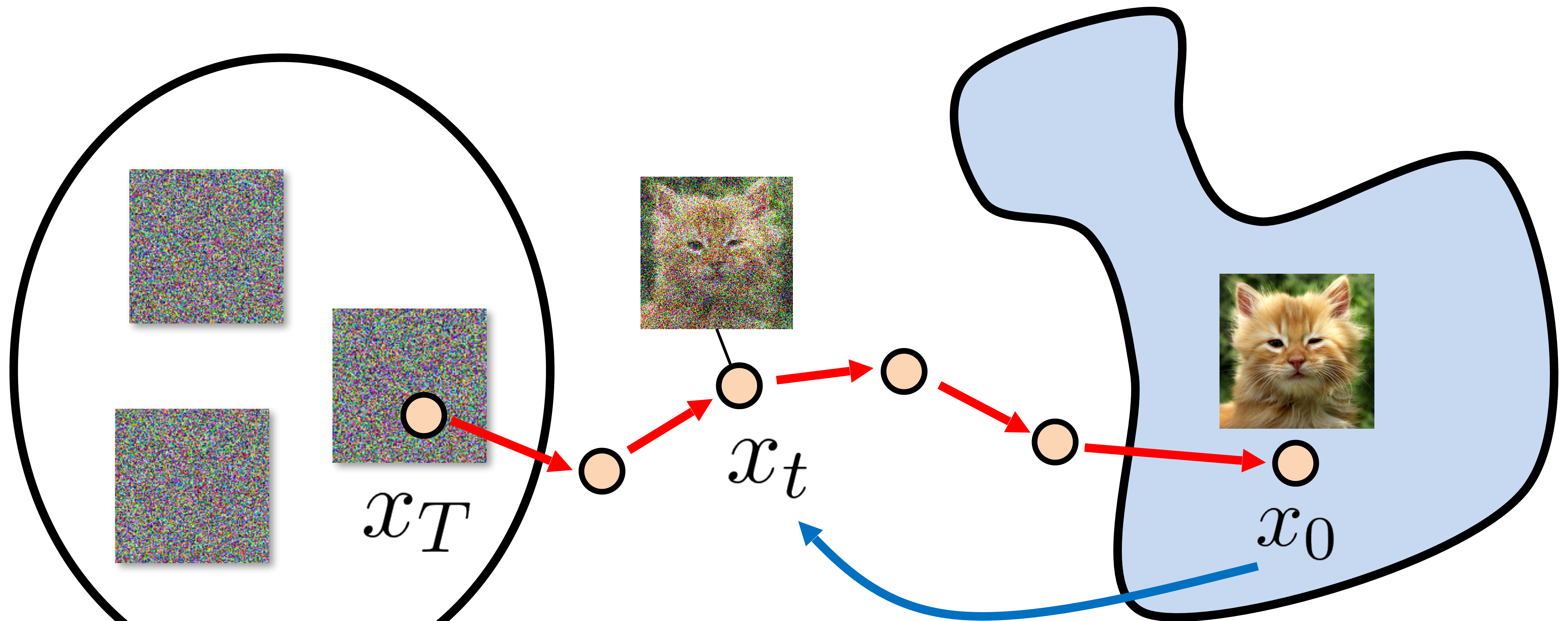
Noise image 2



Output image 2

Let's make this idea more concrete





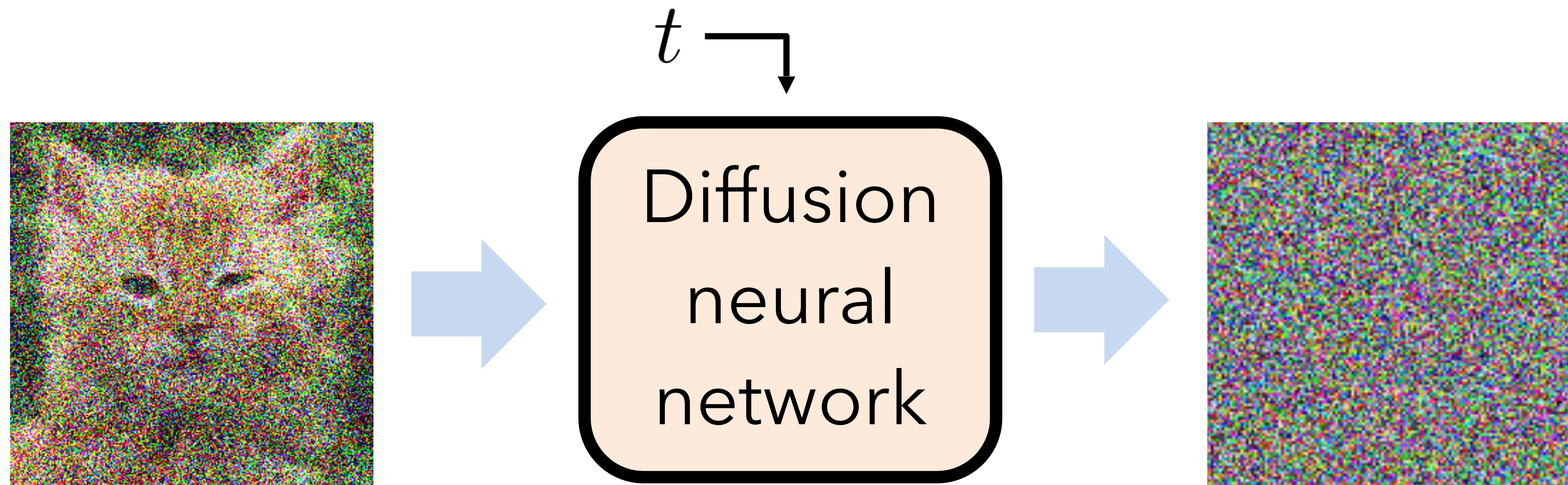
Forward process: $x_t = \sqrt{\bar{\alpha}_t} x_0 + \sqrt{1 - \bar{\alpha}_t} \epsilon$
 where $\epsilon \sim \mathcal{N}(0, 1)$

$\bar{\alpha}_t$: noise coefficients per time step (magic numbers defined by the designers)

(Gaussian noise image)

Adapted from N. Snavely

In practice, we predict the *noise*



x_t

ϵ

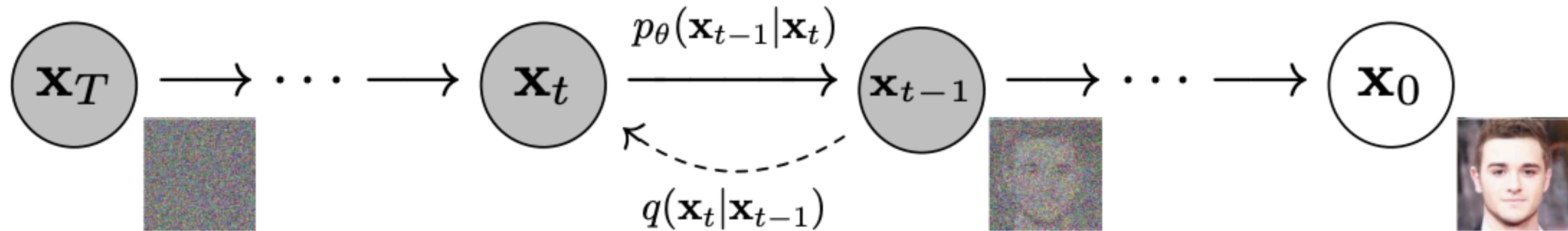
$$x_{t-1} = \frac{1}{\sqrt{\alpha_t}} x_t - \frac{1-\alpha_t}{\sqrt{\alpha_t} \sqrt{1-\bar{\alpha}_t}} \epsilon$$

x_{t-1}

$\frac{1}{\sqrt{\alpha_t}} x_t$

$-\frac{1-\alpha_t}{\sqrt{\alpha_t} \sqrt{1-\bar{\alpha}_t}} \epsilon$

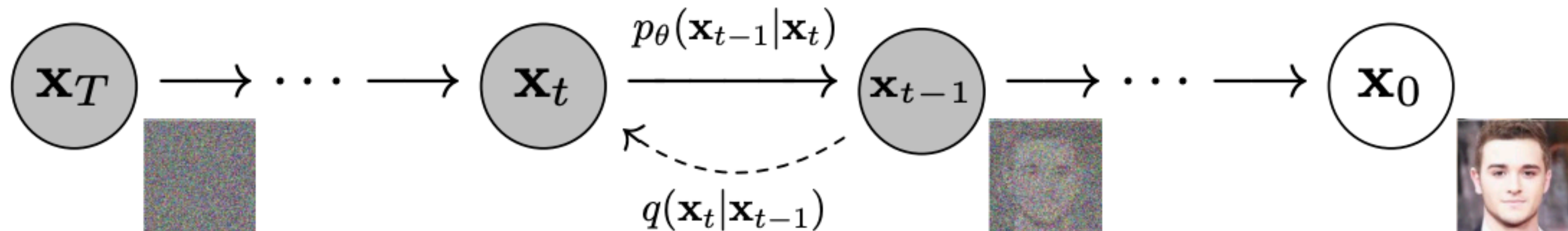
Training a diffusion model



Algorithm 1 Training

- 1: **repeat**
- 2: $\mathbf{x}_0 \sim q(\mathbf{x}_0)$
- 3: $t \sim \text{Uniform}(\{1, \dots, T\})$
- 4: $\epsilon \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$
- 5: Take gradient descent step on
$$\nabla_{\theta} \left\| \epsilon - \epsilon_{\theta}(\sqrt{\bar{\alpha}_t} \mathbf{x}_0 + \sqrt{1 - \bar{\alpha}_t} \epsilon, t) \right\|^2$$
- 6: **until** converged

Training a diffusion model



Algorithm 1 Training

1: **repeat**

2: $\mathbf{x}_0 \sim q(\mathbf{x}_0)$

3: $t \sim \text{Uniform}(\{1, \dots, T\})$

4: $\epsilon \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$

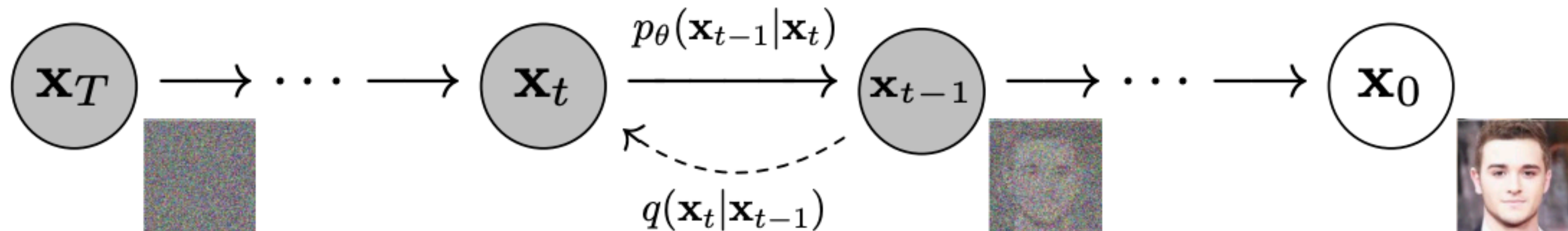
5: Take gradient descent step on

$$\nabla_{\theta} \left\| \epsilon - \epsilon_{\theta}(\sqrt{\bar{\alpha}_t} \mathbf{x}_0 + \sqrt{1 - \bar{\alpha}_t} \epsilon, t) \right\|^2$$

6: **until** converged

Sample x_t by adding noise to a clean image x_0

Training a diffusion model



Algorithm 1 Training

1: **repeat**

2: $\mathbf{x}_0 \sim q(\mathbf{x}_0)$

3: $t \sim \text{Uniform}(\{1, \dots, T\})$

4: $\epsilon \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$

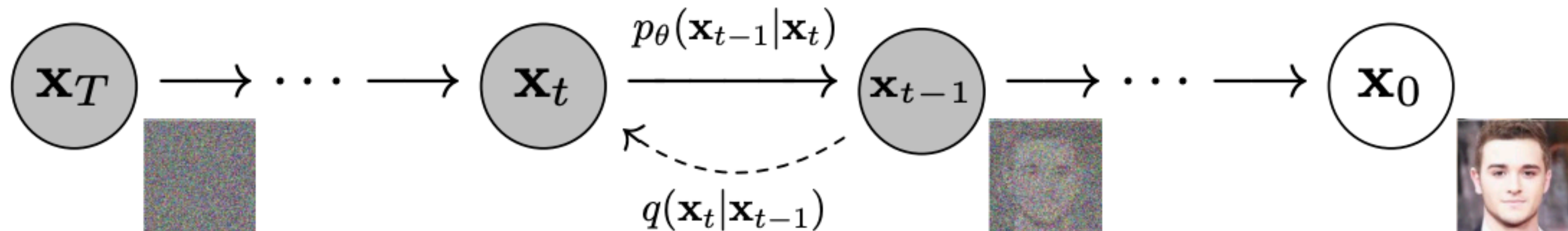
5: Take gradient descent step on

$$\nabla_{\theta} \left\| \epsilon - \epsilon_{\theta}(\sqrt{\bar{\alpha}_t} \mathbf{x}_0 + \sqrt{1 - \bar{\alpha}_t} \epsilon, t) \right\|^2$$

6: **until** converged

Random noise

Training a diffusion model

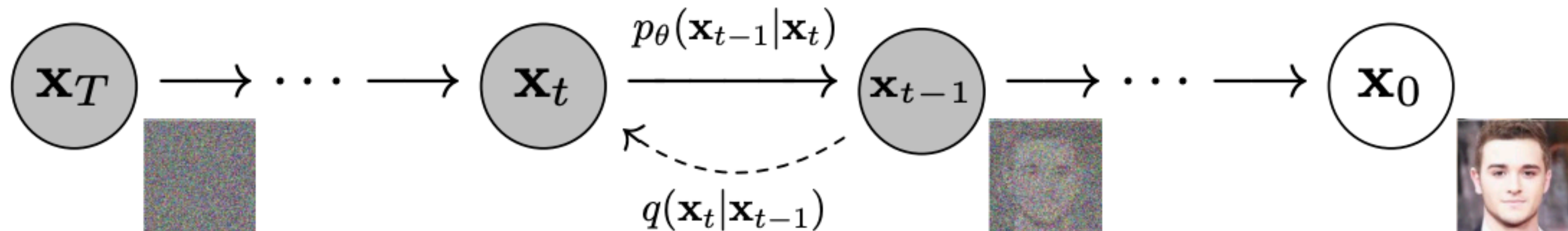


Algorithm 1 Training

- 1: **repeat**
- 2: $\mathbf{x}_0 \sim q(\mathbf{x}_0)$
- 3: $t \sim \text{Uniform}(\{1, \dots, T\})$
- 4: $\epsilon \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$
- 5: Take gradient descent step on
$$\nabla_\theta \left\| \epsilon - \epsilon_\theta(\sqrt{\bar{\alpha}_t} \mathbf{x}_0 + \sqrt{1 - \bar{\alpha}_t} \epsilon, t) \right\|^2$$
- 6: **until** converged

Noisy image

Training a diffusion model



Algorithm 1 Training

1: **repeat**

2: $\mathbf{x}_0 \sim q(\mathbf{x}_0)$

3: $t \sim \text{Uniform}(\{1, \dots, T\})$

Diffusion model ϵ_θ predicts the noise ϵ

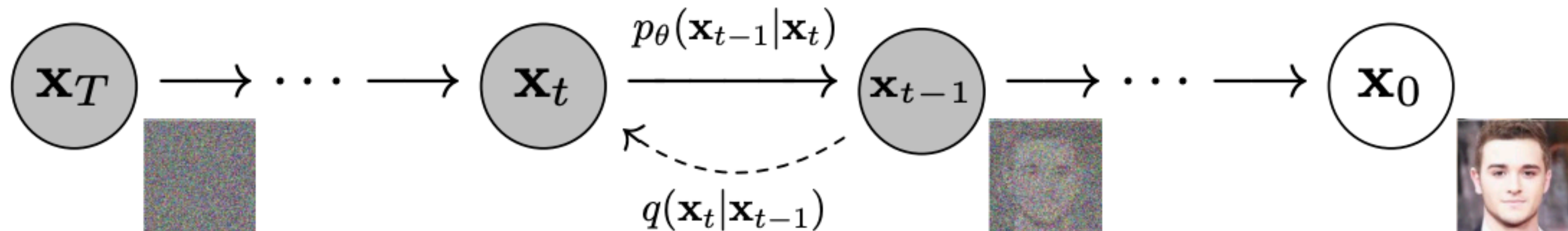
4: $\epsilon \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$

5: Take gradient descent step on

$$\nabla_\theta \left\| \epsilon - \epsilon_\theta(\sqrt{\bar{\alpha}_t} \mathbf{x}_0 + \sqrt{1 - \bar{\alpha}_t} \epsilon, t) \right\|^2$$

6: **until** converged

Training a diffusion model



Algorithm 1 Training

- 1: **repeat**
- 2: $\mathbf{x}_0 \sim q(\mathbf{x}_0)$
- 3: $t \sim \text{Uniform}(\{1, \dots, T\})$
- 4: $\epsilon \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$

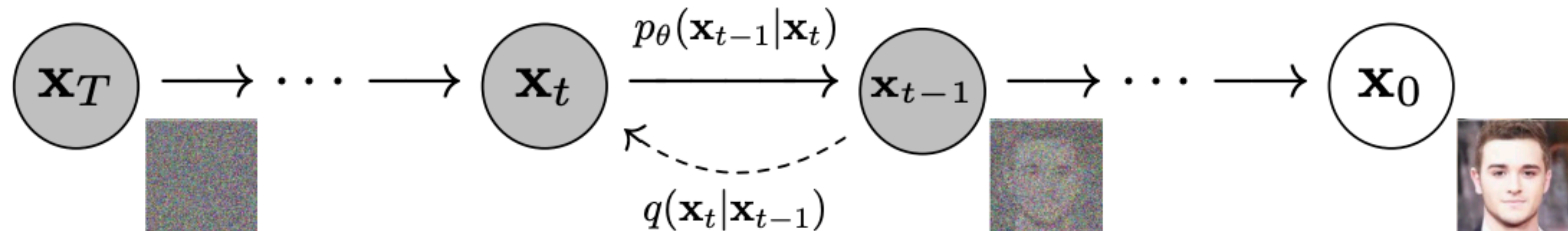
Make the diffusion model better
at denoising this image!

- 5: Take gradient descent step on

$$\nabla_{\theta} \left\| \epsilon - \epsilon_{\theta}(\sqrt{\bar{\alpha}_t} \mathbf{x}_0 + \sqrt{1 - \bar{\alpha}_t} \epsilon, t) \right\|^2$$

- 6: **until** converged

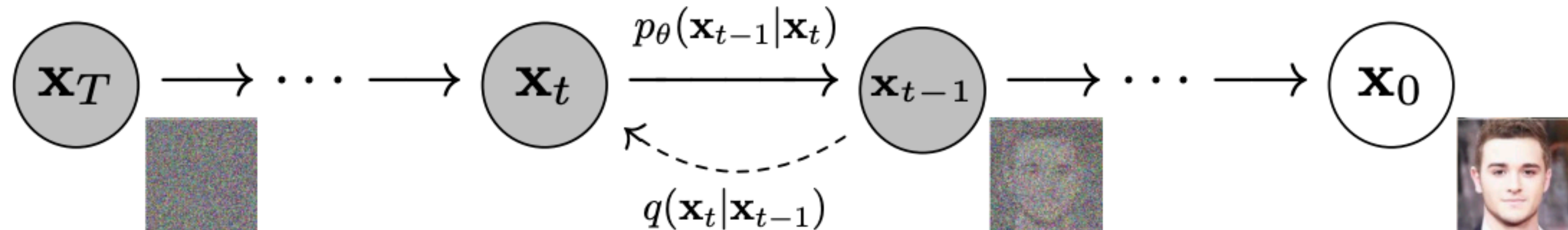
Sampling from a diffusion model



Algorithm 2 Sampling

- 1: $\mathbf{x}_T \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$
 - 2: **for** $t = T, \dots, 1$ **do** Remove a little bit of the noise
 - 3: $\mathbf{z} \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$ if $t > 1$, else $\mathbf{z} = \mathbf{0}$
 - 4: $\mathbf{x}_{t-1} = \frac{1}{\sqrt{\alpha_t}} \left(\mathbf{x}_t - \frac{1-\alpha_t}{\sqrt{1-\bar{\alpha}_t}} \boldsymbol{\epsilon}_\theta(\mathbf{x}_t, t) \right) + \sigma_t \mathbf{z}$ ↙
 - 5: **end for**
 - 6: **return** $\mathbf{x}_0$
-

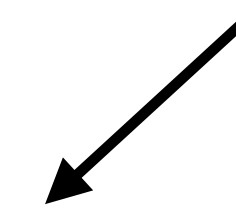
Sampling from a diffusion model



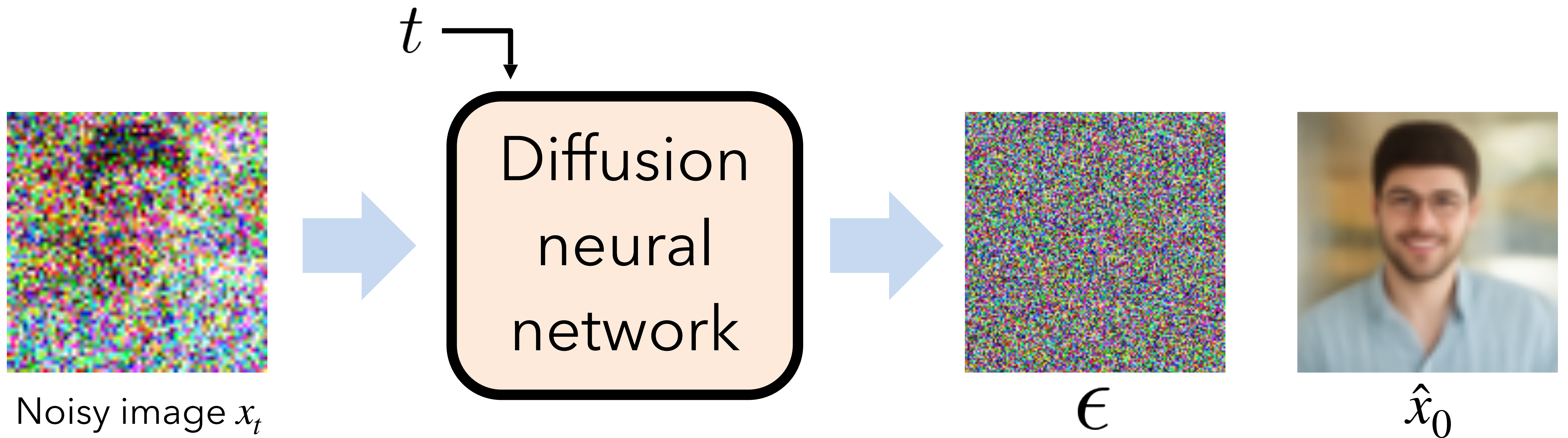
Algorithm 2 Sampling

- 1: $\mathbf{x}_T \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$
- 2: **for** $t = T, \dots, 1$ **do**
- 3: $\mathbf{z} \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$ if $t > 1$, else $\mathbf{z} = \mathbf{0}$
- 4: $\mathbf{x}_{t-1} = \frac{1}{\sqrt{\alpha_t}} \left(\mathbf{x}_t - \frac{1-\alpha_t}{\sqrt{1-\bar{\alpha}_t}} \boldsymbol{\epsilon}_\theta(\mathbf{x}_t, t) \right) + \sigma_t \mathbf{z}$
- 5: **end for**
- 6: **return** $\mathbf{x}_0$

Add back a bit of noise, too.
(some diffusion variants don't do this)

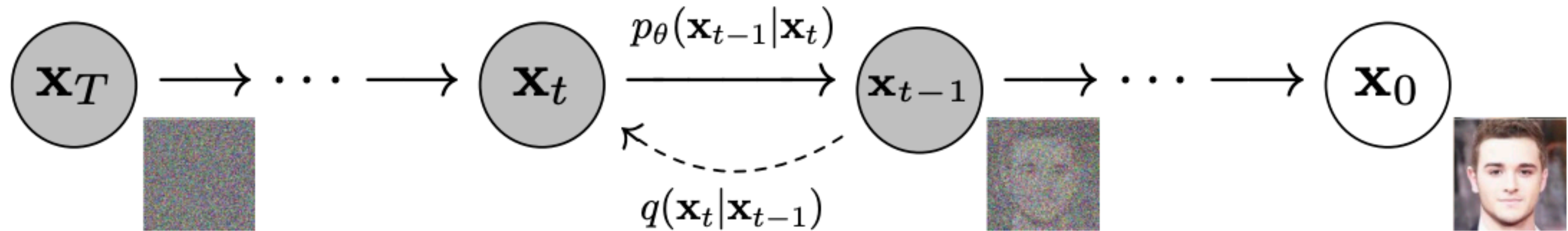


One-step denoising



where $\hat{x}_0 = \frac{1}{\sqrt{\alpha_t}}(x_t - \sqrt{(1 - \alpha_t)}\epsilon_{\theta}(x_t, t)) = \mathbb{E}[x_0 | x_t]$ is "one step" prediction.

Sampling from a diffusion model



Algorithm 2 Sampling

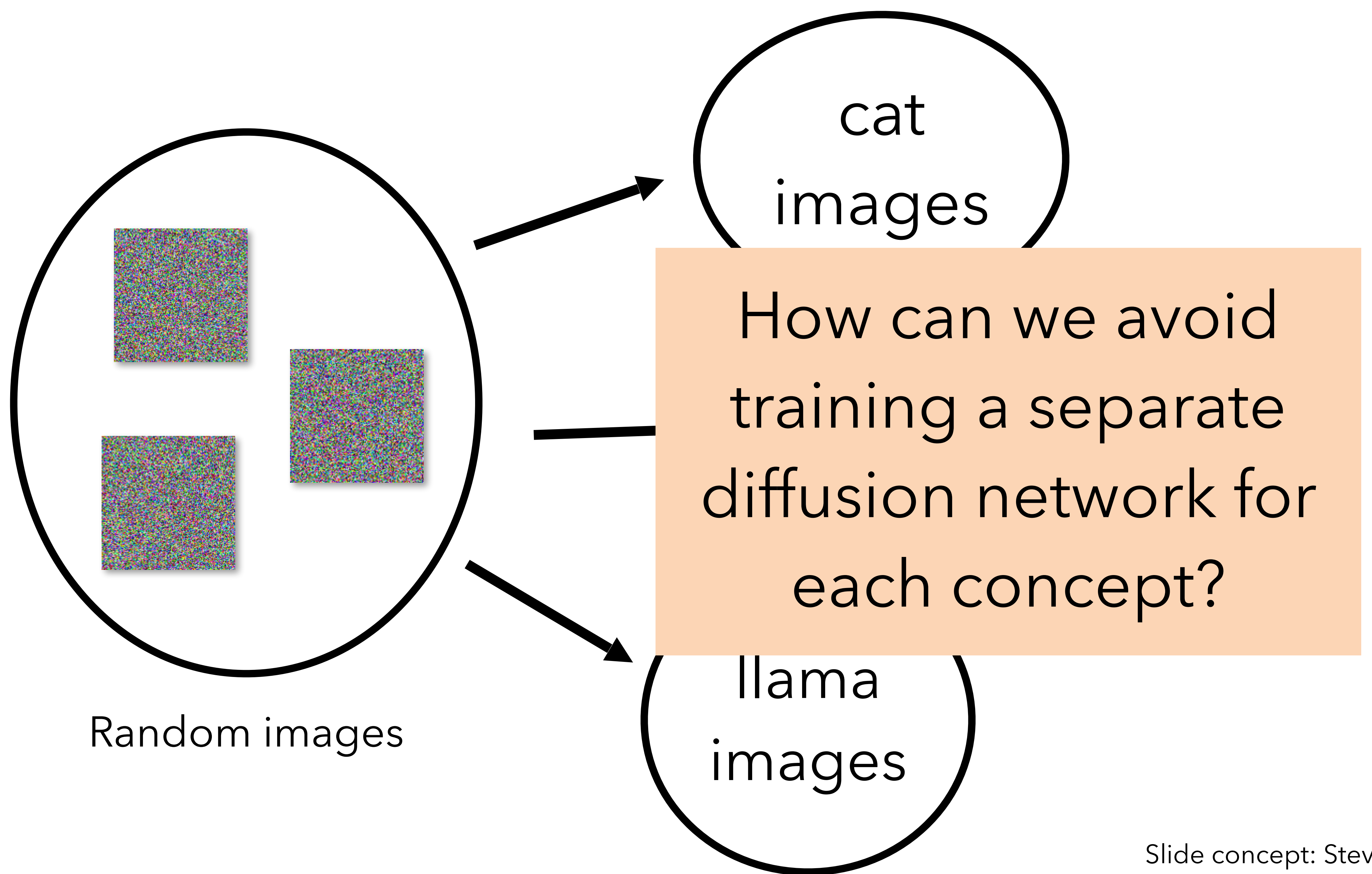
```
1:  $\mathbf{x}_T \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$ 
2: for  $t = T, \dots, 1$  do
3:    $\mathbf{z} \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$  if  $t > 1$ , else  $\mathbf{z} = \mathbf{0}$ 
4:    $\mathbf{x}_{t-1} = \frac{1}{\sqrt{\alpha_t}} \left( \mathbf{x}_t - \frac{1-\alpha_t}{\sqrt{1-\bar{\alpha}_t}} \boldsymbol{\epsilon}_\theta(\mathbf{x}_t, t) \right) + \sigma_t \mathbf{z}$ 
5: end for
6: return  $\mathbf{x}_0$ 
```

Can write update as:

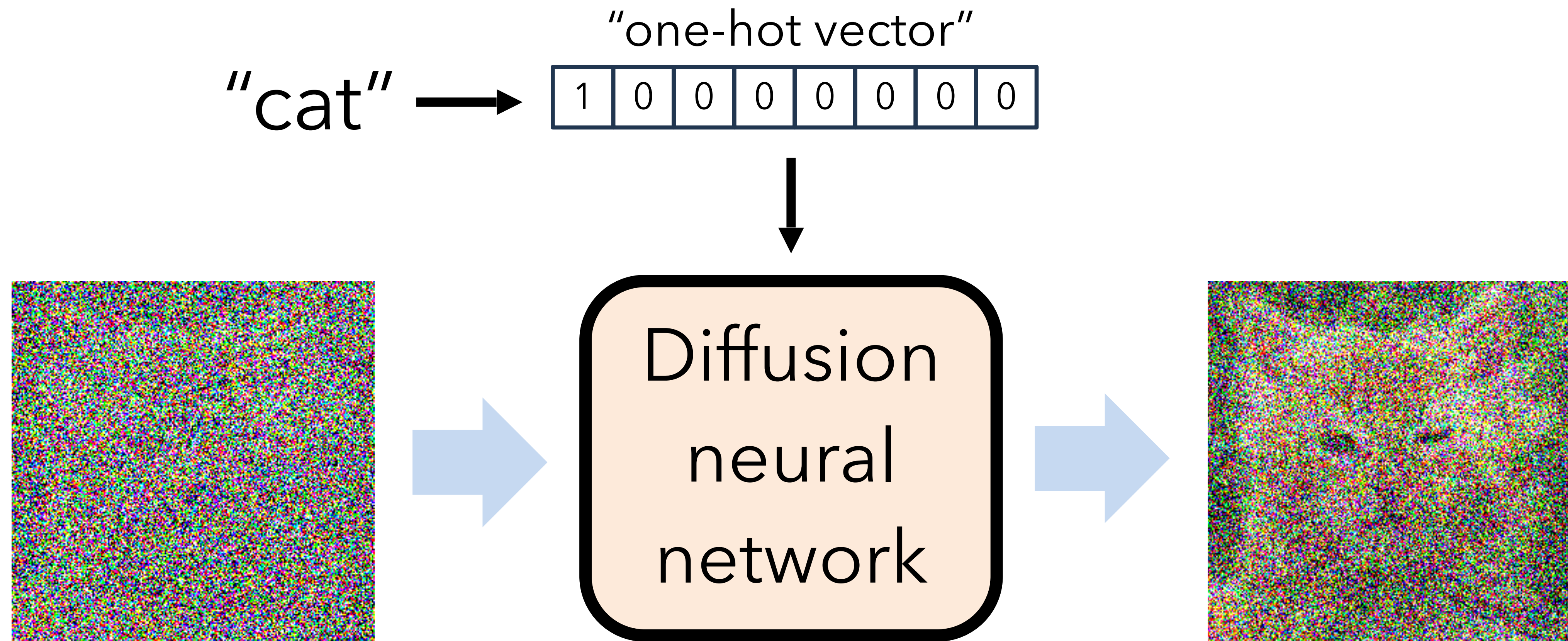
$$x_{t-1} = w_0 \hat{x}_0 + w_1 x_t + \sigma_t \mathbf{z}$$

for constants w_0, w_1 where $\hat{x}_0$ is "one step" estimate of clean image:

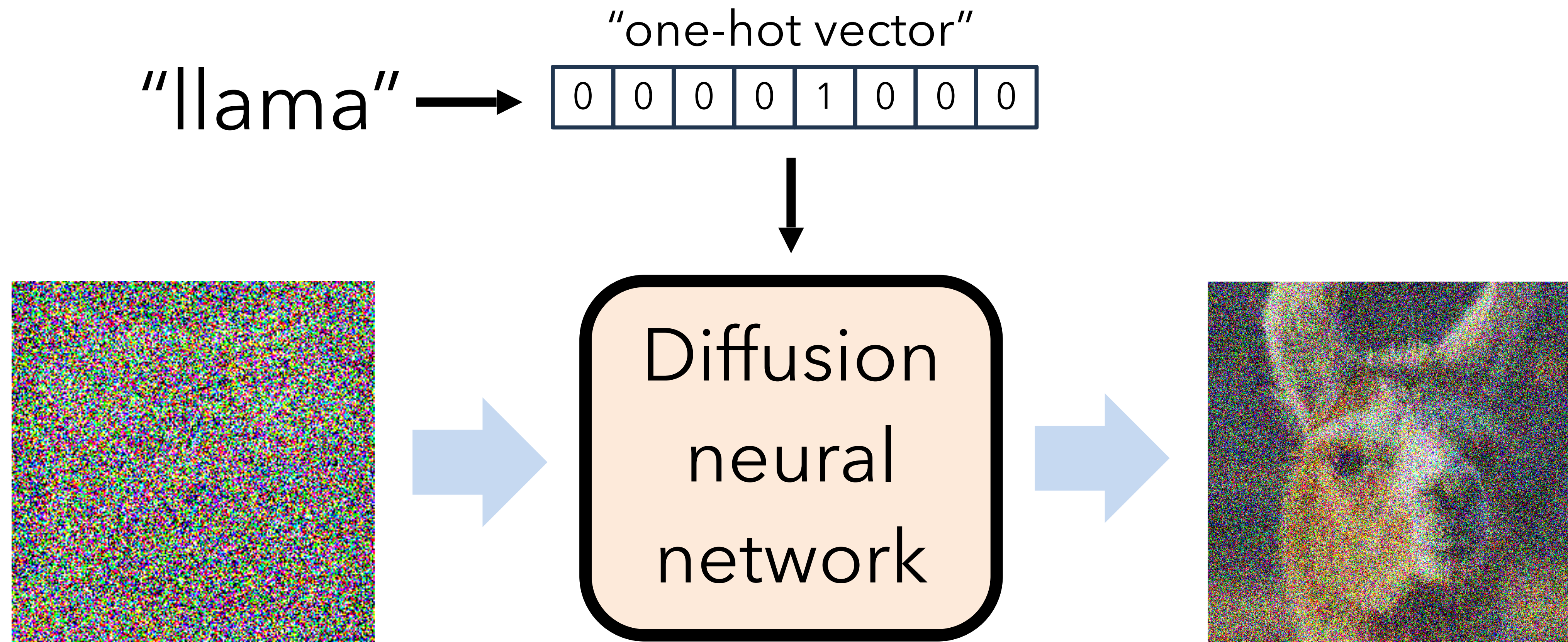
$$\hat{x}_0 = \frac{1}{\sqrt{\alpha_t}} (x_t - \sqrt{(1-\alpha_t)} \boldsymbol{\epsilon}_\theta(x_t, t))$$



Idea 1: add a class label as conditioning



Idea 1: add a class label as conditioning



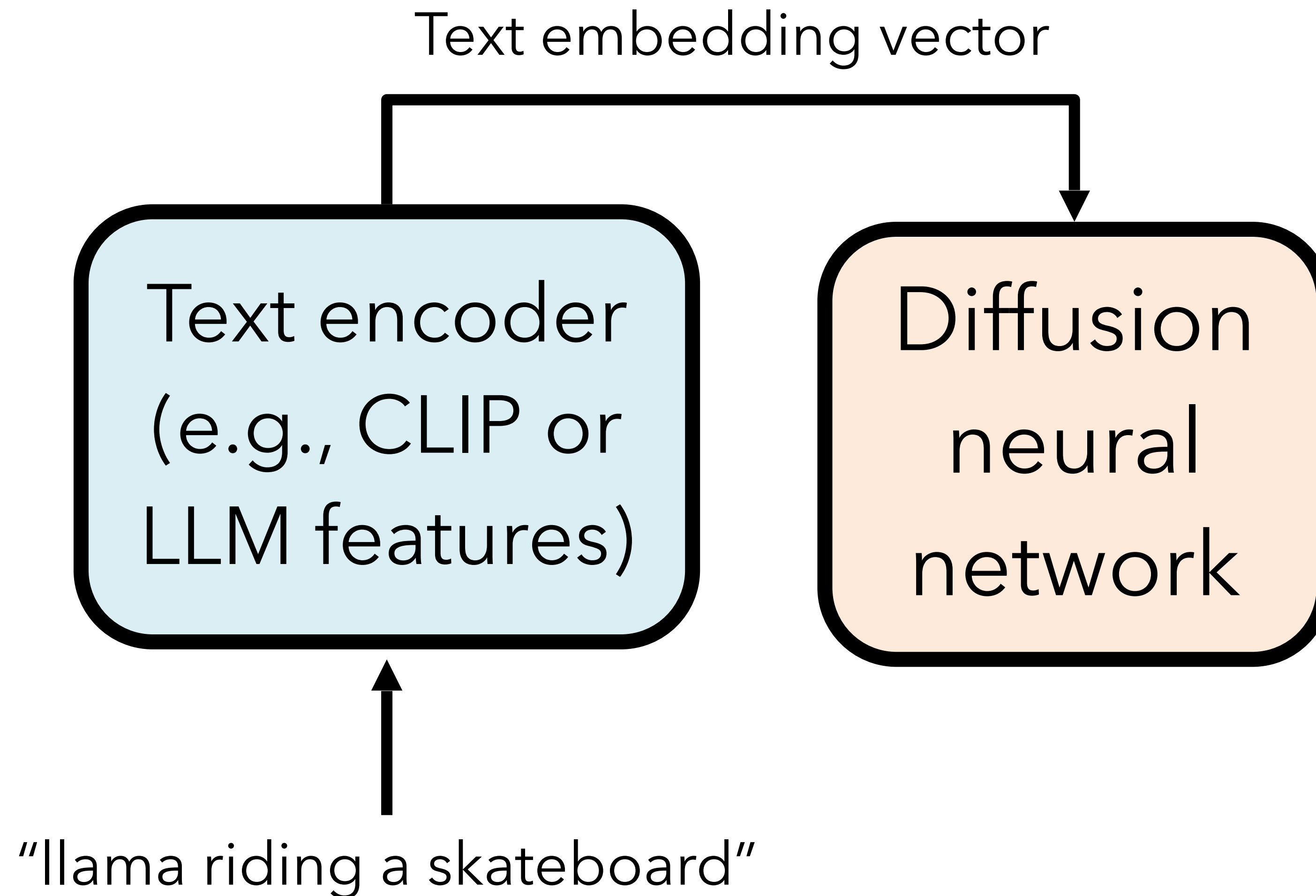
"Class conditioning" – only allows us to generate a fixed set of classes

Works but doesn't always produce high quality images

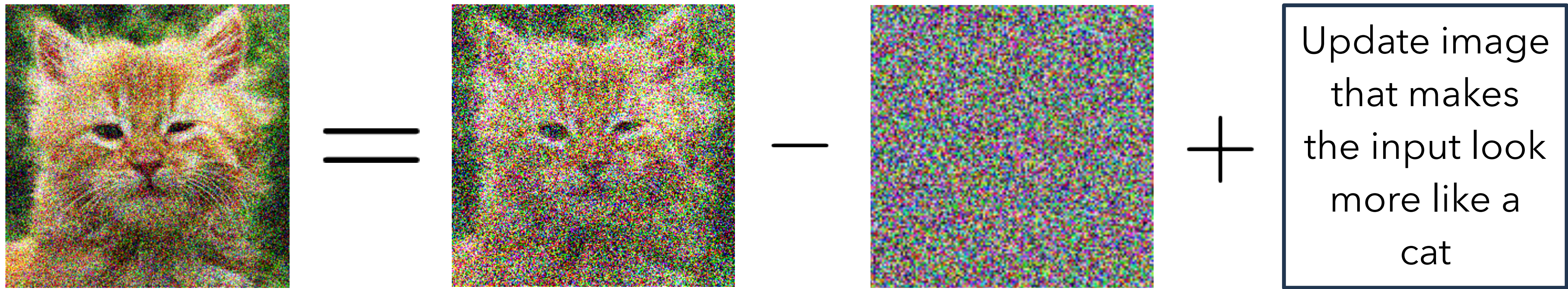
“Malamute” class



Idea 2: add a text prompt as conditioning



Idea 3: use an image classifier to guide the denoising process
("classifier guidance")


$$x_{t-1} = \frac{1}{\sqrt{\alpha_t}} x_t - \frac{1 - \alpha_t}{\sqrt{\alpha_t} \sqrt{1 - \bar{\alpha}_t}} \epsilon + \text{Update image that makes the input look more like a cat}$$

Idea 3: use an image classifier to guide the denoising process
("classifier guidance")

Update image
that makes
the input look
more like a
cat

$$= \nabla F_{\theta}(x_t | y = \text{"cat"})$$

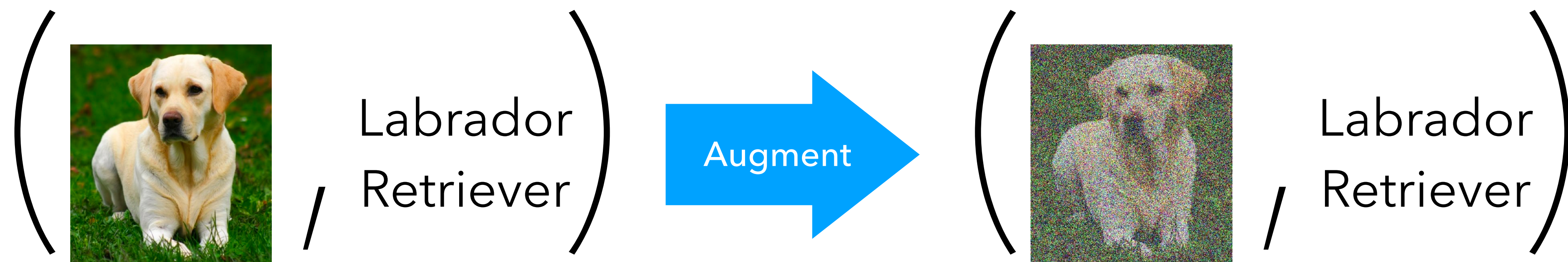
F_{θ} : Image classifier, e.g., AlexNet
trained on ImageNet, but fine-
tuned for noisy images

$$\tilde{\epsilon}_t(x_t, t, y) = \epsilon_{\theta}(x_t, t) - \gamma \nabla_{x_t} \log p(x_t | y)$$

Classifier Guidance

Problem: Classifier isn't trained on noisy images!

Solution: Finetune the classifier on noisy images



Classifier Guidance

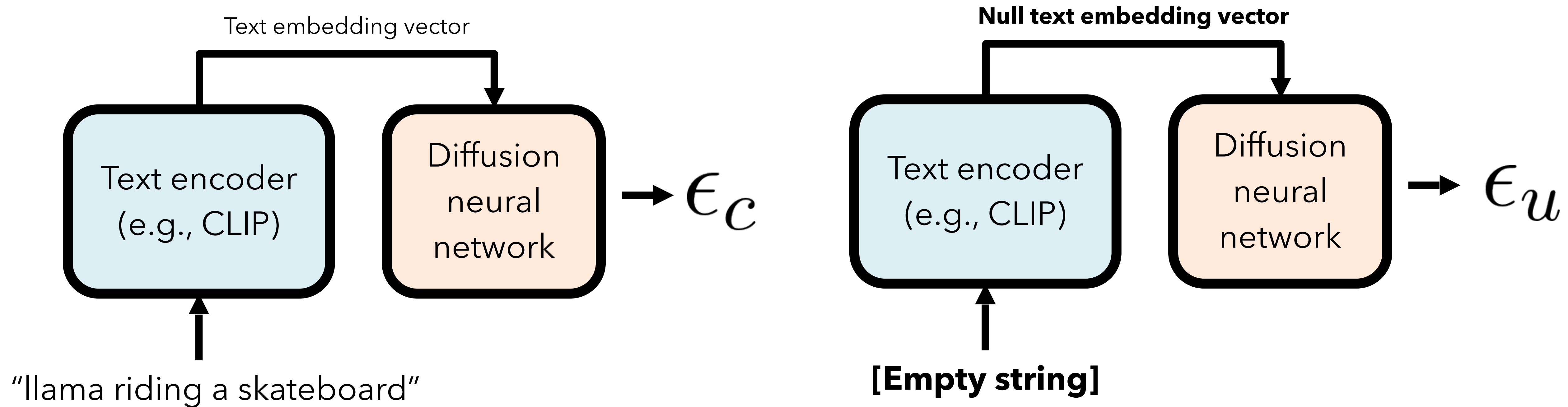


Guidance Weight 1.0



Guidance Weight 10.0

Idea 4: Classifier-free guidance (CFG)



Mix conditional and unconditional noise:

$$\epsilon = \epsilon_u + \gamma(\epsilon_c - \epsilon_u) \quad \text{where } \gamma > 1$$

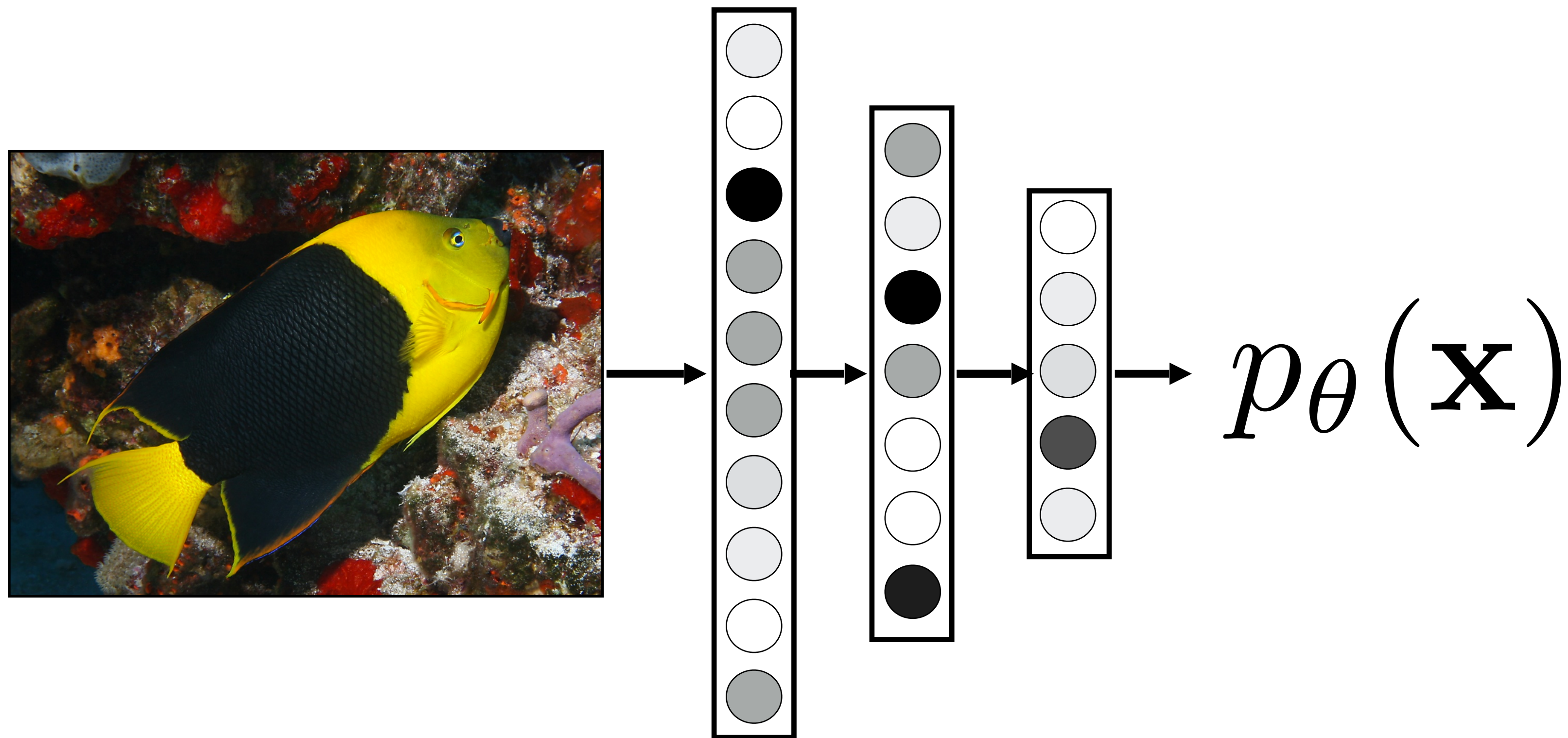
Classifier-free Guidance



Low γ

High γ

Another perspective on diffusion models



How would such a model work?

Energy-based model (EBM)

Energy function implemented
by neural net, $E_{\theta}(\mathbf{x}) : \mathbb{R}^n \rightarrow \mathbb{R}$

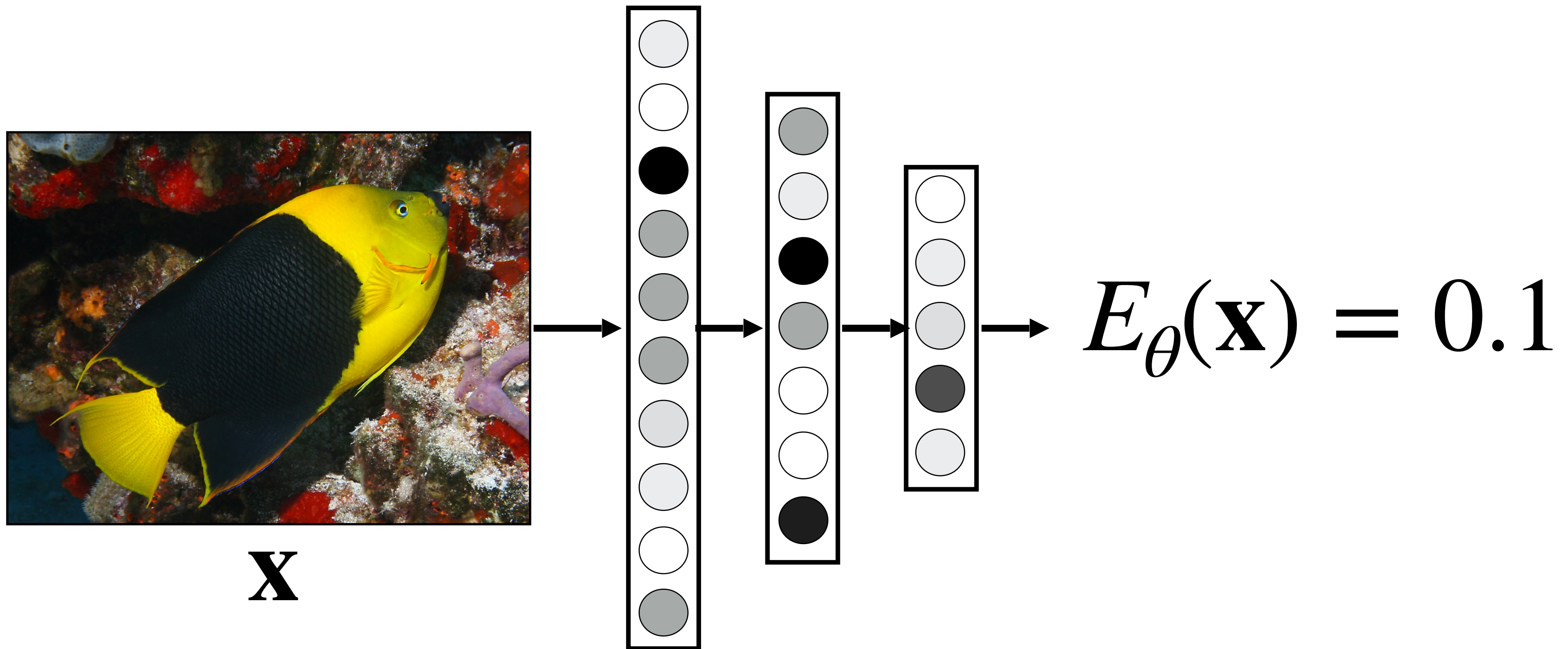
$$p_{\theta}(\mathbf{x}) = \frac{\exp(-E_{\theta}(\mathbf{x}))}{Z(\theta)}$$

Normalization constant
a.k.a. **partition function**

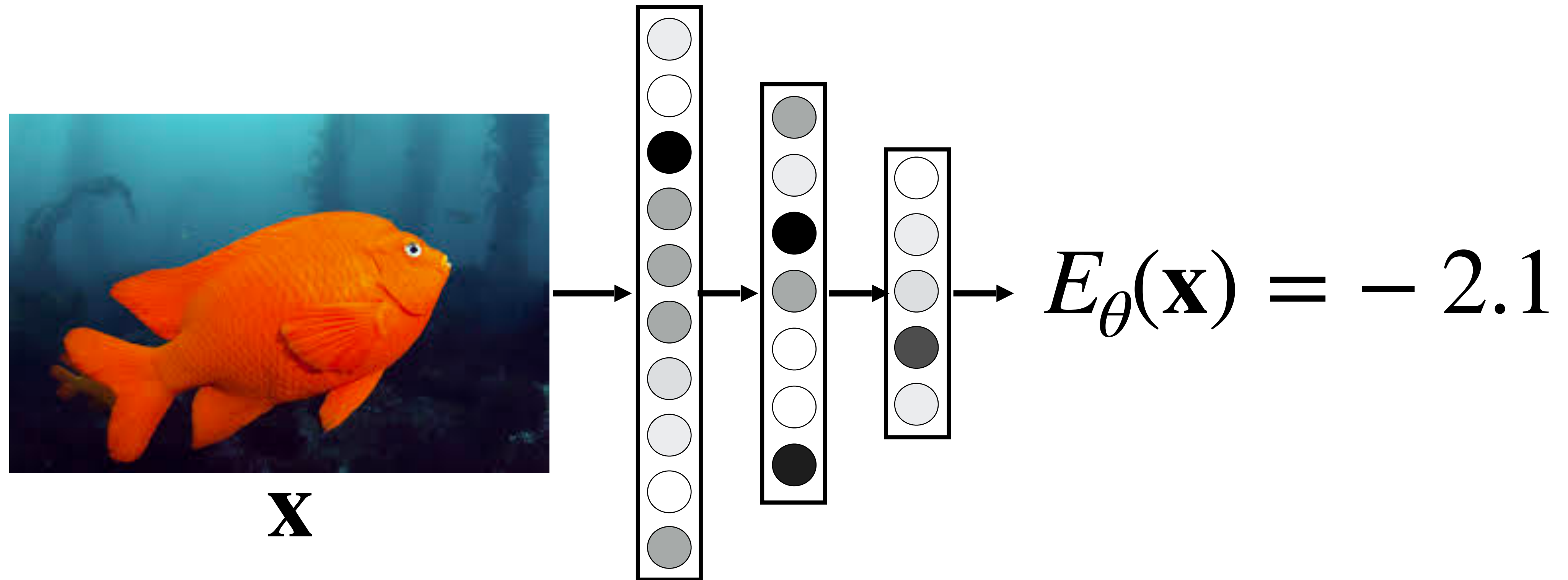
where $Z(\theta) = \int_{\mathbf{x}} \exp(-E_{\theta}(\mathbf{x})) d\mathbf{x}$

What makes this idea challenging? Needs to sum to 1!

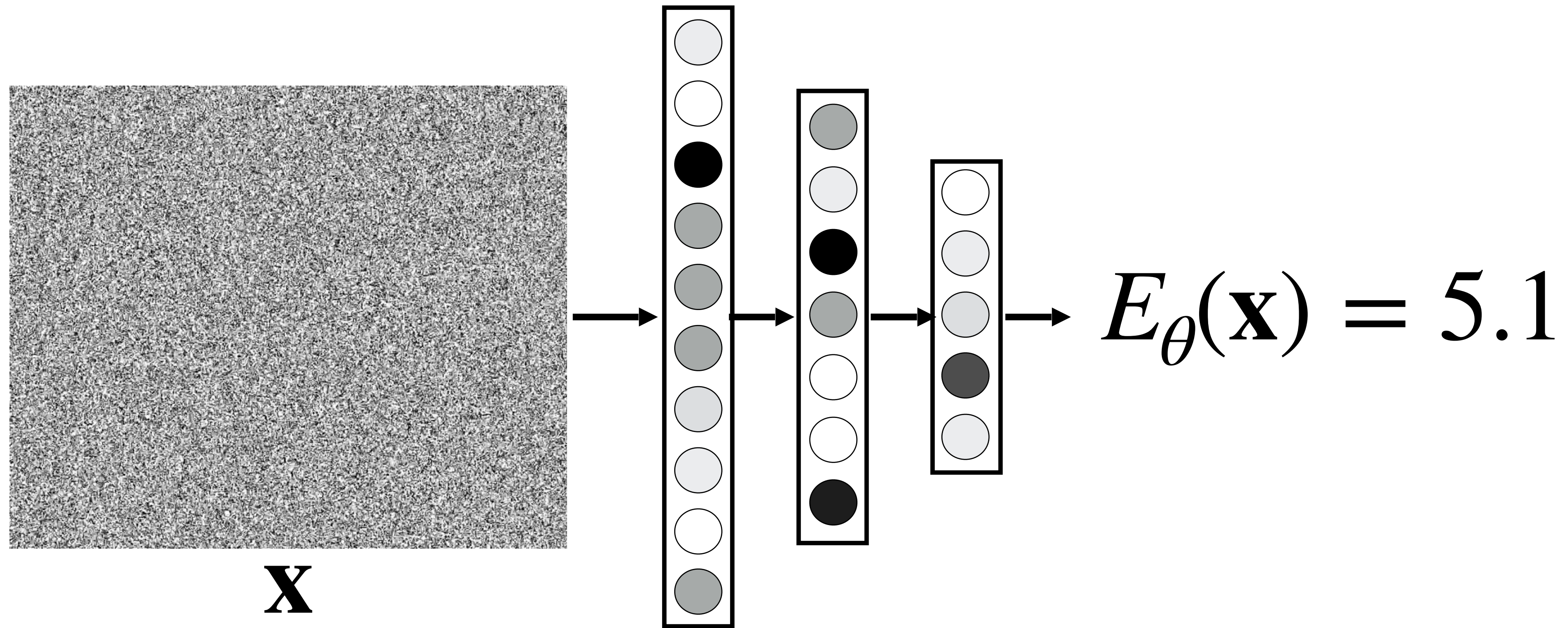
Energy function



Energy function



Energy function



When computing $Z(\theta)$, we need to integrate over the full input space, which makes it challenging.

Denoisers as score functions

- The **score function** $s_\theta(\mathbf{x}) = \nabla_{\mathbf{x}} \log(p_\theta(\mathbf{x}))$ generally much easier to work with.

For example, in EBM's:

$$\nabla_{\mathbf{x}} \log(p_\theta(\mathbf{x})) = \nabla_{\mathbf{x}} \log \left(\frac{\exp(-E_\theta(\mathbf{x}))}{Z(\theta)} \right) = \nabla_{\mathbf{x}} \exp(-E_\theta(\mathbf{x}))$$

- So, if we could learn the score function, that would circumvent the need for computing the normalization constant.
- You can show that the denoiser learns the score function of the noised data distribution by Tweedie's formula [Robbins, 1956]:

$$\epsilon_\theta(\mathbf{x}_t) \approx -\sigma_t \nabla_{\mathbf{x}_t} \log p(\mathbf{x}_t)$$

where $\mathbf{x}_t$ is the data noised at step t of the diffusion model with Gaussian noise σ_t .

- So, when we remove noise, we are (roughly speaking) following the gradient of log prob. and moving to higher density regions.

Score function perspective on classifier-free guidance

Classifier-free guidance:

$$\epsilon_{\theta}(\mathbf{x}_t) + \gamma(\epsilon_{\theta}(\mathbf{x}_t, c) - \epsilon_{\theta}(\mathbf{x}_t))$$

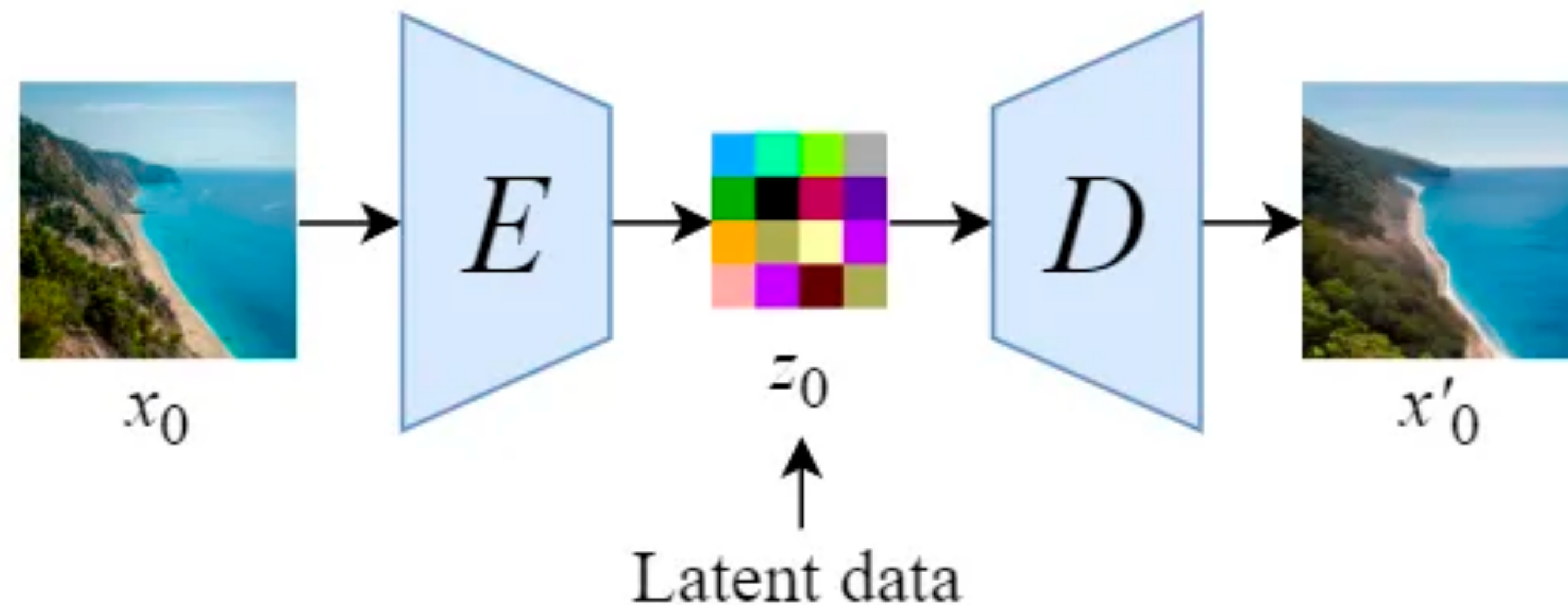
From a score function perspective this is (approximately):

$$\approx -\nabla_{\mathbf{x}_t} [\log p_{\theta}(\mathbf{x}_t) + \gamma (\log p_{\theta}(\mathbf{x}_t \mid y) - \log p_{\theta}(\mathbf{x}_t))]$$

$$= -\nabla_{\mathbf{x}_t} \log(p_{\theta}(\mathbf{x}_t)^{1-\gamma} p_{\theta}(\mathbf{x}_t \mid y)^{\gamma})$$

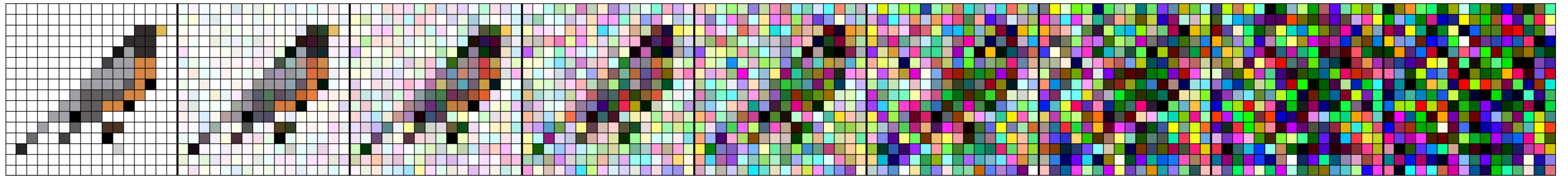
Latent diffusion models (LDMs)

- Do diffusion in latent space for speed and potentially for better perceptual quality (just like the VQ-GAN).

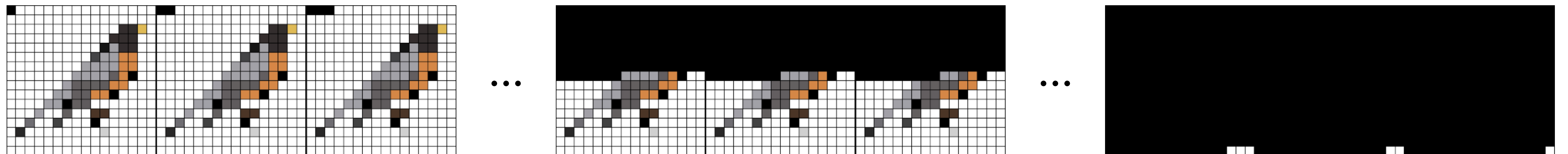


Autoregressive models vs. diffusion models

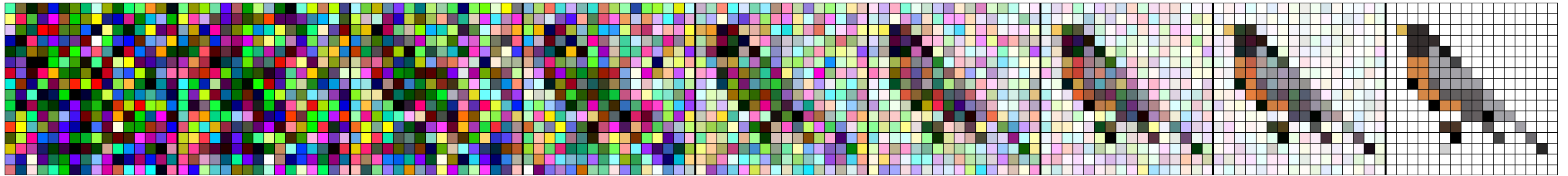
Forward diffusion process



Reverse autoregressive sequence



Diffusion model



Autoregressive model

