Projective Geometry Redux & Image Based Rendering

By Abe Davis

Announcements

- Project 4 is released
  - Due Friday, April 17 by 11:59pm.
  - The project will be done in groups of two, with groups defaulting to the Project 3 groups (though groups can be changed on CMSX)
- New Grading Policy
  - Check email for more information

Today's Lecture

- Homogeneous Coordinates and Geometry in n-dimensions
  - New slides, following my own derivations, intended to help with confusion I've noticed in the first part of the course
  - Mostly re-derives stuff you know, but hopefully with stronger motivation, rigor, and intuition

- Image Based Rendering and Light Fields
  - Not in previous versions of the course, but an active area of work in computer vision with many applications (e.g., AR/VR, film special effects, etc.)

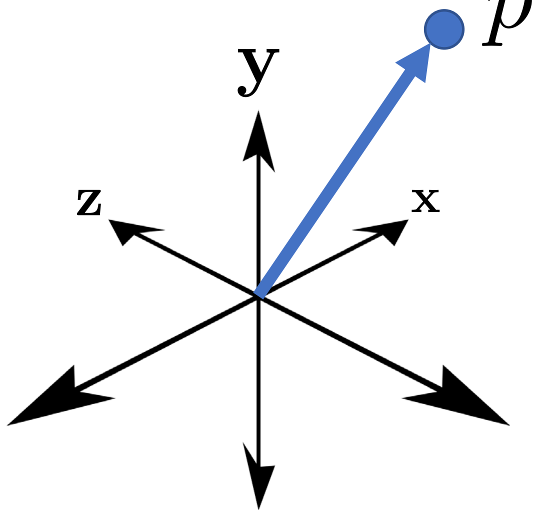
Part 1:
Building a Geometry of Points & Views

By Abe Davis

Or “Let’s derive homogeneous coordinates from scratch!”

Motivation

- We may observe geometry in different ways
  - e.g., different views or cameras
- How do we separate our description of geometry from our choice of reference frame?



$$p = \begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix}$$

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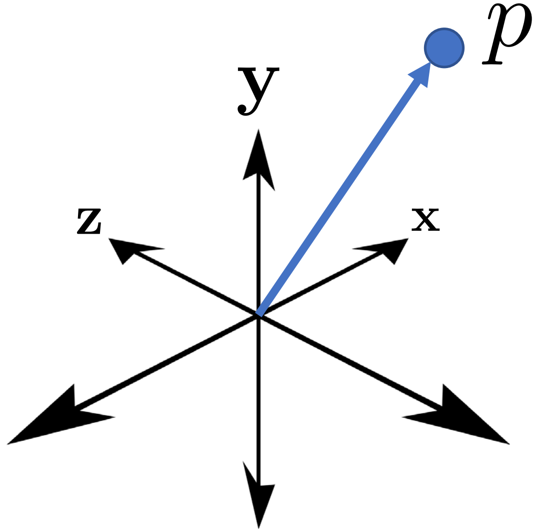

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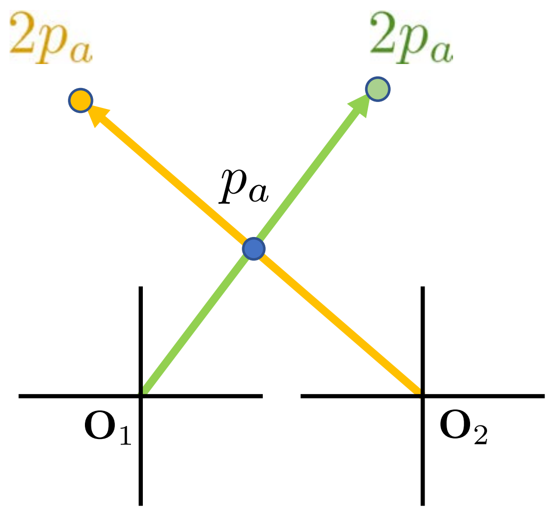

Multiplying a point by a constant

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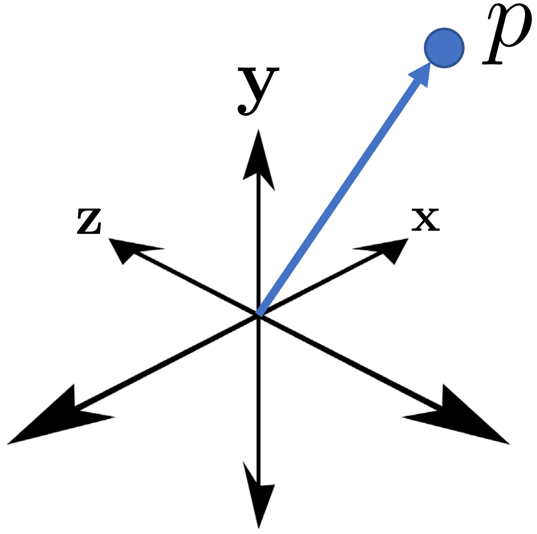

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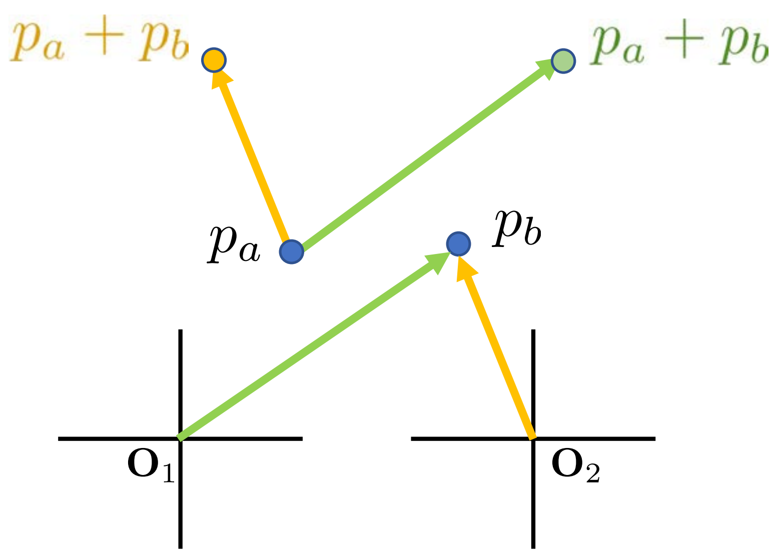

Adding two points

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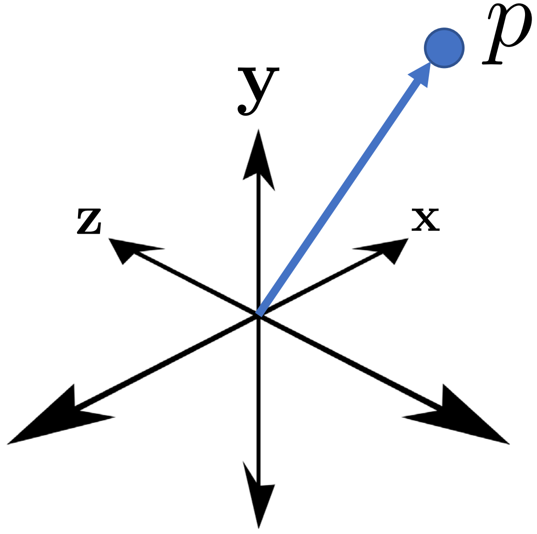


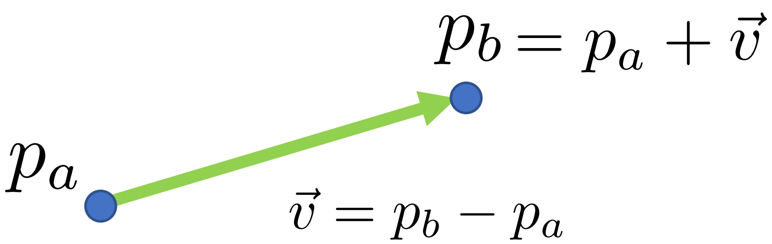
$$p = \begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix}$$

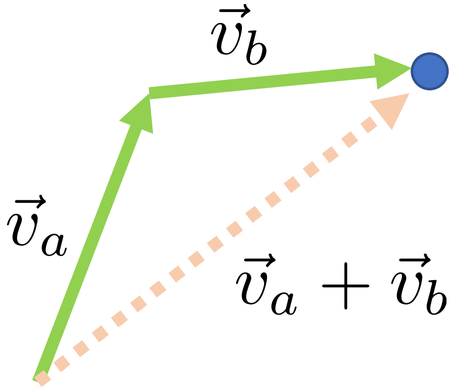
Problem:
Standard representation treats points like vectors from the origin

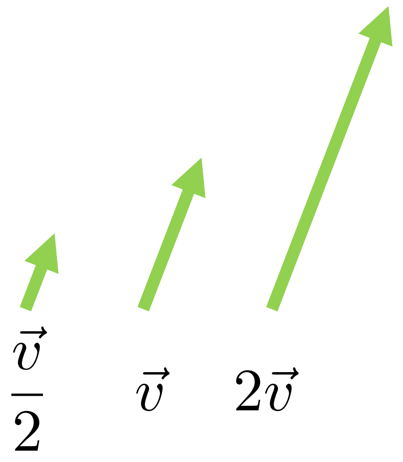
Points and vectors are NOT the same thing!

Relating Points and Vectors

- A point is a unique location
- A vector is the difference between two locations
- We can add vectors to points and to other vectors
- We can scale vectors







Homogeneous Values

- A point is a unique location
- A vector is the difference between two locations
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Homogeneous Points and Vectors:

$$\mathbf{p}_a = \begin{bmatrix} p_a \\ 1 \end{bmatrix} \quad \mathbf{p}_b = \begin{bmatrix} p_b \\ 1 \end{bmatrix} \qquad \mathbf{v} = \begin{bmatrix} v \\ 0 \end{bmatrix}$$

Set an extra value to 1 for points

And to 0 for vectors

For now, let's only consider homogeneous values that are 0 or 1

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point – point = vector $\begin{bmatrix} p_b \\ 1 \end{bmatrix} - \begin{bmatrix} p_a \\ 1 \end{bmatrix} = \begin{bmatrix} p_b - p_a \\ 0 \end{bmatrix}$

point + vector = point $\begin{bmatrix} p \\ 1 \end{bmatrix} + \begin{bmatrix} v \\ 0 \end{bmatrix} = \begin{bmatrix} p + v \\ 1 \end{bmatrix}$

vector + vector = vector $\begin{bmatrix} v_a \\ 0 \end{bmatrix} + \begin{bmatrix} v_b \\ 0 \end{bmatrix} = \begin{bmatrix} v_a + v_b \\ 0 \end{bmatrix}$

vector $\times$ constant = vector $c \begin{bmatrix} v_a \\ 0 \end{bmatrix} = \begin{bmatrix} cv_b \\ 0 \end{bmatrix}$

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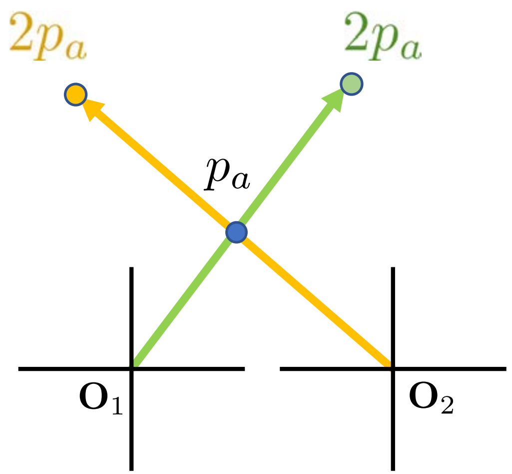
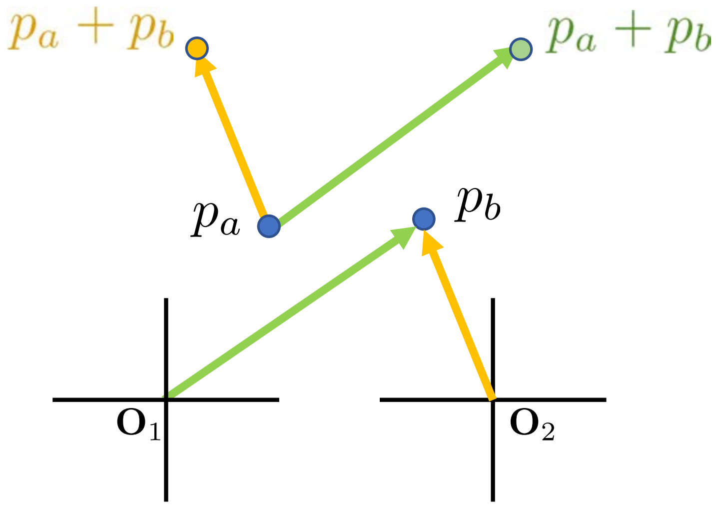
Homogeneous Values

- What should it mean to multiply a point by a constant?
- What should it mean to add points?

Recall: The Problem With Regular Coordinates



Multiplying a point by a constant
With regular coordinates



Adding two points
With regular coordinates

Homogeneous Values

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$$c\mathbf{p}_a = \begin{bmatrix} cp_a \\ \boxed{c} \end{bmatrix}$$

$$\mathbf{p}_a + \mathbf{p}_b = \begin{bmatrix} p_a + p_b \\ \boxed{2} \end{bmatrix}$$

What does it mean to have a homogeneous value that is not 0 or 1?

Homogeneous Values

- What should it mean to multiply a point by a constant?
- What should it mean to add points?

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$$\alpha\mathbf{p}_a + \beta\mathbf{p}_b = \begin{bmatrix} \alpha p_a + \beta p_b \\ (\alpha + \beta) \end{bmatrix}$$

When $\beta = -\alpha$, this becomes $\alpha(p_a - p_b)$, a vector

When $\beta = 1 - \alpha$ this becomes $\begin{bmatrix} \alpha p_a + (1-\alpha)p_b \\ 1 \end{bmatrix}$, a point

When can these operations be combined to get valid points or vectors?

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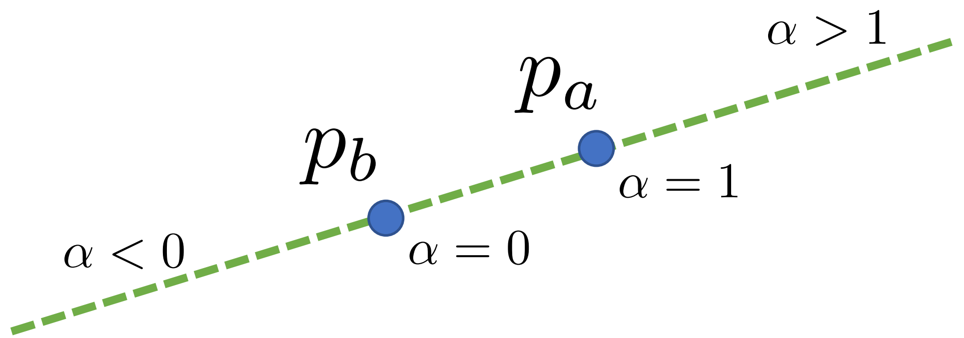
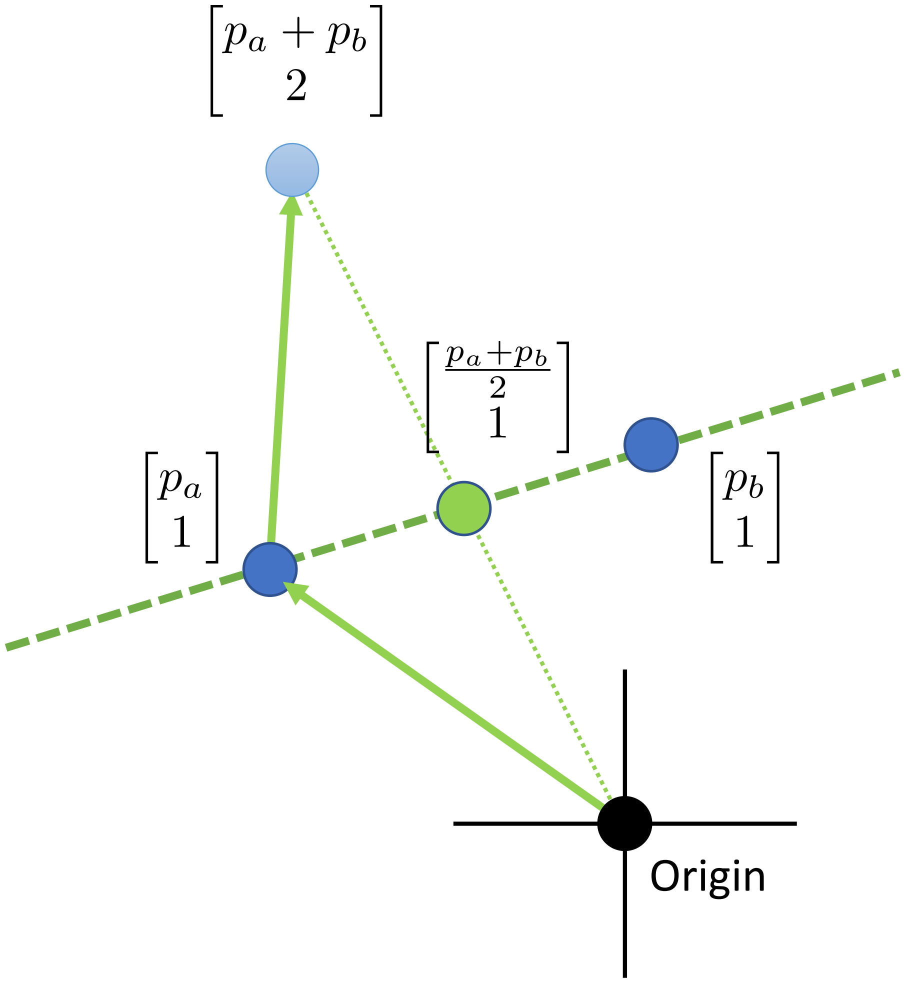


Homogeneous ~~Values~~ Coordinates

- Homogeneous values keep track of how much our choice of origin has influenced our coordinates
- We can correct for the influence on a point by dividing all coordinates by the homogeneous value



Barycentric Coordinates, Homogenization, & Center of Mass

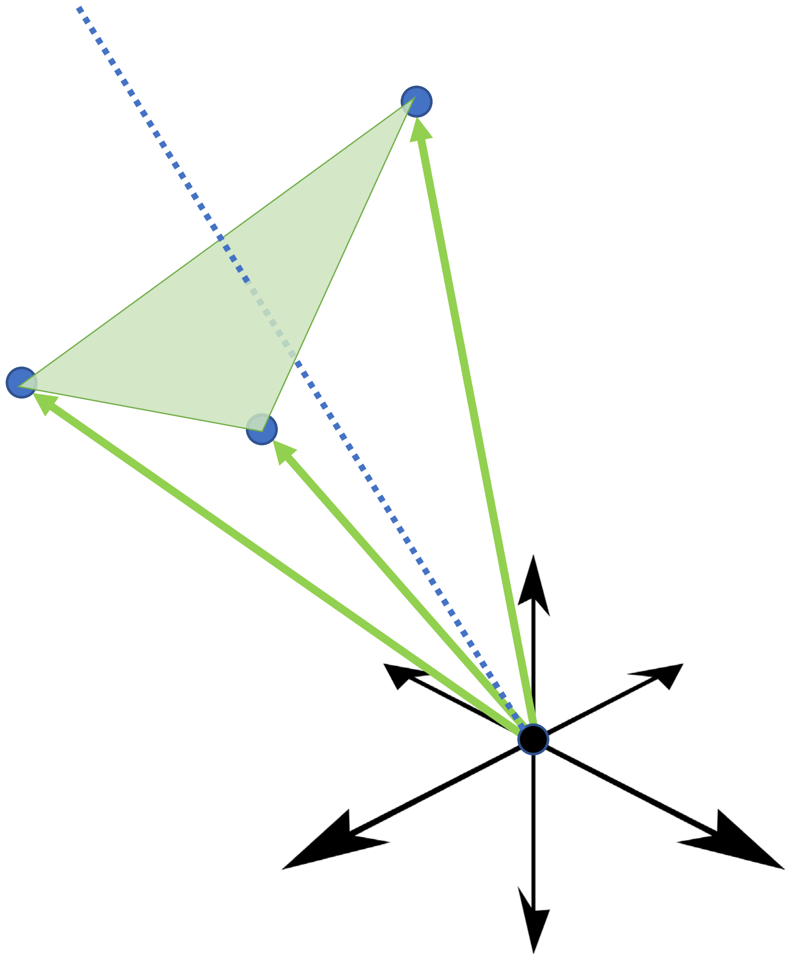
- If we homogenize the weighted sum of k points, we get get their center of mass

$$p = \frac{\sum \alpha_i p_i}{\sum \alpha_i}$$

Weighted sum of points (weights given by alphas)

$$\mathbf{c} = \frac{\sum m_i p_i}{\sum m_i}$$

Equation for center of mass (masses given by m's)

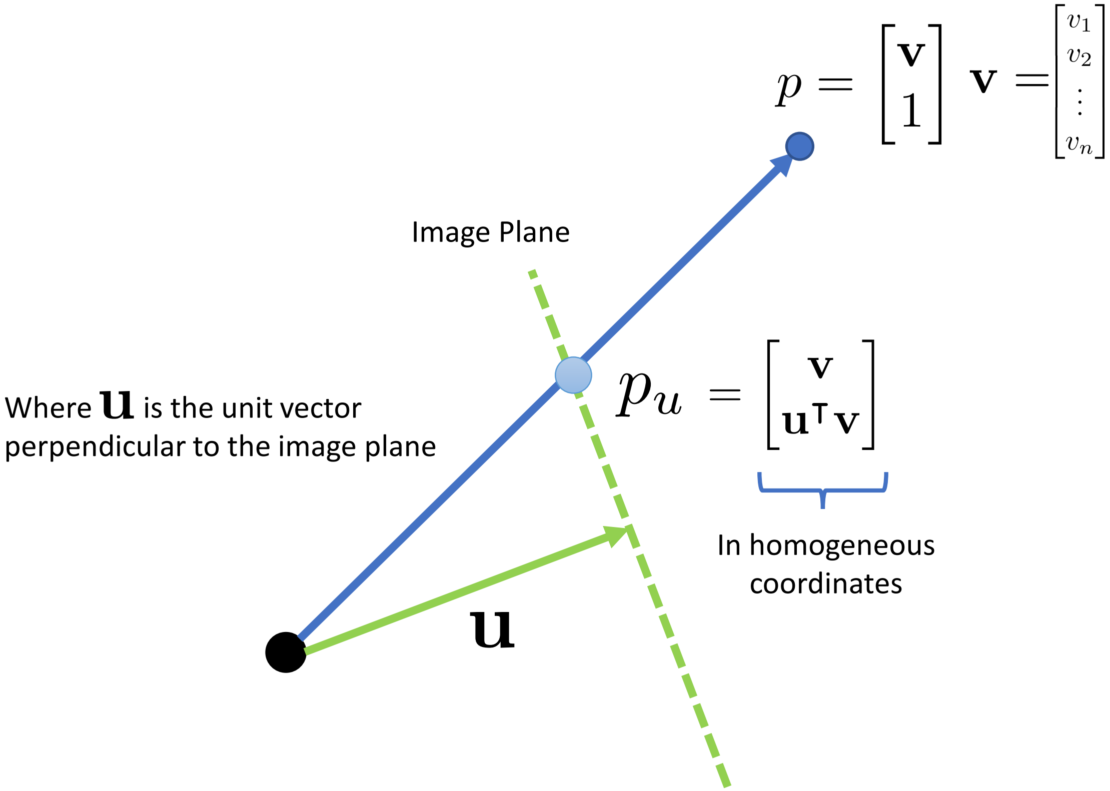


Homogeneous Coordinates & Projection

- How to project an n-dimensional vector onto an image plane?

Homogeneous Coordinates & Projection

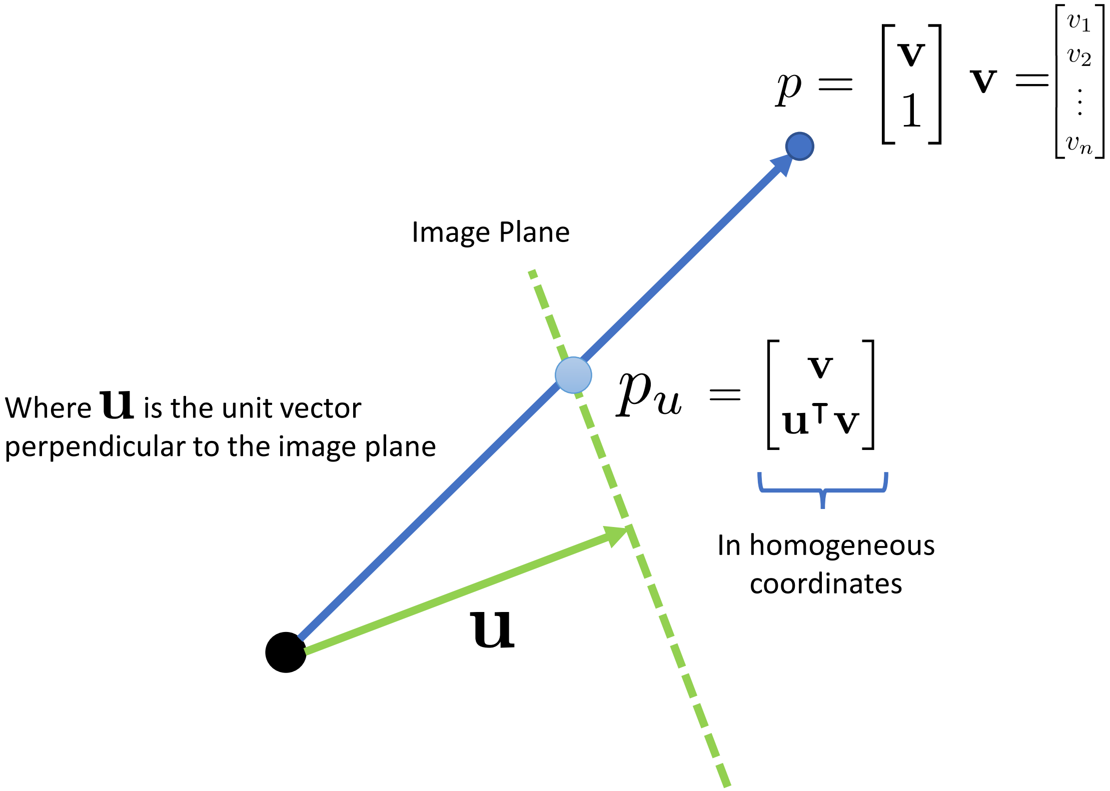
- How to project an n-dimensional vector onto an image plane?



What is the projection p_u of p onto the image plane?

Homogeneous Coordinates & Projection

- How to project an n-dimensional vector onto an image plane?



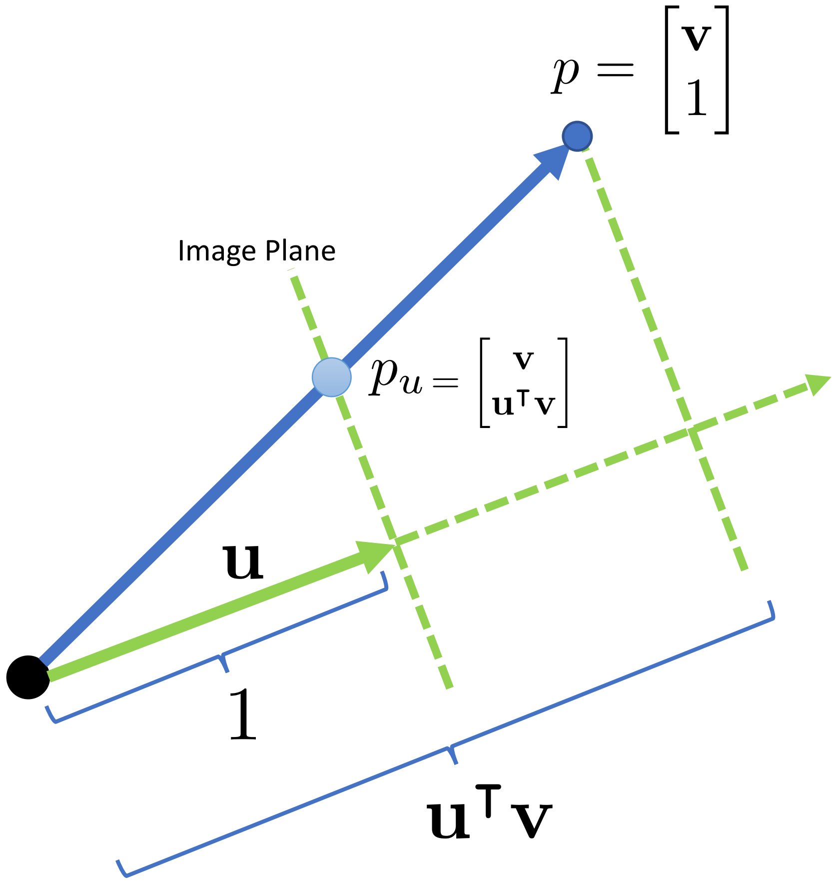
How do we express this as a matrix?

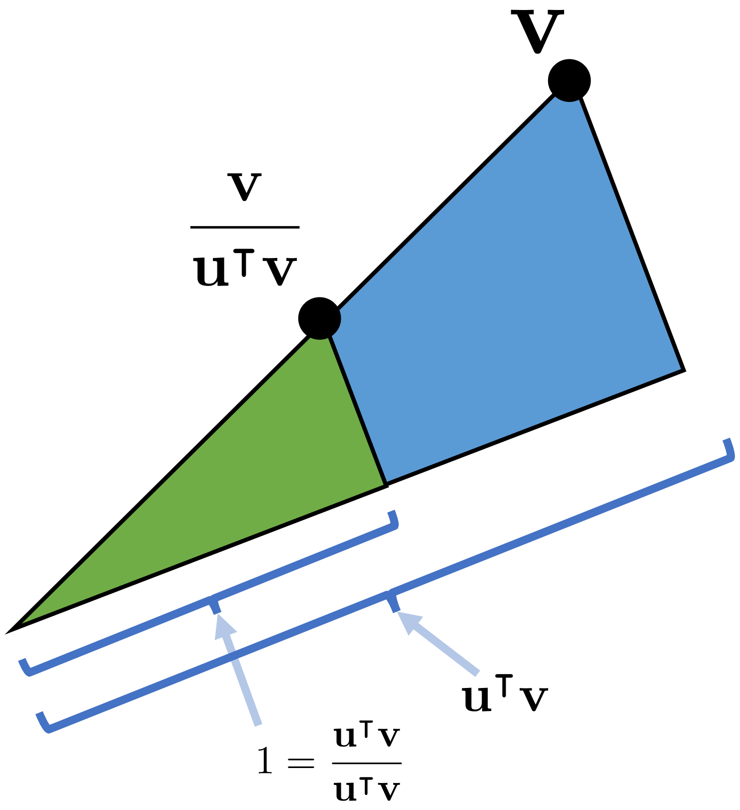
$$\mathbf{P}p = \begin{bmatrix} \mathbf{v} \\ \mathbf{u}^\intercal \mathbf{v} \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} \mathbf{I_{n \times n}} & \mathbf{0}_{n \times 1} \\ \mathbf{u}^\intercal & 0 \end{bmatrix}}_{\text{Projection onto image plane defined by } \mathbf{u}} \begin{bmatrix} \mathbf{v} \\ 1 \end{bmatrix} = \begin{bmatrix} \mathbf{v} \\ \mathbf{u}^\intercal \mathbf{v} \end{bmatrix}$$

Projection onto image plane defined by $\mathbf{u}$

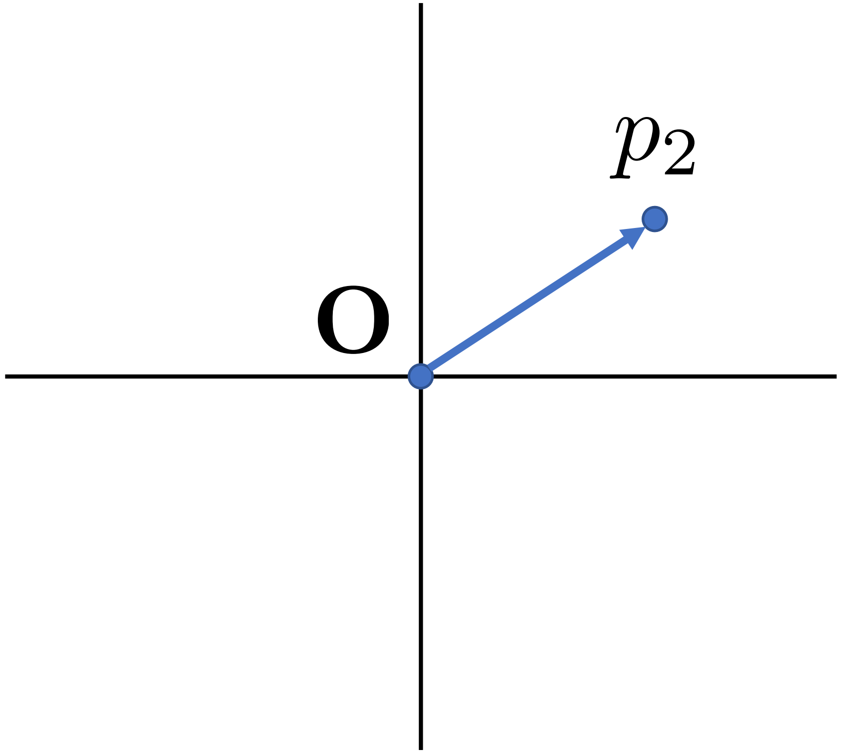
Similar triangles used to compute p_u (Extra Slide)





Homogeneous Coordinates & Translation

- 3D translation is not linear in regular 3D coordinates



$$\mathbf{M}\begin{bmatrix}0\\ \vdots\\ 0\end{bmatrix} = \begin{bmatrix}0\\ \vdots\\ 0\end{bmatrix}$$

In regular coordinates, no matrix can take the origin away from the origin…

Homogeneous Coordinates & Translation

- Translation is not linear in regular coordinates

$$\begin{bmatrix} \mathbf{x}_x & \mathbf{y}_x & \mathbf{z}_x \\ \mathbf{x}_y & \mathbf{y}_y & \mathbf{z}_y \\ \mathbf{x}_z & \mathbf{y}_z & \mathbf{z}_z \end{bmatrix} \begin{bmatrix} \mathbf{p}_x \\ \mathbf{p}_y \\ \mathbf{p}_z \end{bmatrix} = \mathbf{p}_x\mathbf{x} + \mathbf{p}_y\mathbf{y} + \mathbf{p}_z\mathbf{z}$$

Homogeneous Coordinates & Translation

- Translation is not linear in regular coordinates

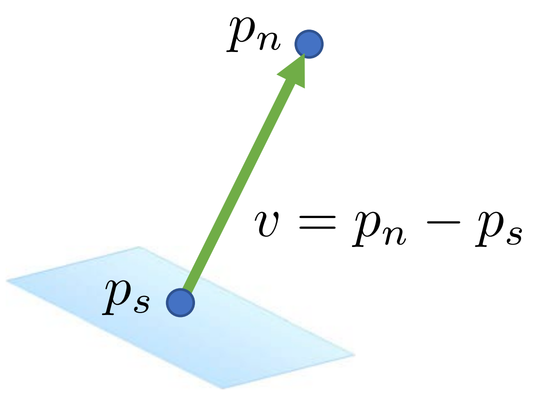
$$\begin{bmatrix} \mathbf{x}_x & \mathbf{y}_x & \mathbf{z}_x & \mathbf{t}_x \\ \mathbf{x}_y & \mathbf{y}_y & \mathbf{z}_y & \mathbf{t}_y \\ \mathbf{x}_z & \mathbf{y}_z & \mathbf{z}_z & \mathbf{t}_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{p}_x \\ \mathbf{p}_y \\ \mathbf{p}_z \\ 1 \end{bmatrix} = \mathbf{p}_x \mathbf{x} + \mathbf{p}_y \mathbf{y} + \mathbf{p}_z \mathbf{z} + \mathbf{t}$$

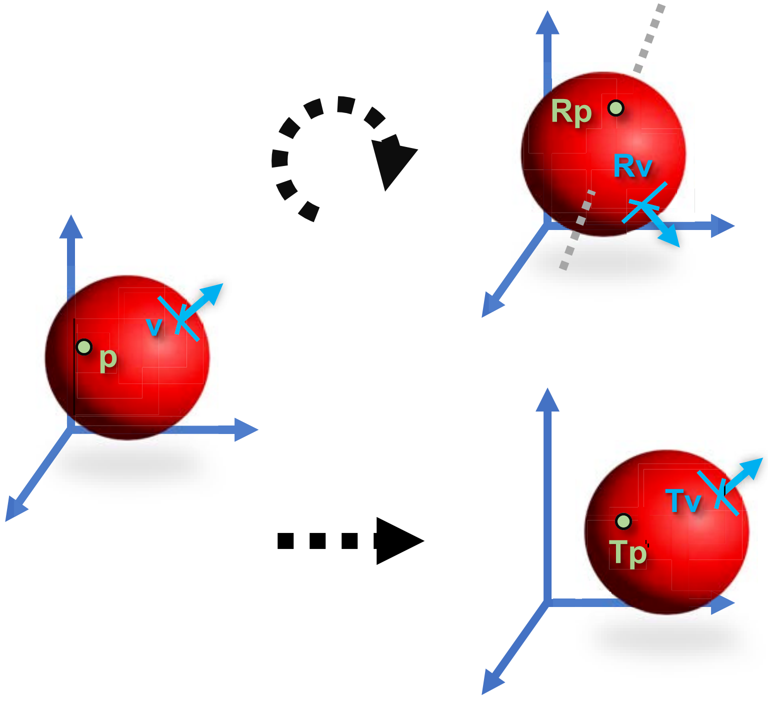
Homogeneous Coordinates & Translation

- Tra

Translation & Rotation: Vectors vs Points

- Points rotate and translate
- Vectors rotate but **do not translate**
  - Consider the surface normal of an object
  - If we translate the object, the surface normal direction does not change





Homogeneous Coordinates & Translation

- Translating vectors (e.g., surface normals)

$$\begin{bmatrix} \mathbf{x}_x & \mathbf{y}_x & \mathbf{z}_x & \mathbf{t}_x \\ \mathbf{x}_y & \mathbf{y}_y & \mathbf{z}_y & \mathbf{t}_y \\ \mathbf{x}_z & \mathbf{y}_z & \mathbf{z}_z & \mathbf{t}_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{n}_x \\ \mathbf{n}_y \\ \mathbf{n}_z \\ 0 \end{bmatrix} = \mathbf{n}_x \mathbf{x} + \mathbf{n}_y \mathbf{y} + \mathbf{n}_z \mathbf{z}$$

Homogeneous Coordinates: Putting It All Together

Change of Basis/Rotation

Translation

Location

$$\begin{bmatrix} \mathbf{x}_x & \mathbf{y}_x & \mathbf{z}_x & \mathbf{t}_x \\ \mathbf{x}_y & \mathbf{y}_y & \mathbf{z}_y & \mathbf{t}_y \\ \mathbf{x}_z & \mathbf{y}_z & \mathbf{z}_z & \mathbf{t}_z \\ & \mathbf{u}^\mathsf{T} & & 0 \end{bmatrix} \begin{bmatrix} \mathbf{v}_x \\ \mathbf{v}_y \\ \mathbf{v}_z \\ 1 \end{bmatrix} = \begin{bmatrix} \mathbf{v}_x\mathbf{x} + \mathbf{v}_y\mathbf{y} + \mathbf{v}_z\mathbf{z} + \mathbf{t} \\ \mathbf{u}^\mathsf{T}\mathbf{v} \end{bmatrix}$$

Image Plane for Projection

0 if projection is done, 1 otherwise

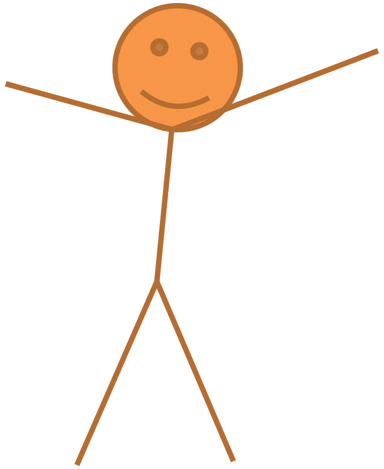
Whether Point or Vector

Part 2:
Image-Based Rendering

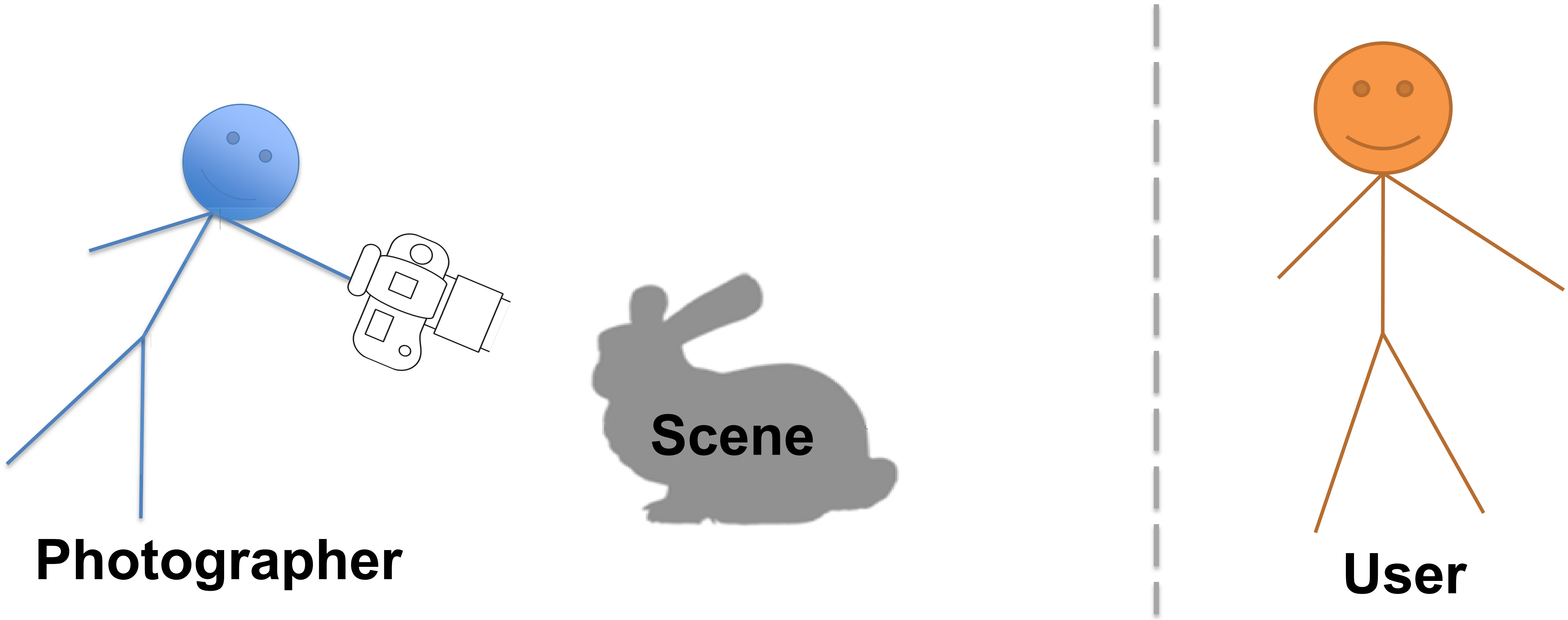
By Abe Davis

Light Fields & Image-Based Rendering

With Stick Figures!



Traditional Photography



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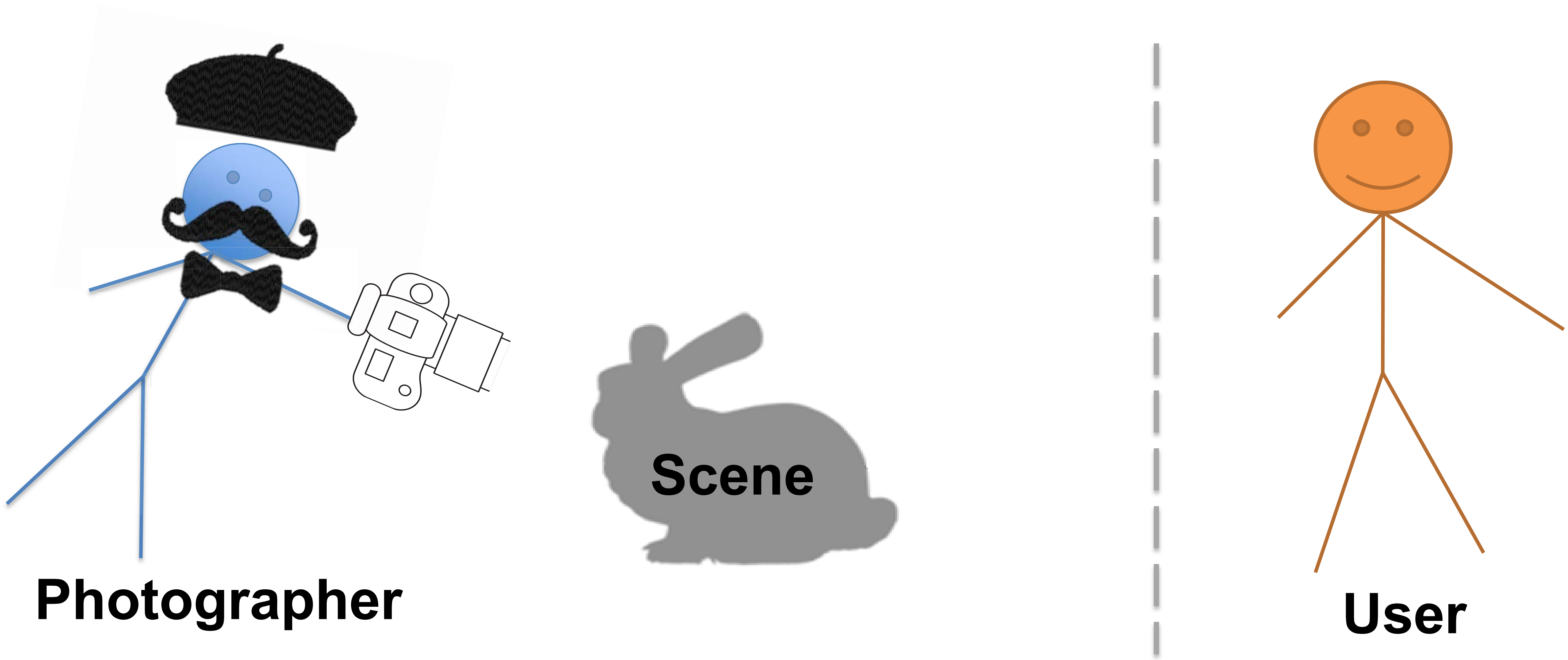
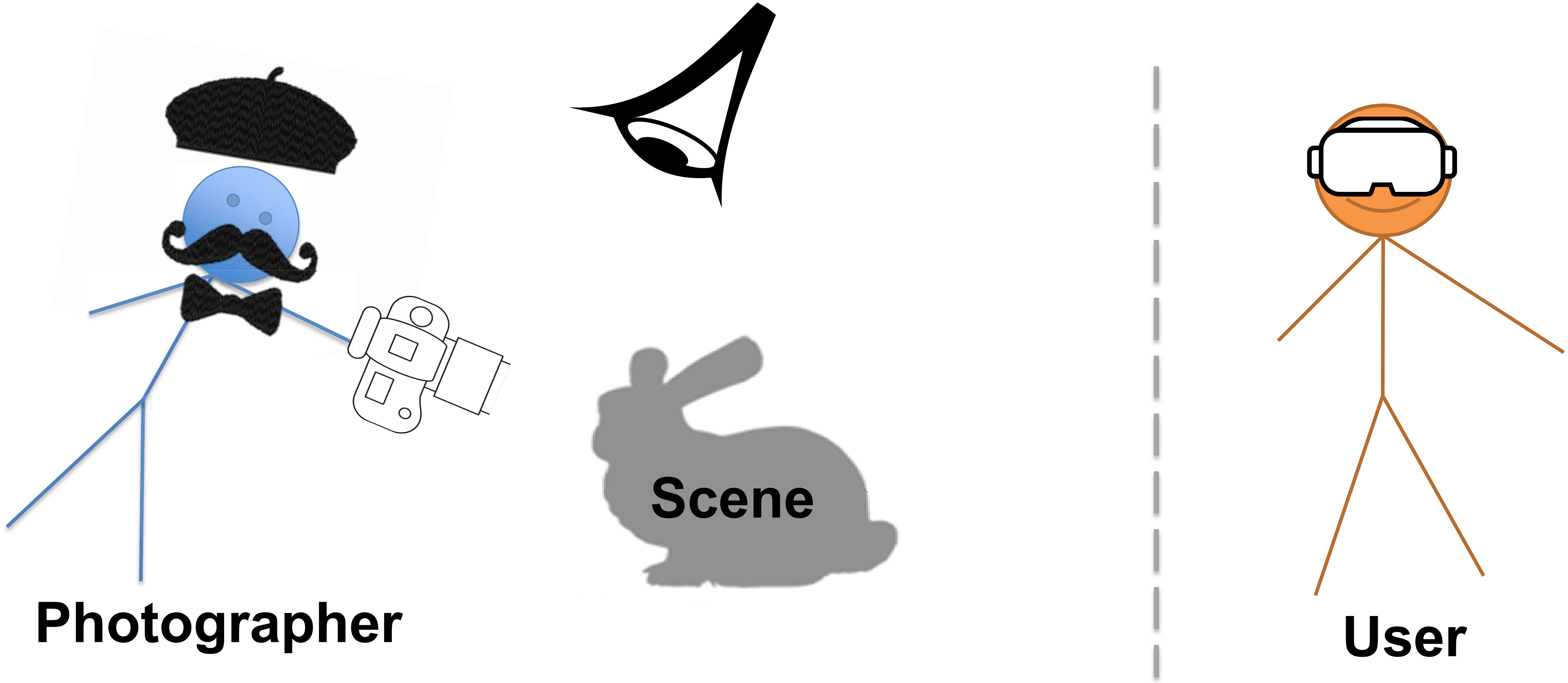
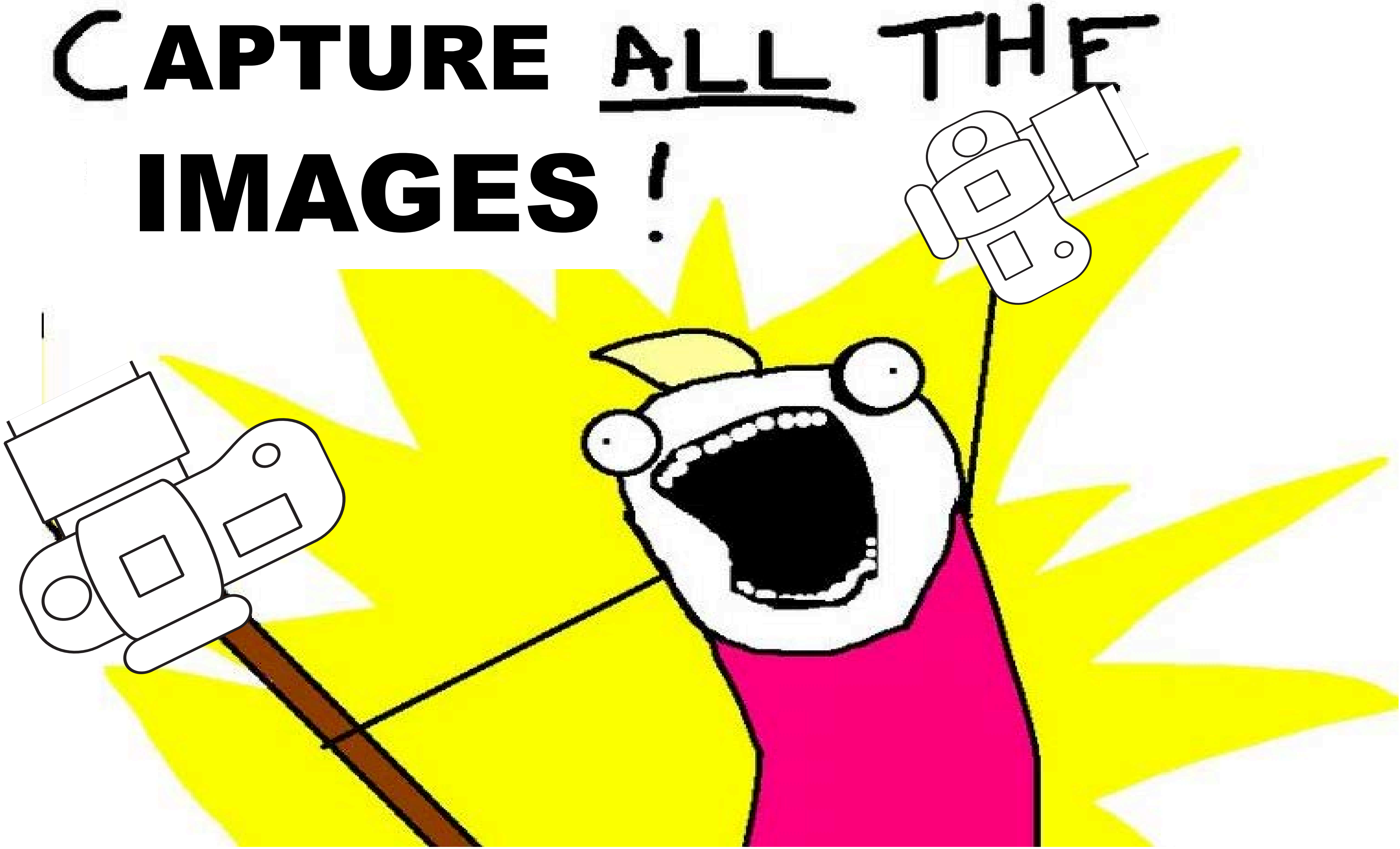


Image-Based Rendering

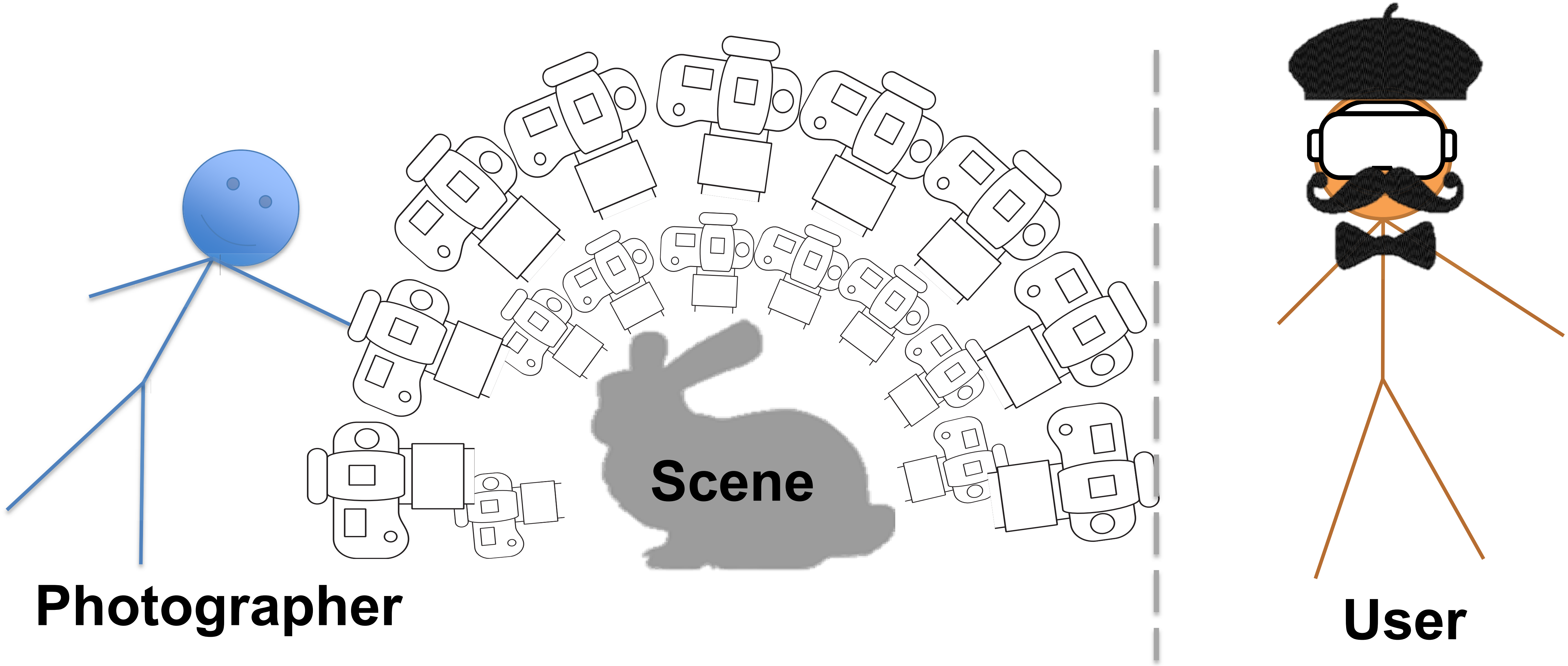


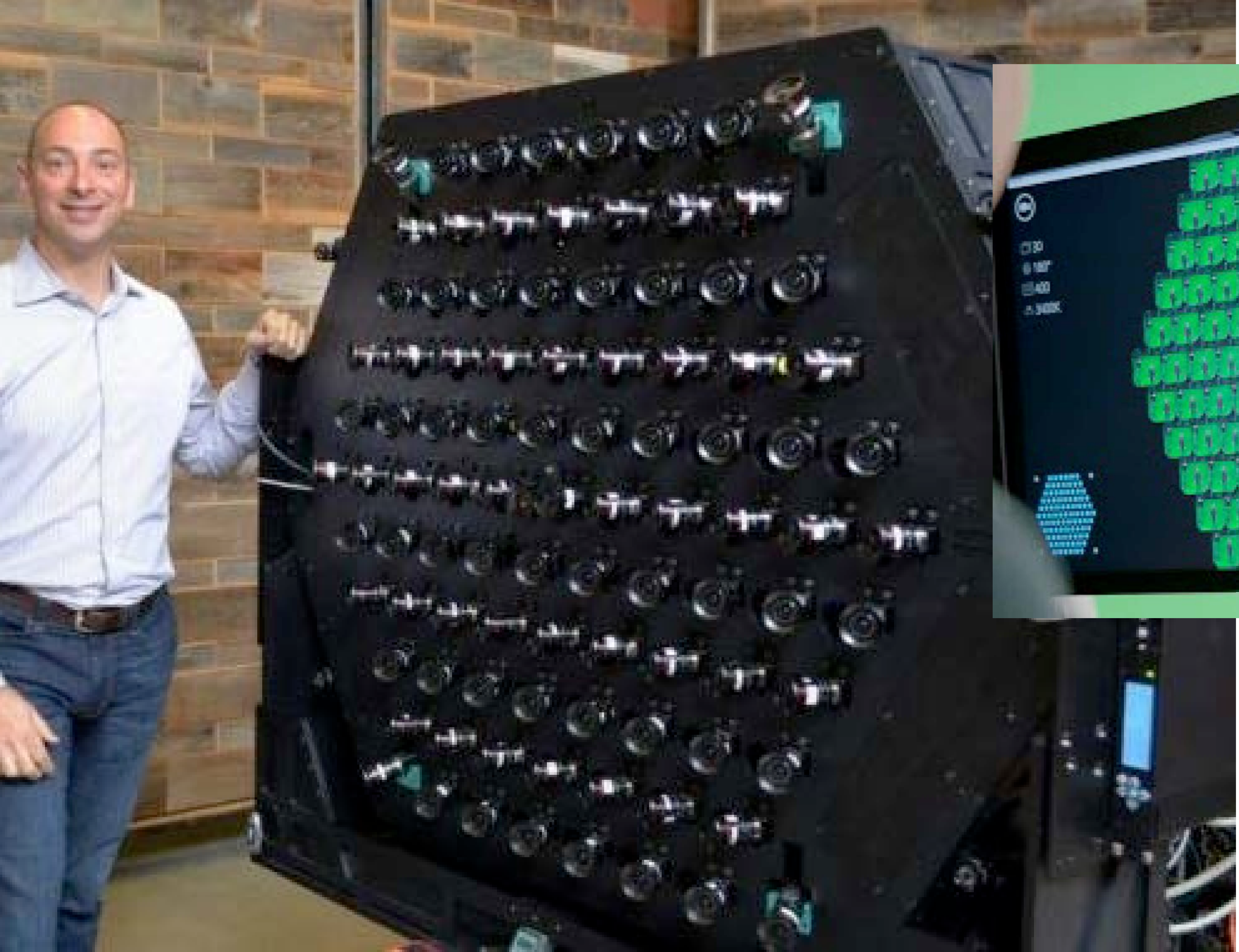
What would be the simplest, most naïve, brute force approach to give the viewer control of the camera?



HyperboleAndAHalf

“Light Field” Photography



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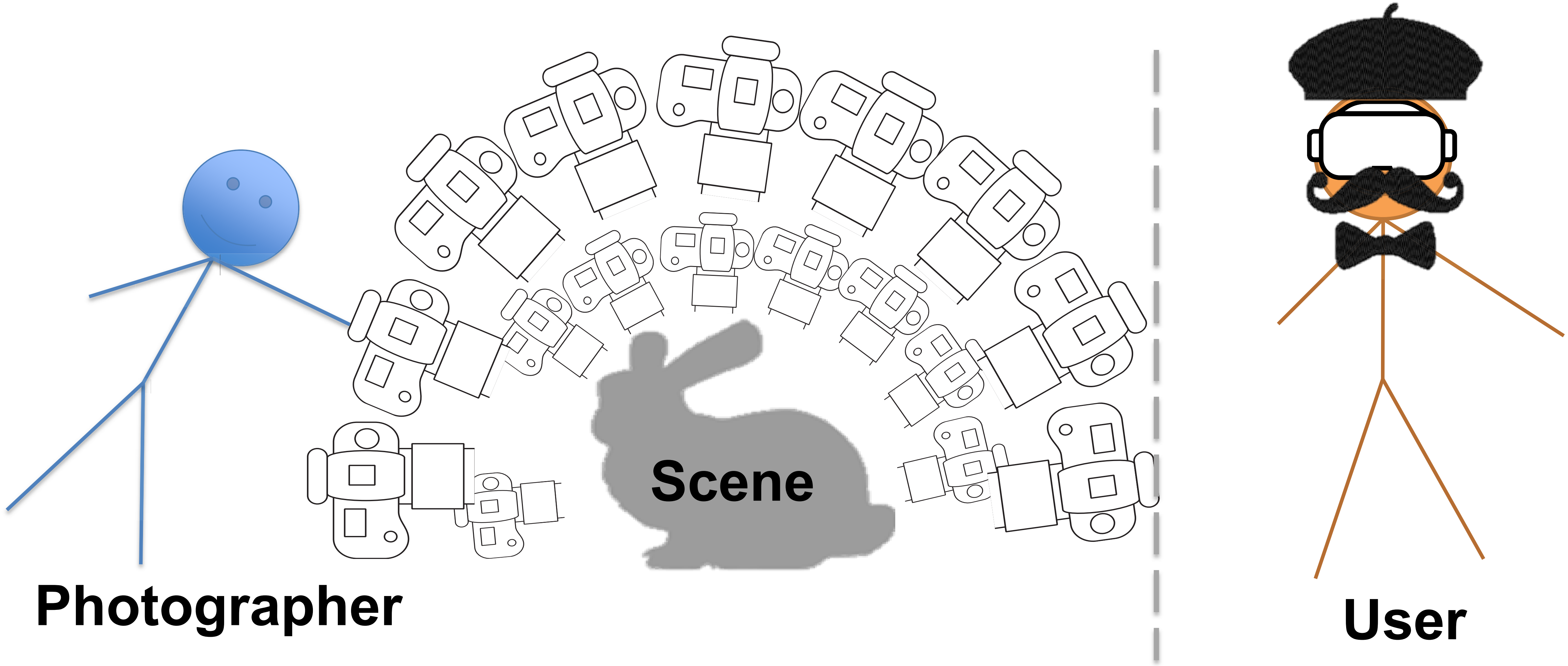
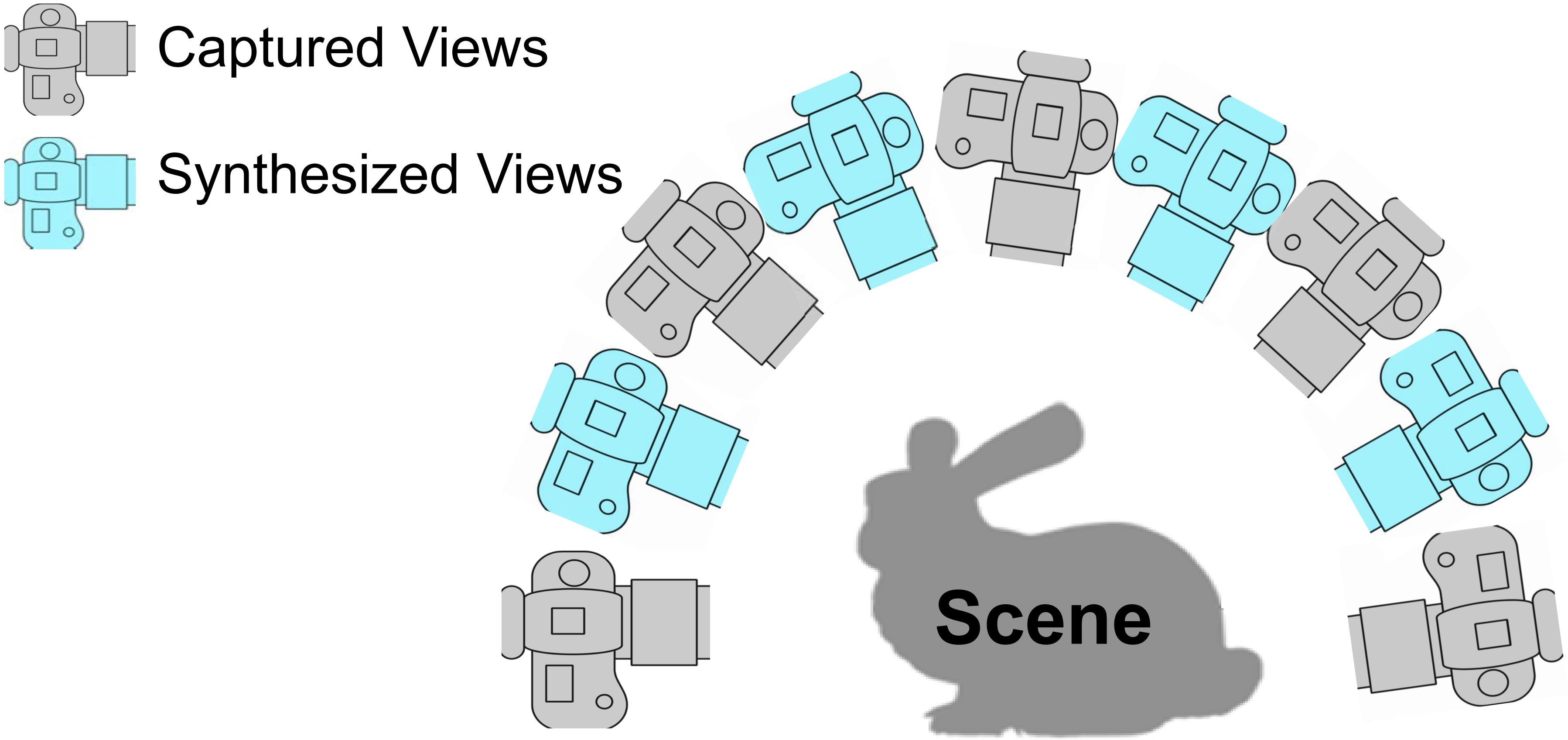


We can’t capture *all* the images

Light Field Photography



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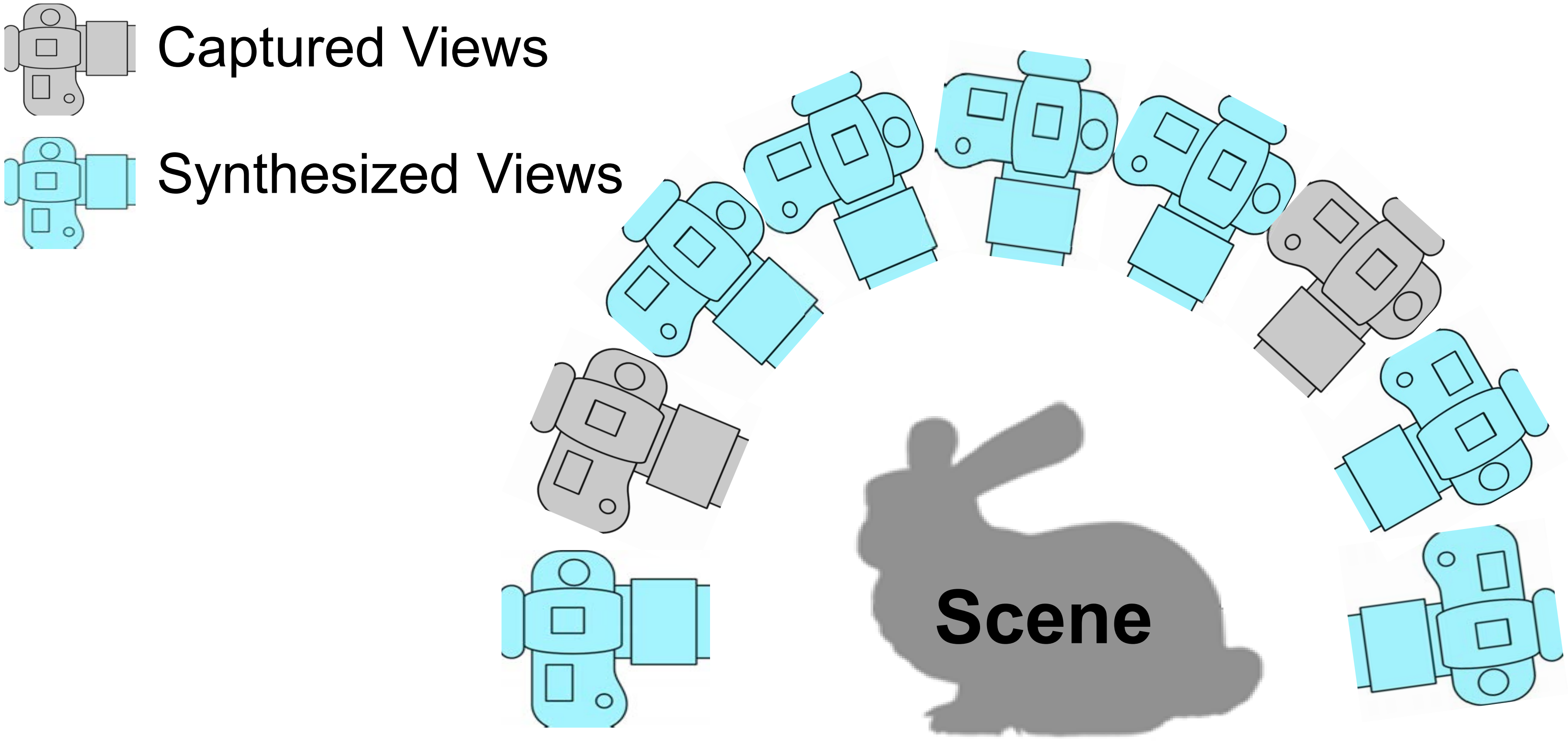
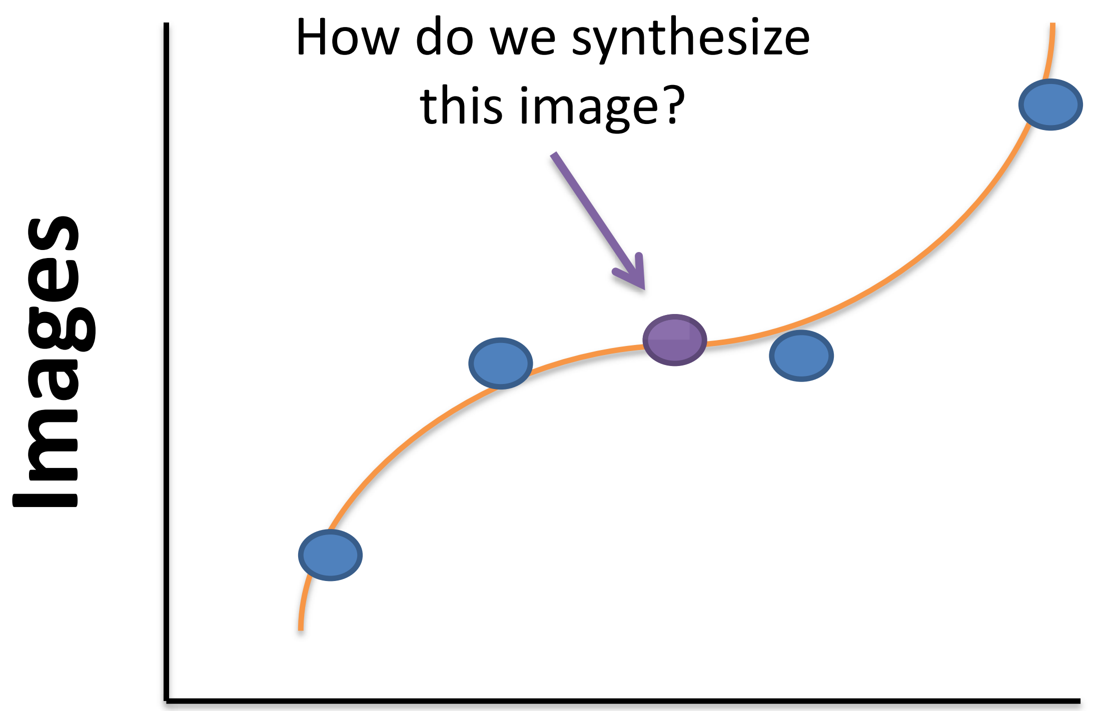


Sampling and Reconstruction

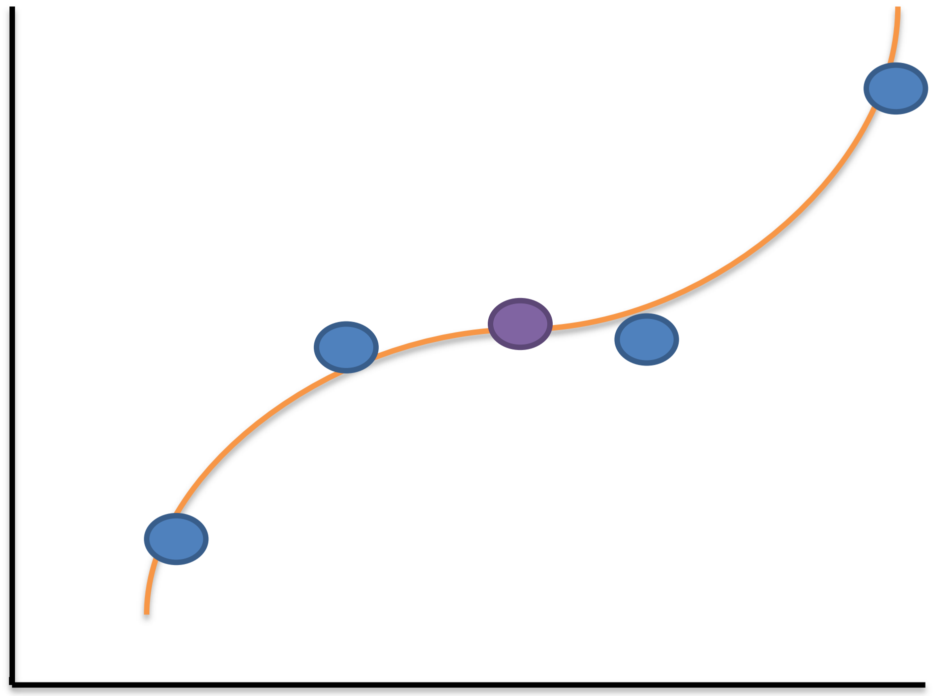


Camera parameters

(e.g. position, orientation, focus, depth of field…)

Sampling and Reconstruction

- How do we sample?
- What space do we use to represent our data?
- How do we Interpolate in that space?
- How do we extract images from that space?



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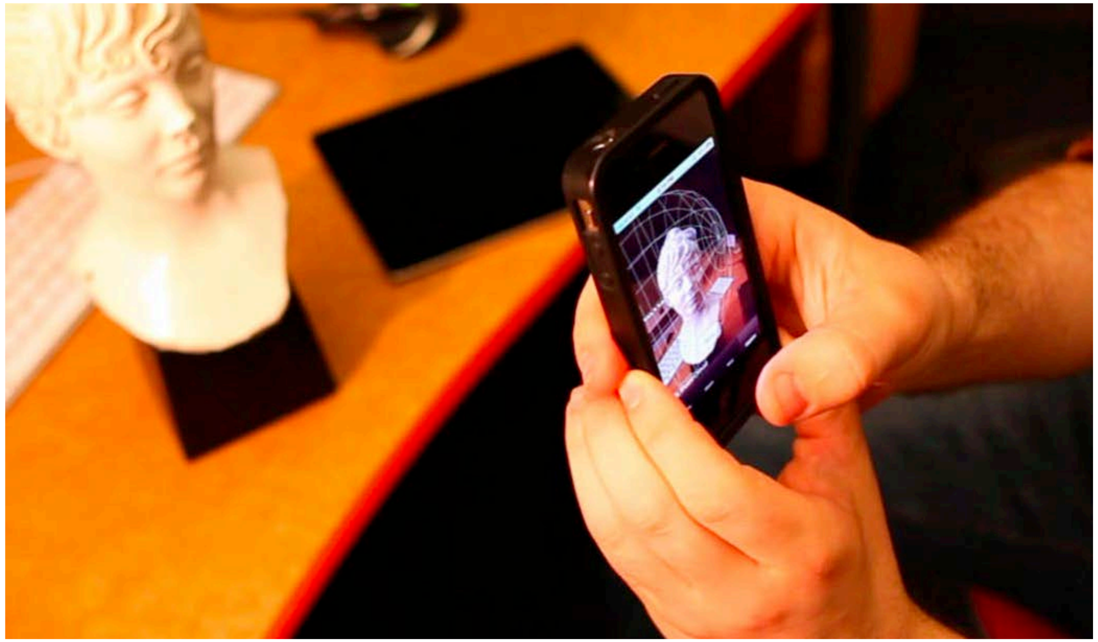

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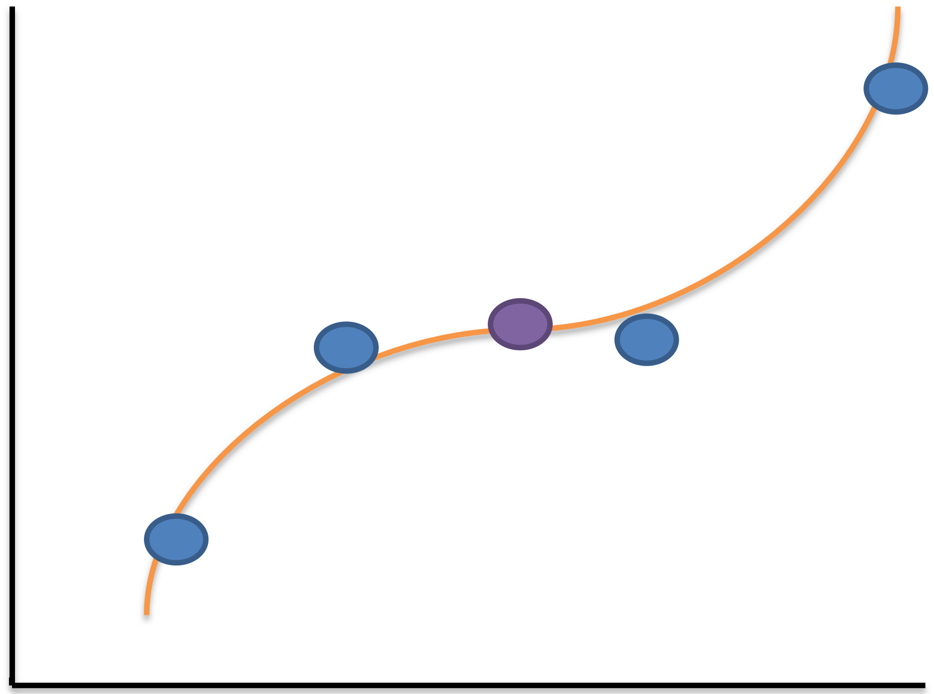
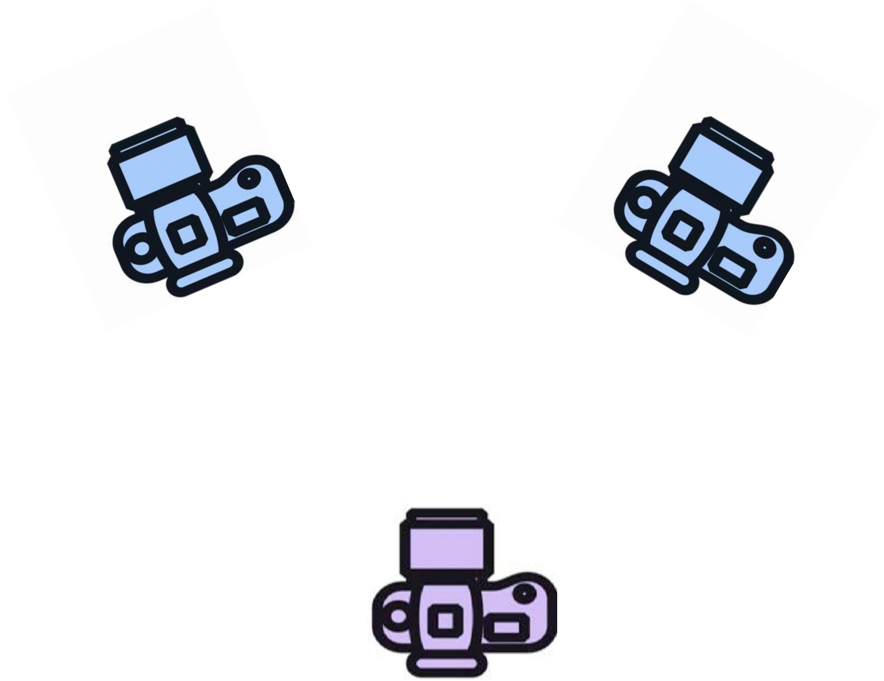
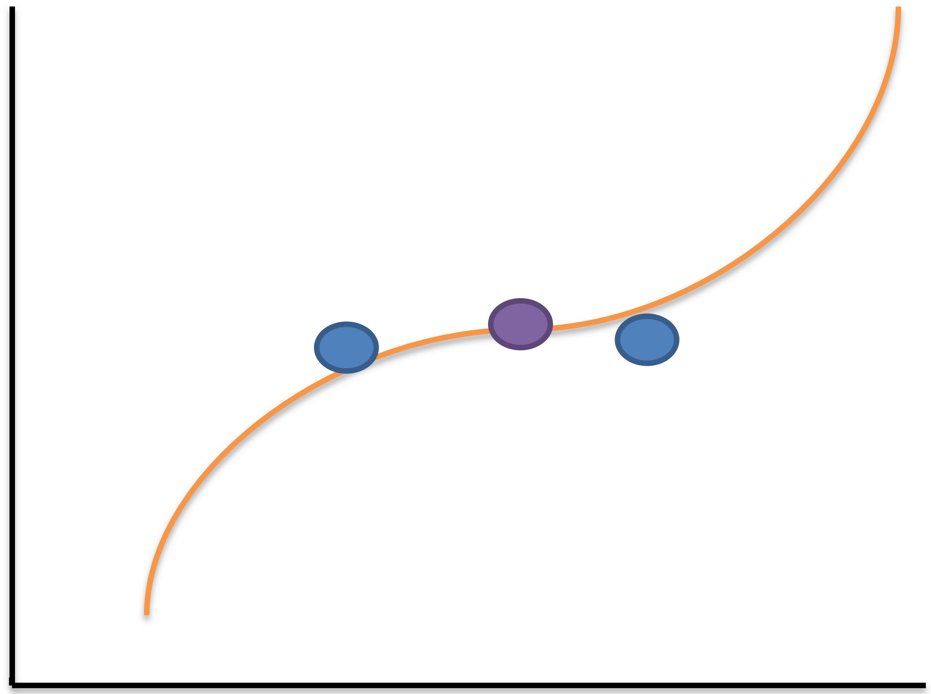
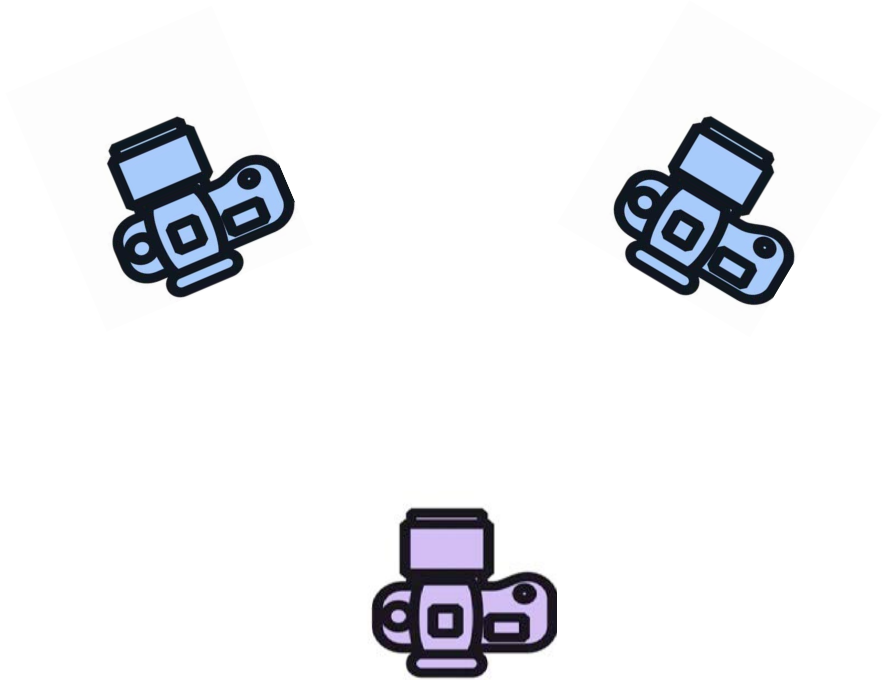


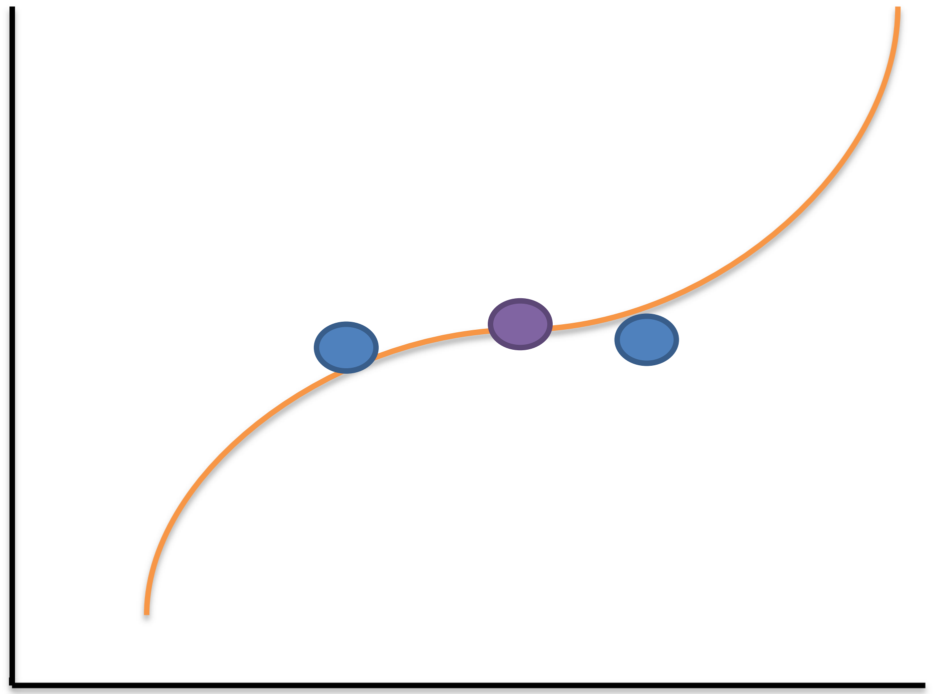
Sampling and Reconstructing Rays



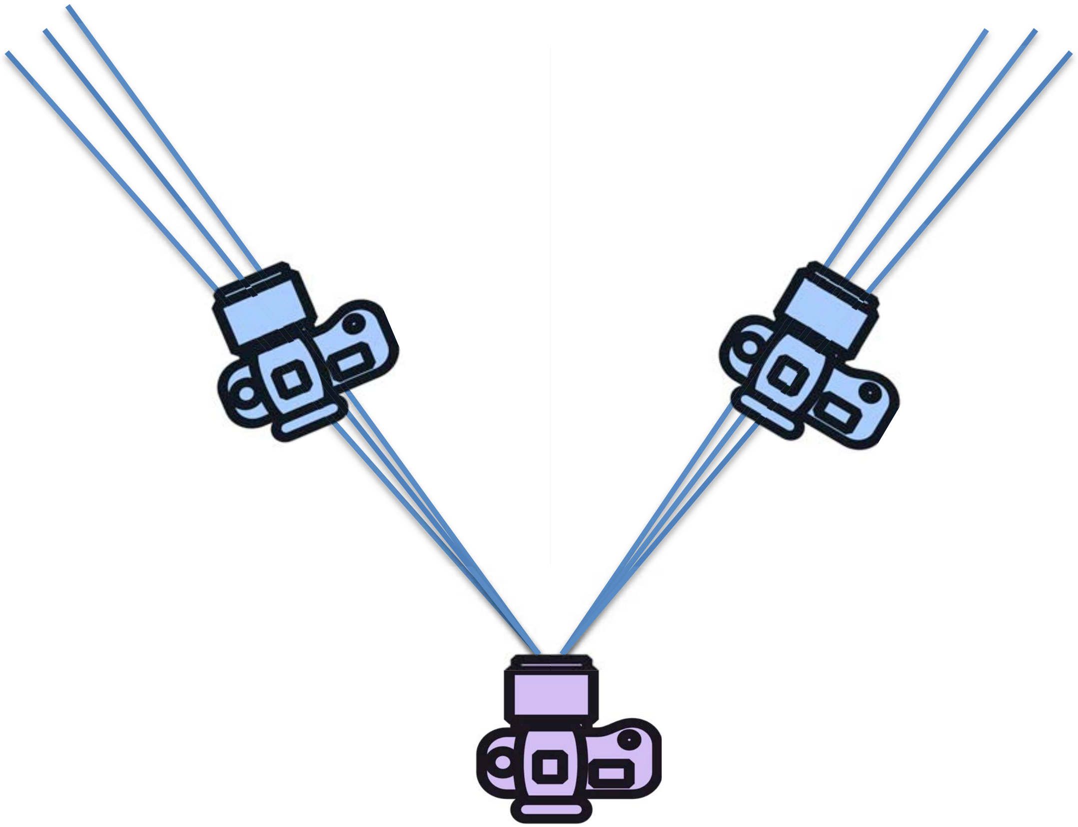


Sampling and Reconstructing Rays

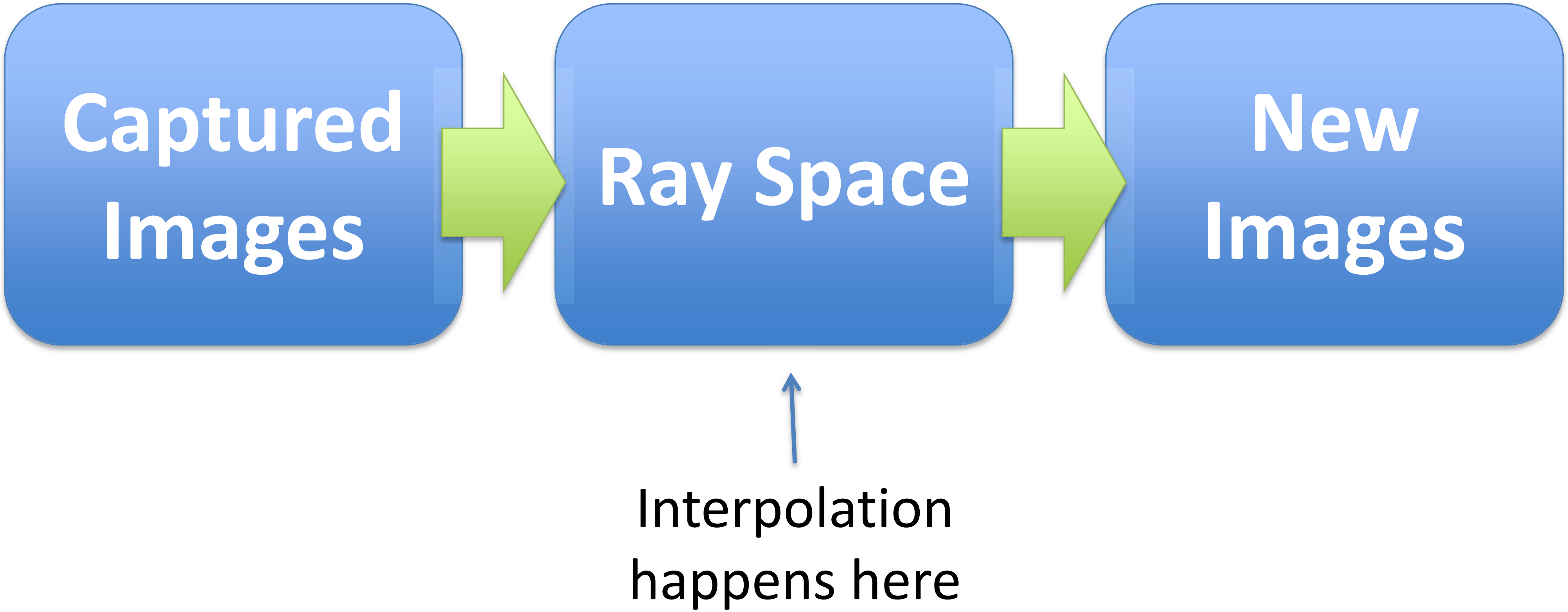




Sampling and Reconstructing Rays



Sample ≈ Pixel ≈ 1 Ray of Light

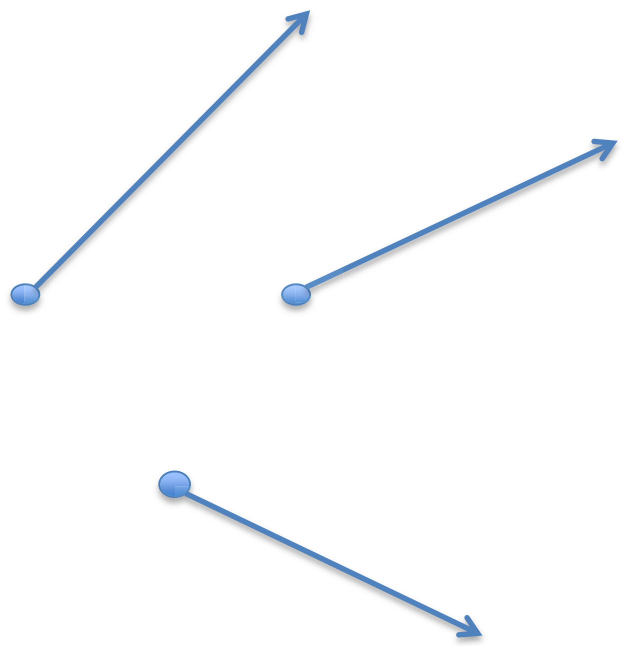

Captured Images
Ray Space
New Images
Interpolation happens here

How should we parameterize light?

Light ray = f(?)

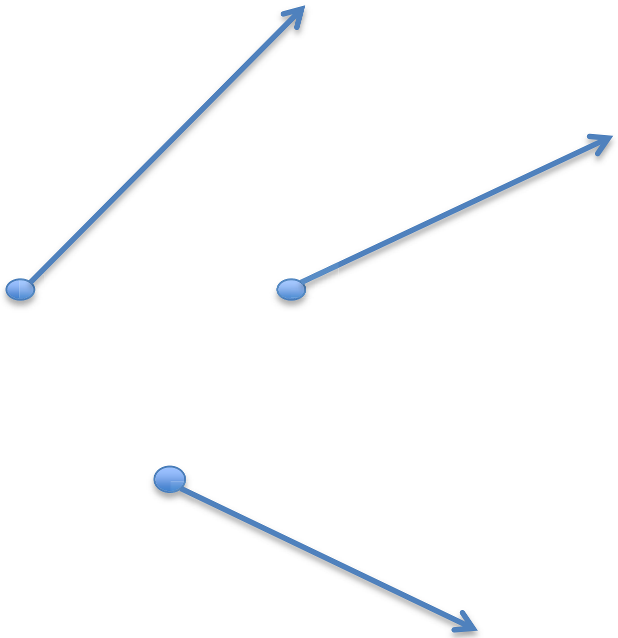
Light

- Radiance:
  - R(position, angle)
  - How many dimensions?



Light

- Radiance:
  - R(position, angle)
  - Position = (x,y,z)
  - Angle = (theta, phi)
  - 5 dimensions



What is a good parameterization for light?

- The Light Field

What is a good parameterization for light?

- The Light Field
  - Unobstructed light
  - Each ray defined by intersection with 2 planes

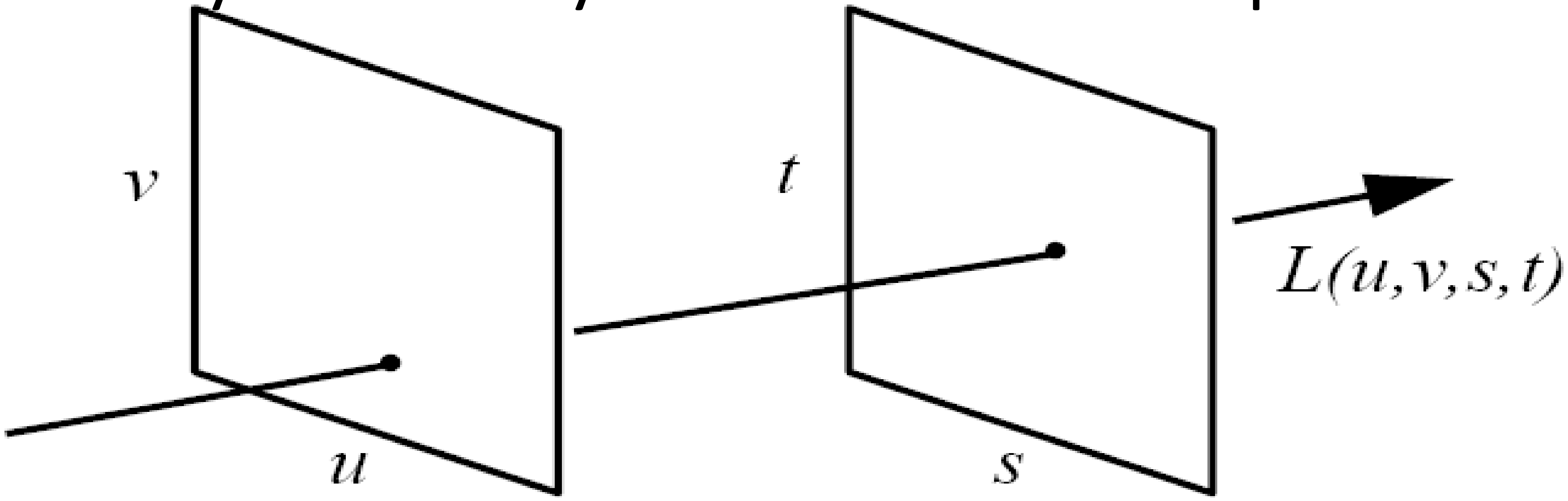


Figure 1: The light slab representation.

What is a good parameterization for light?

- The Light Field
  - Unobstructed light
  - Each ray defined by intersection with 2 planes

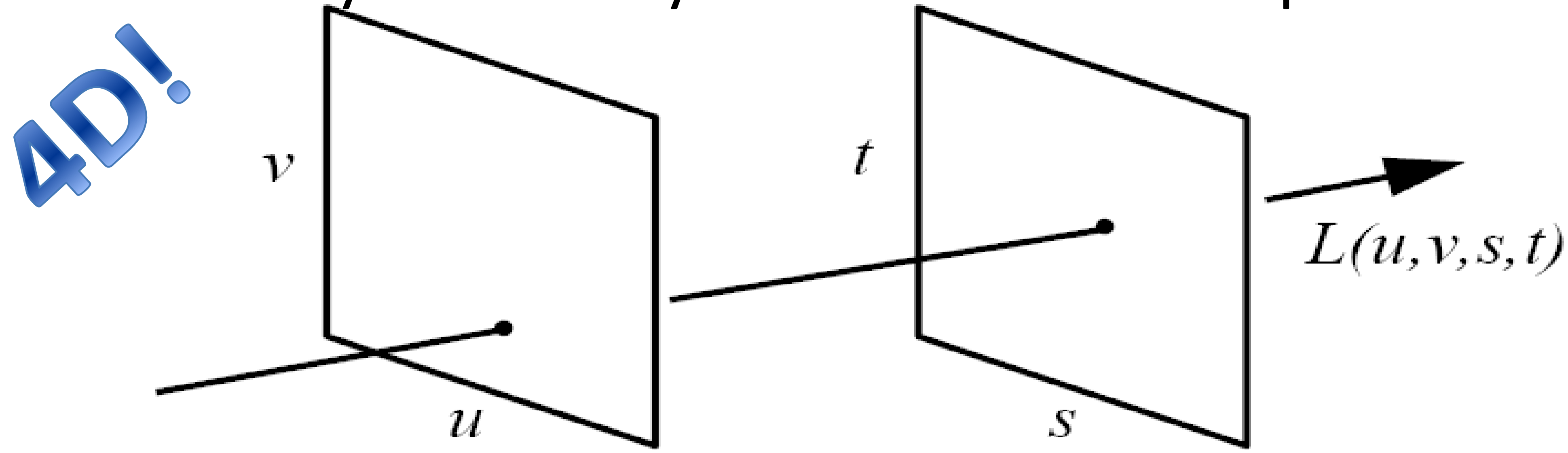
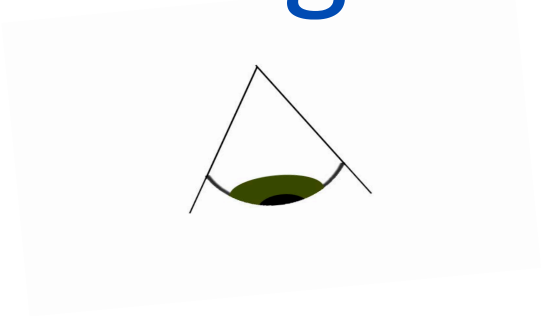


Figure 1: The light slab representation.

The Light Field



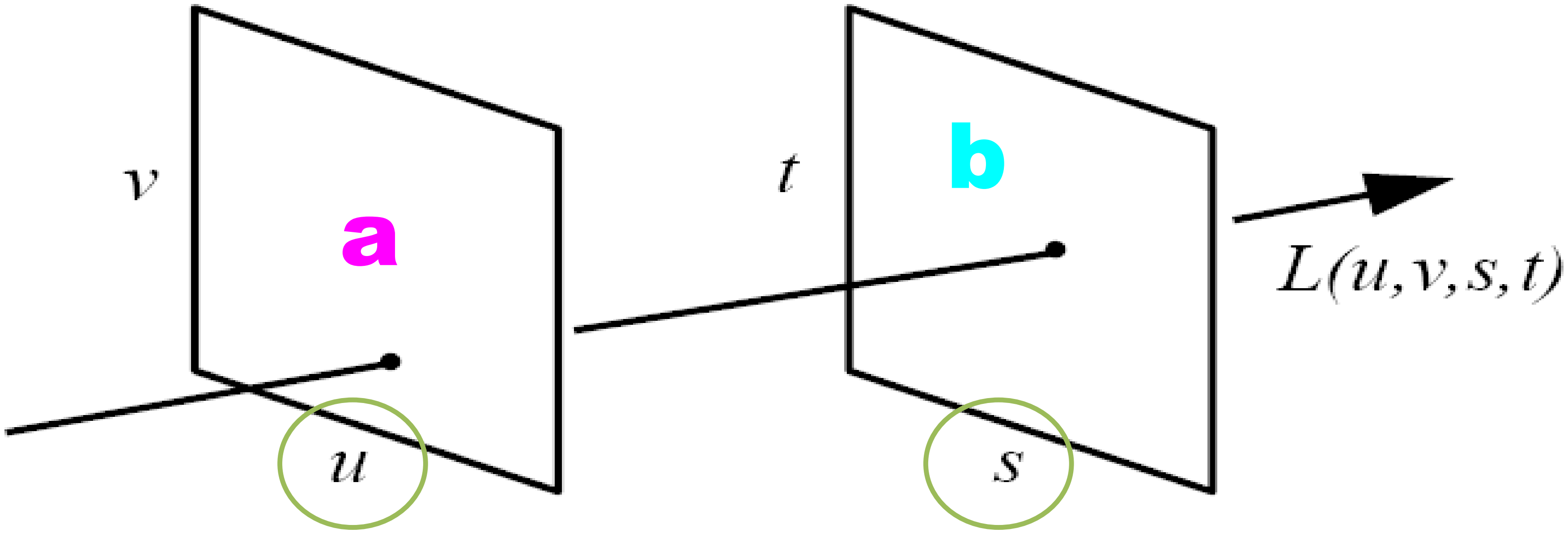
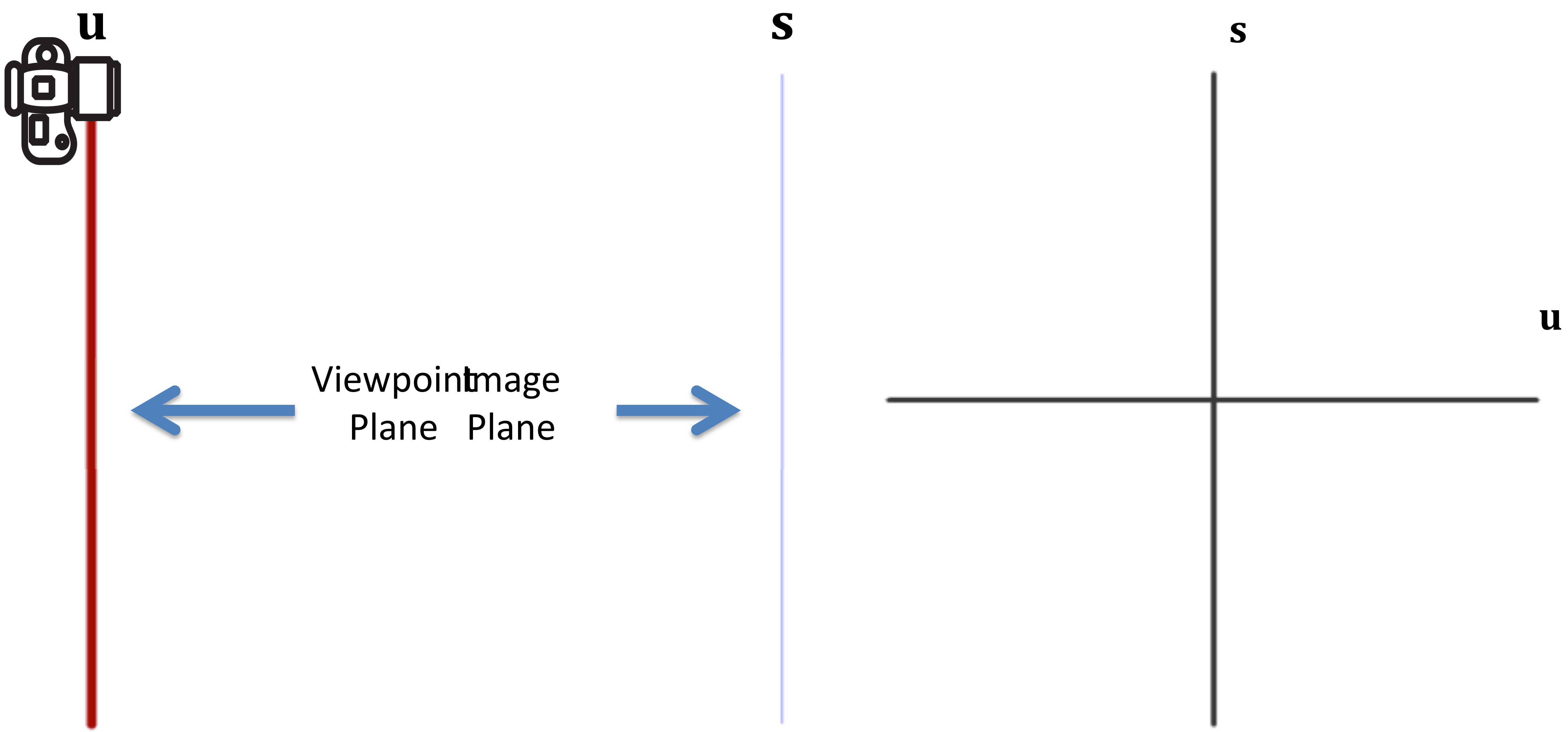
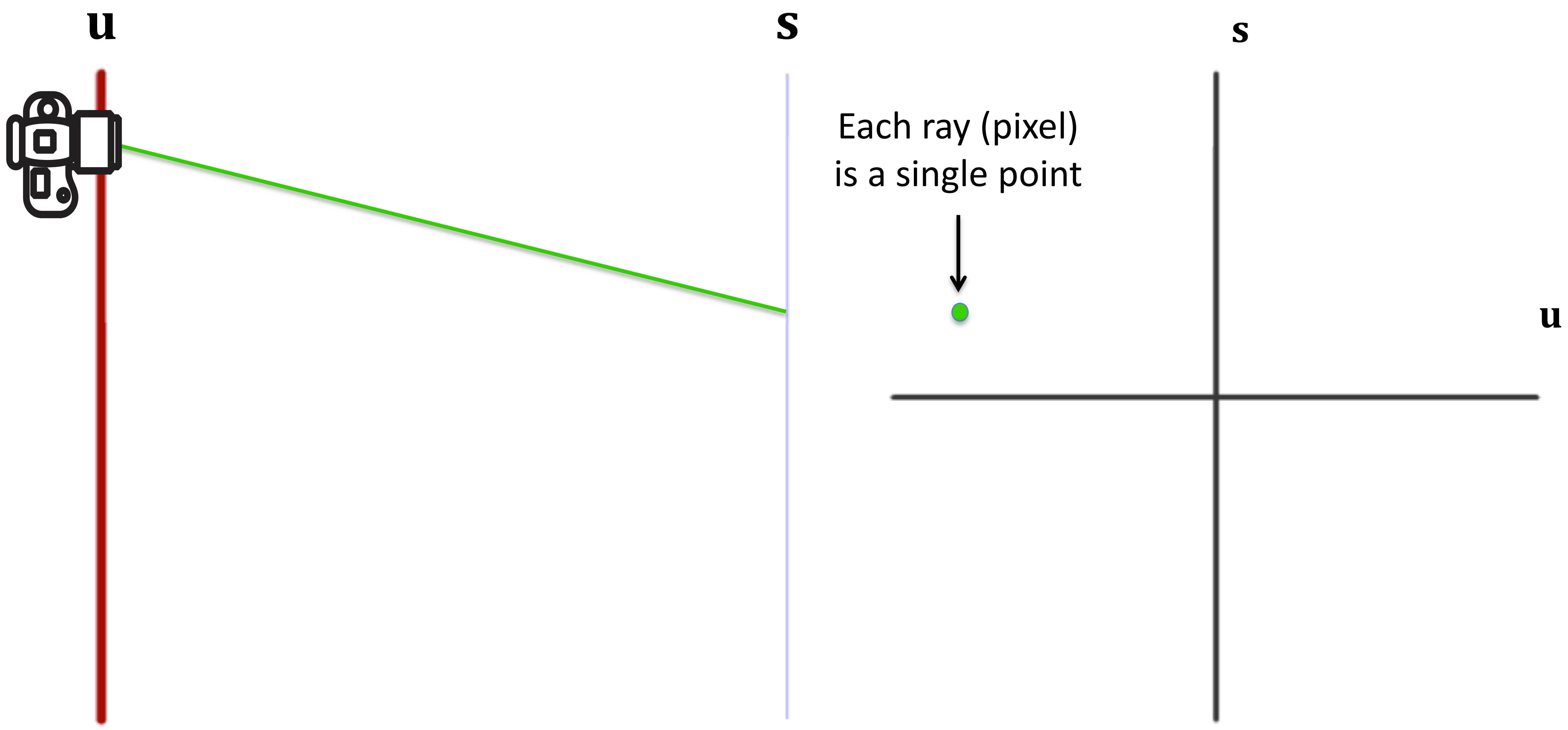


Figure 1: The light slab representation.

Ray Space



Ray Space



Ray Space

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Ray Space

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Ray Space

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Ray Space

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Ray Space

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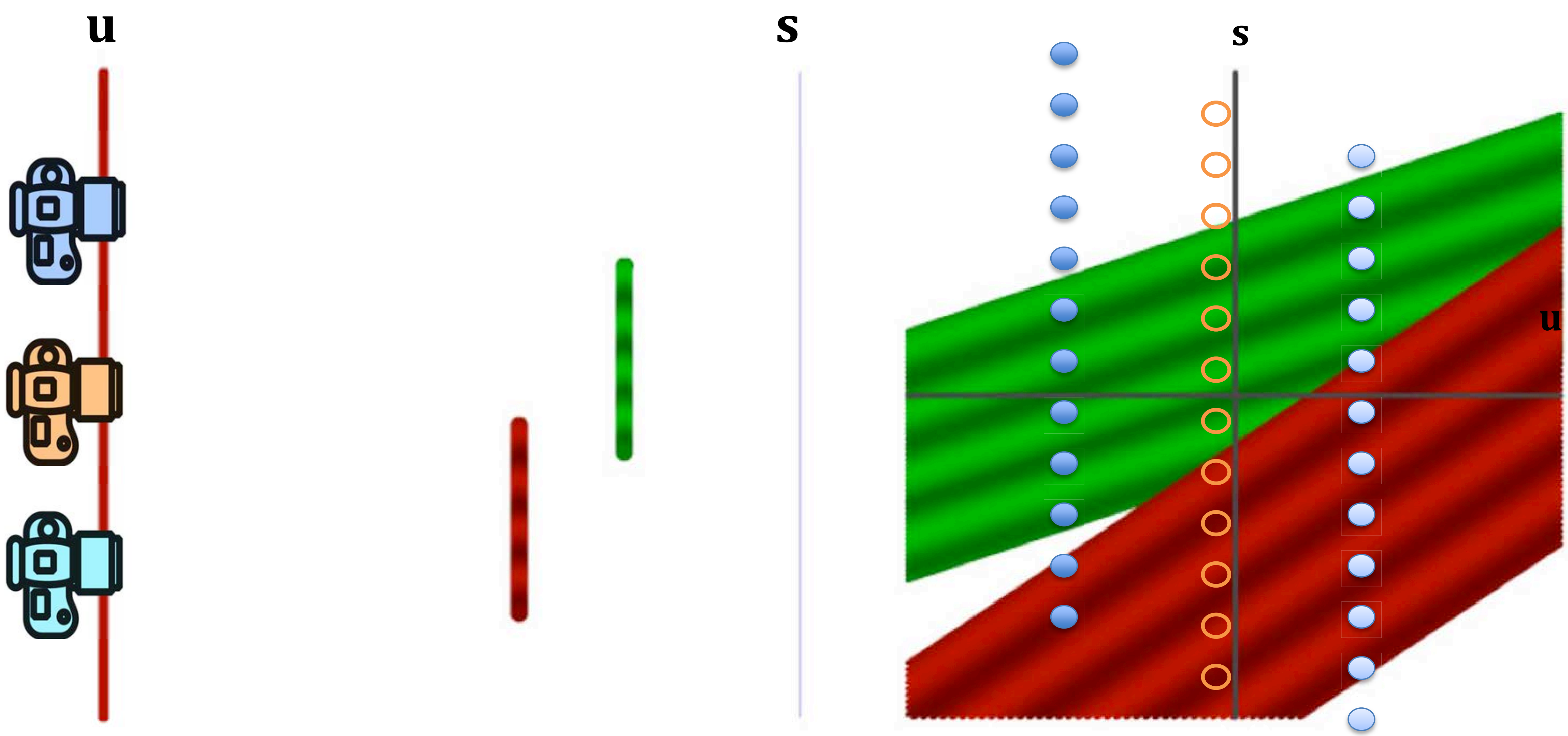
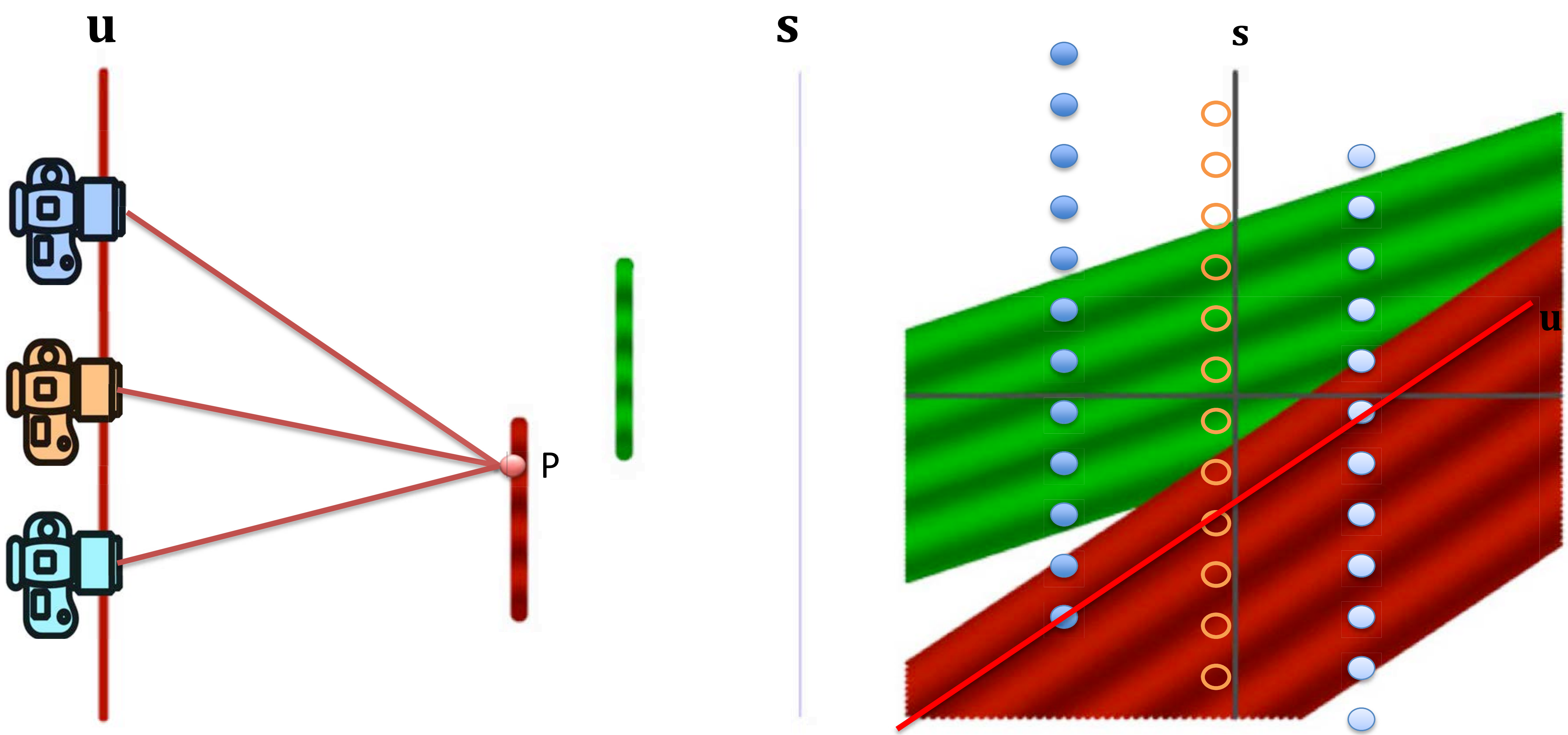
Ray Space



Ray Space



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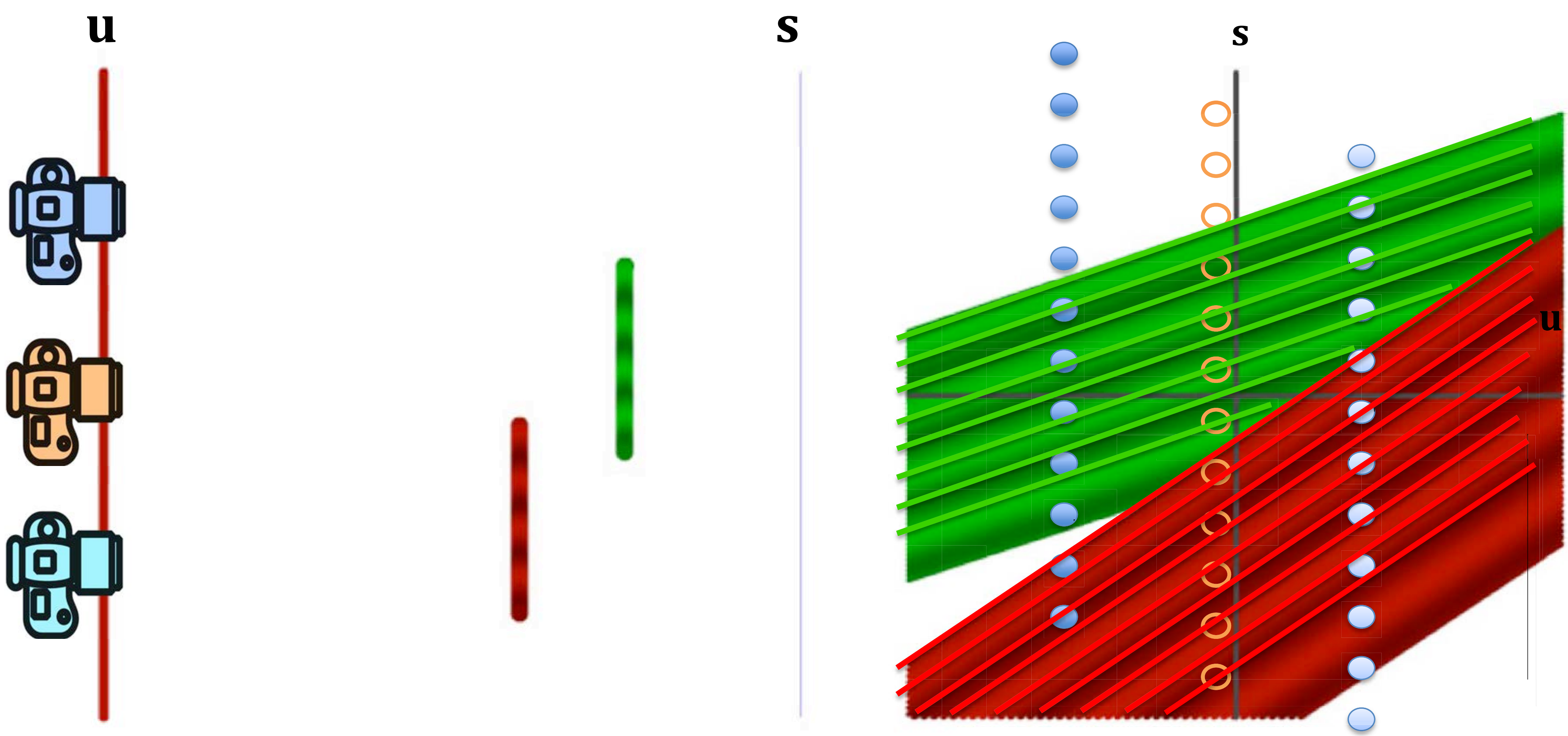

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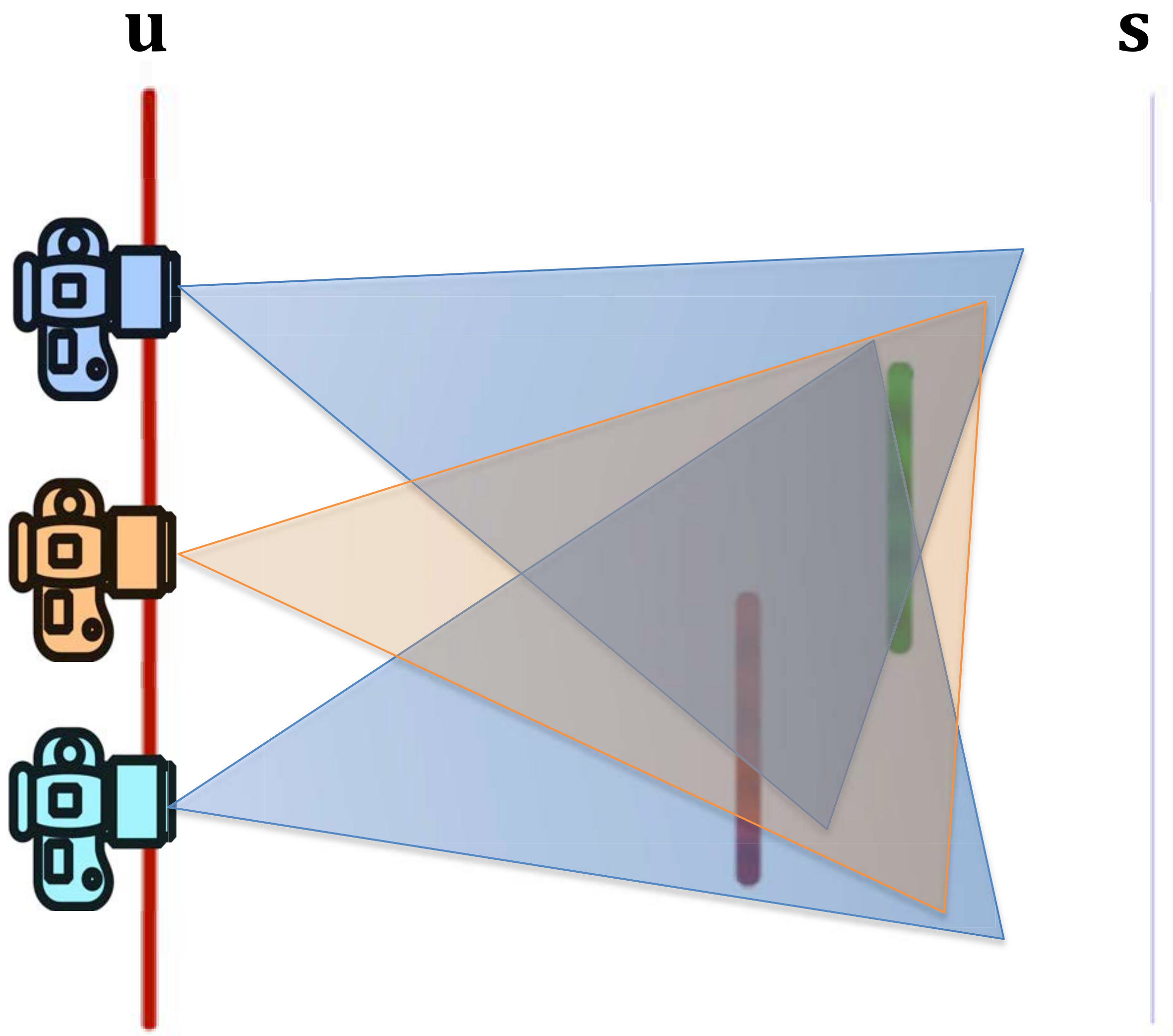
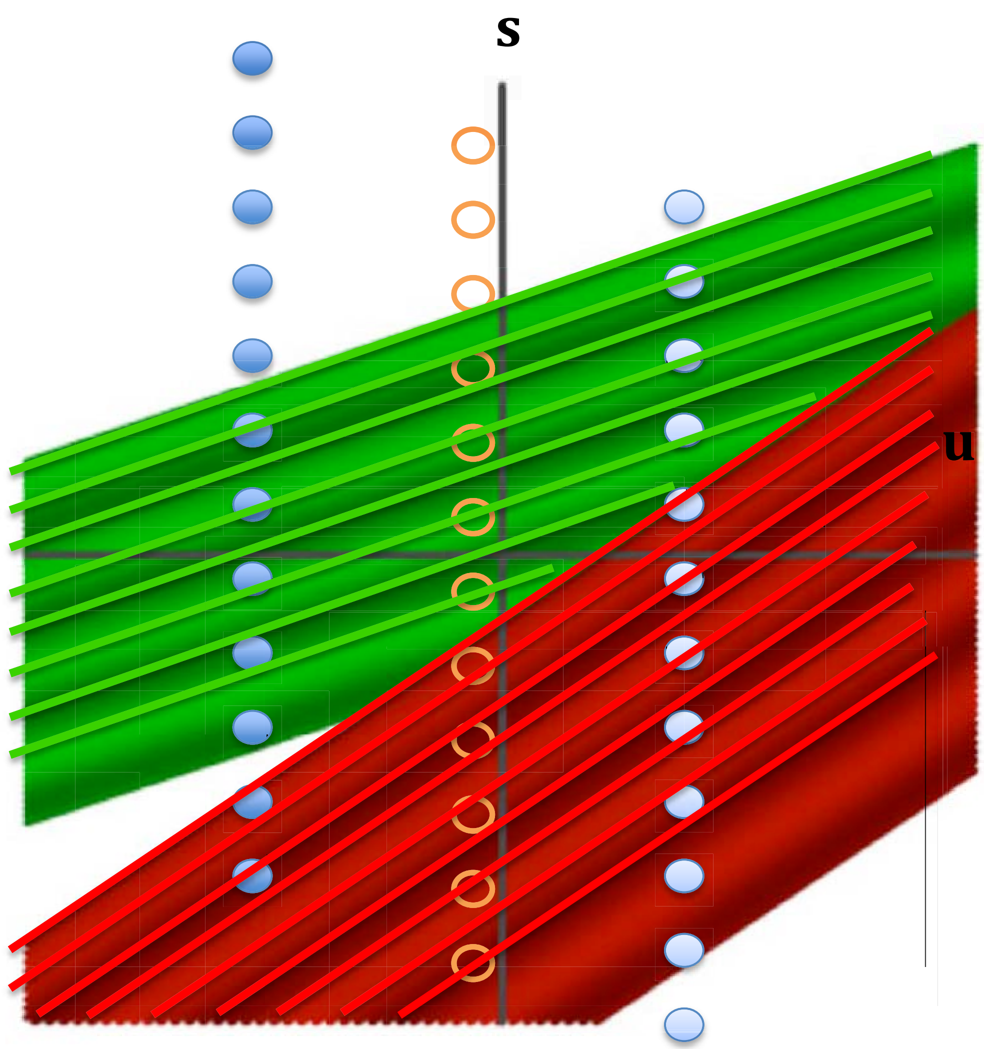
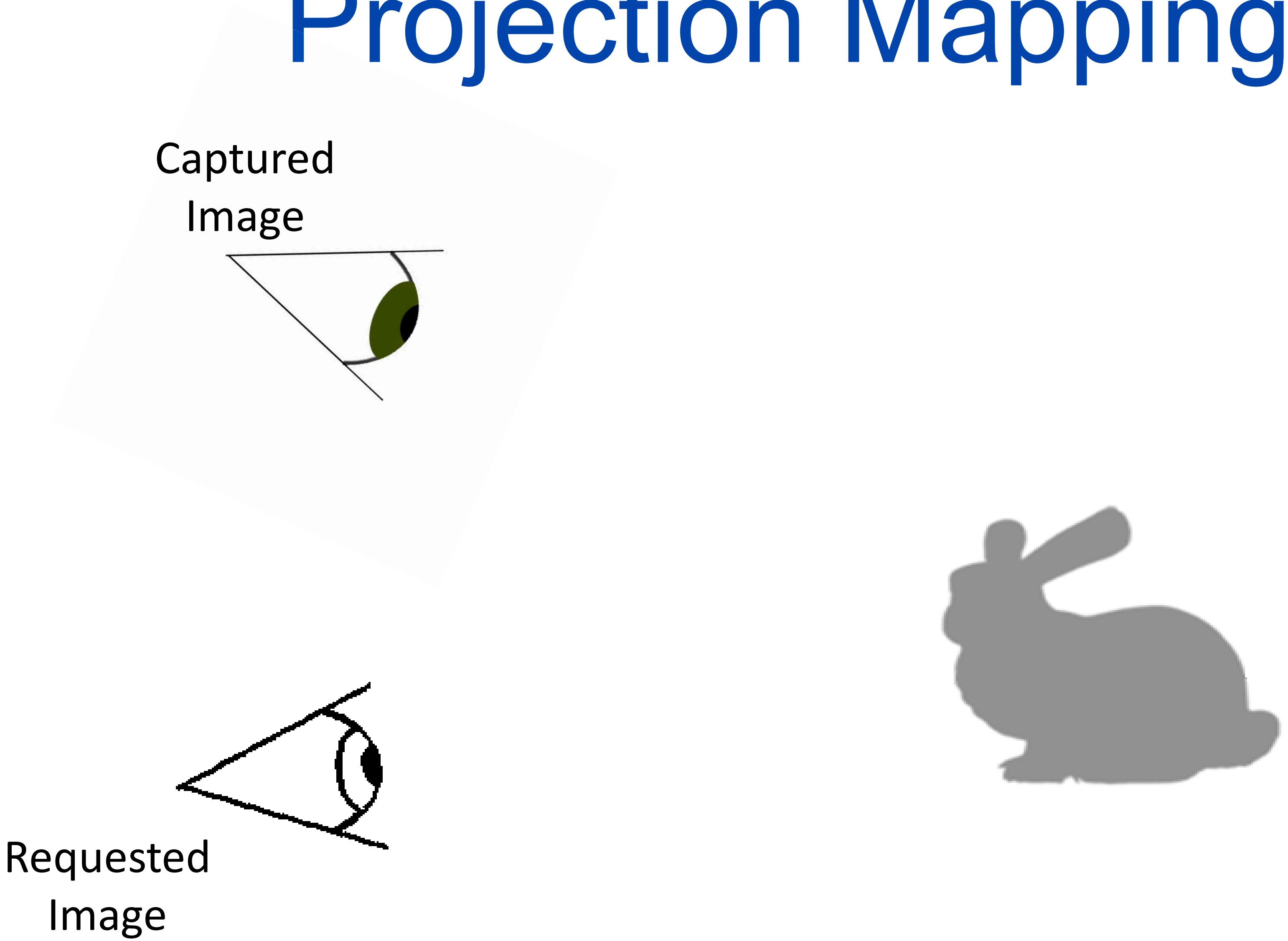


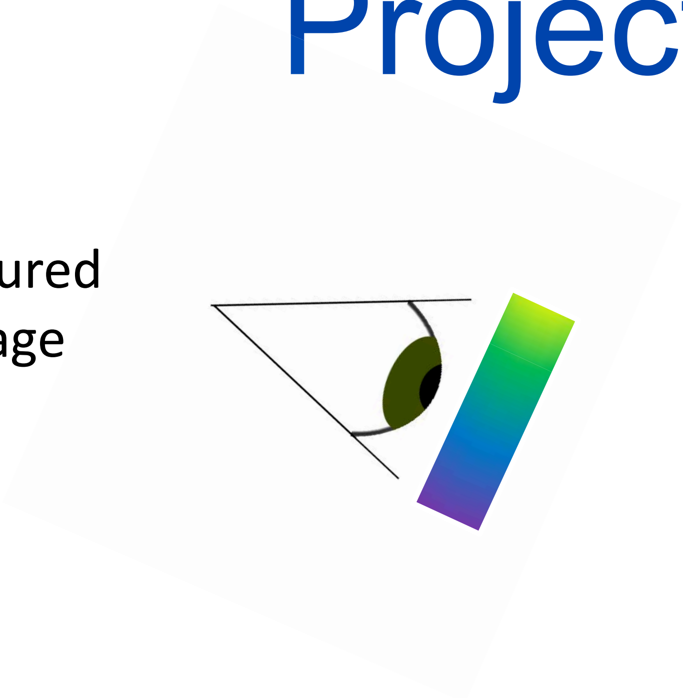


Projection Mapping

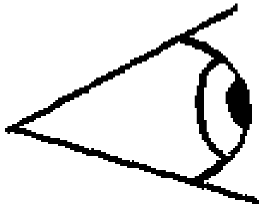


Projection Mapping

Captured Image

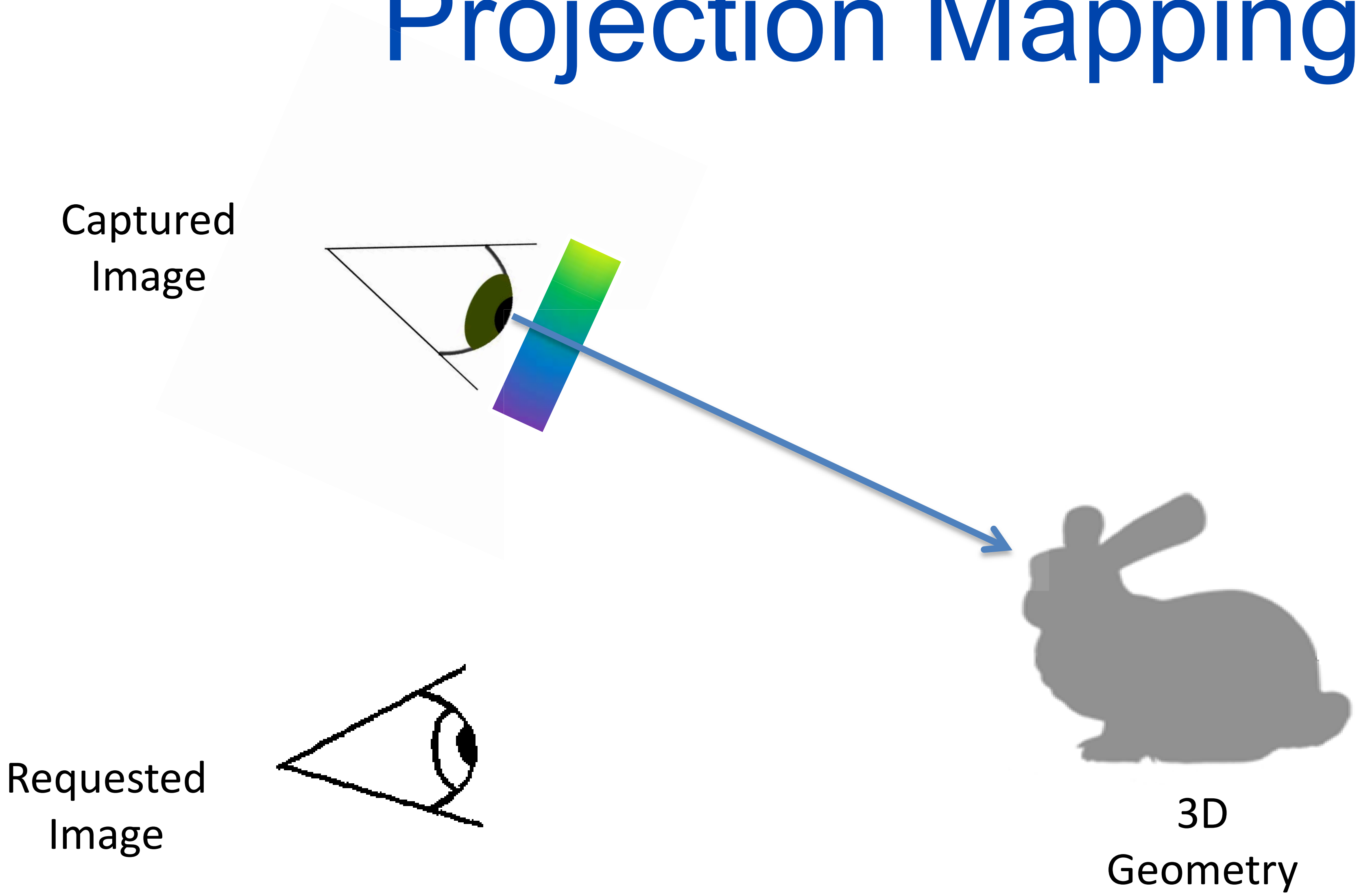


Requested Image

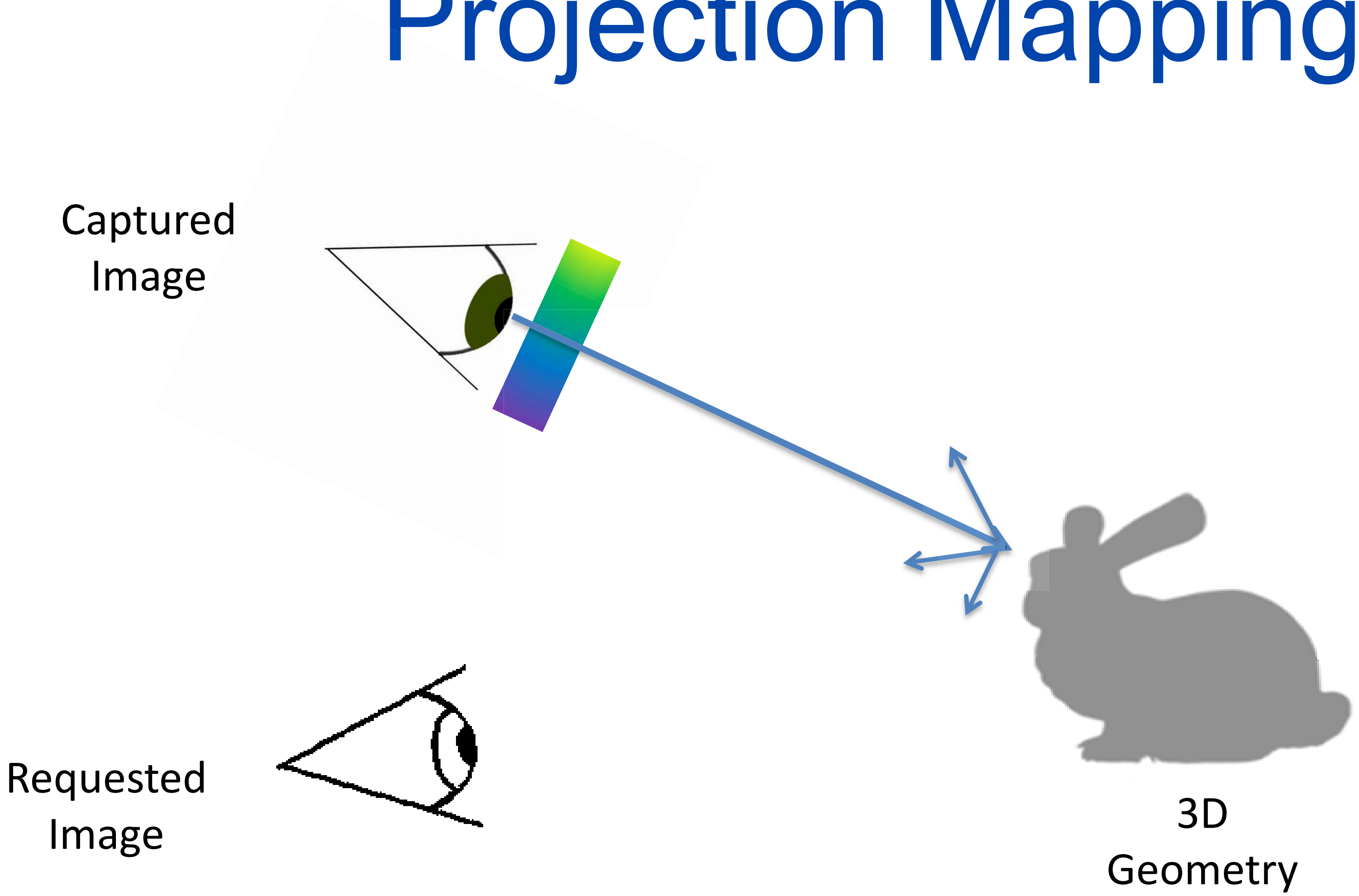


3D Geometry

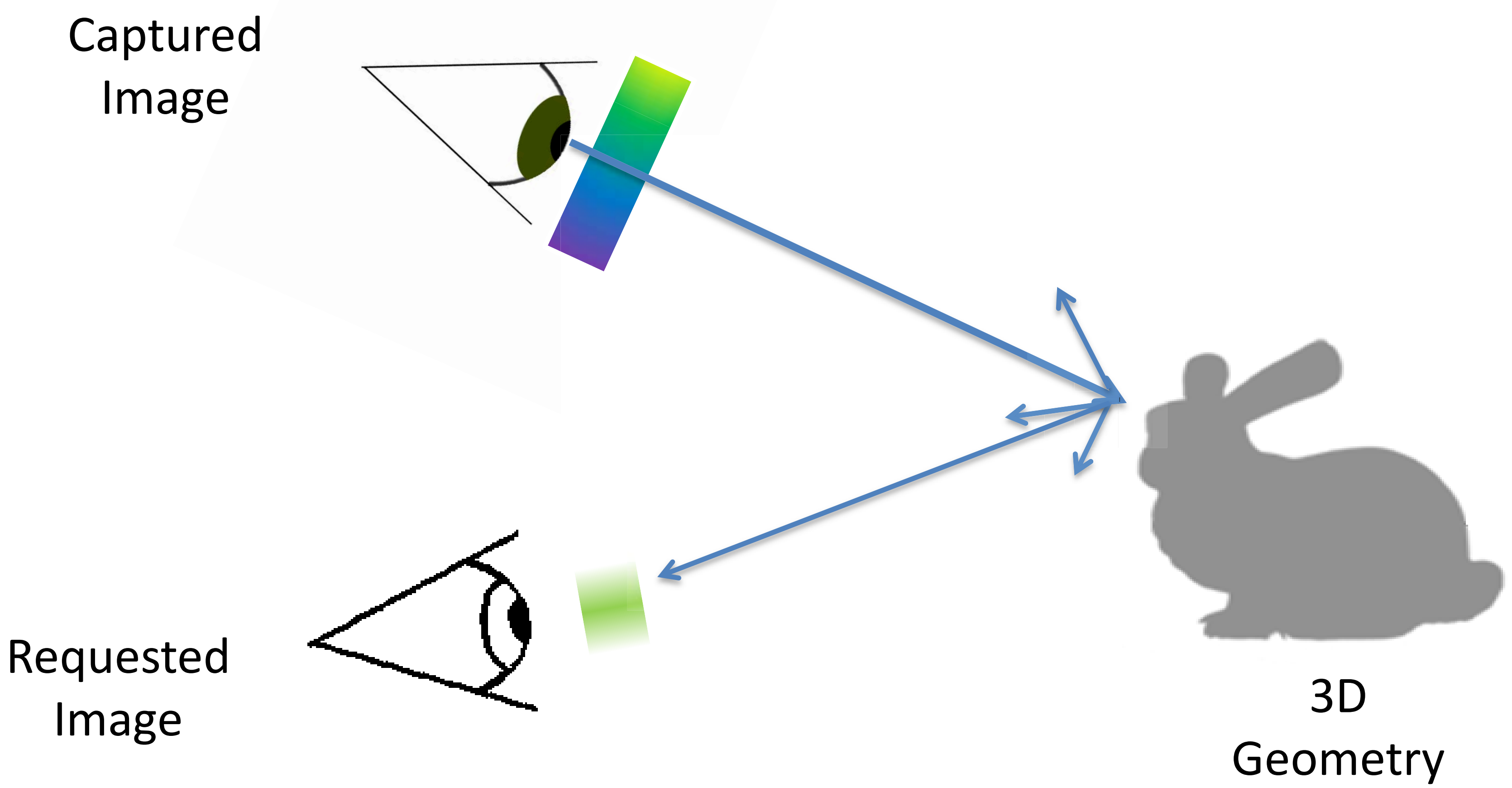
Projection Mapping



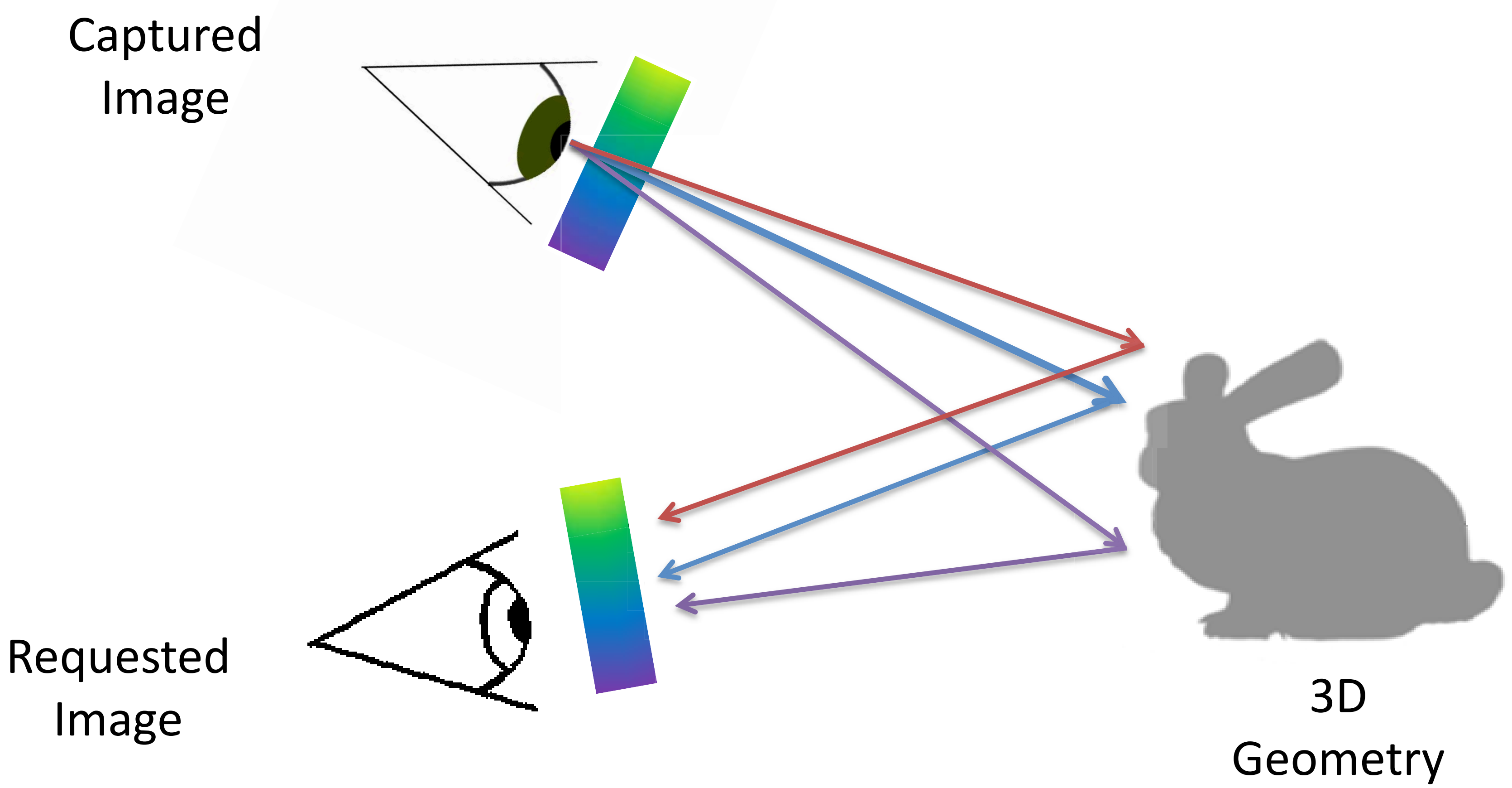
Projection Mapping



Projection Mapping

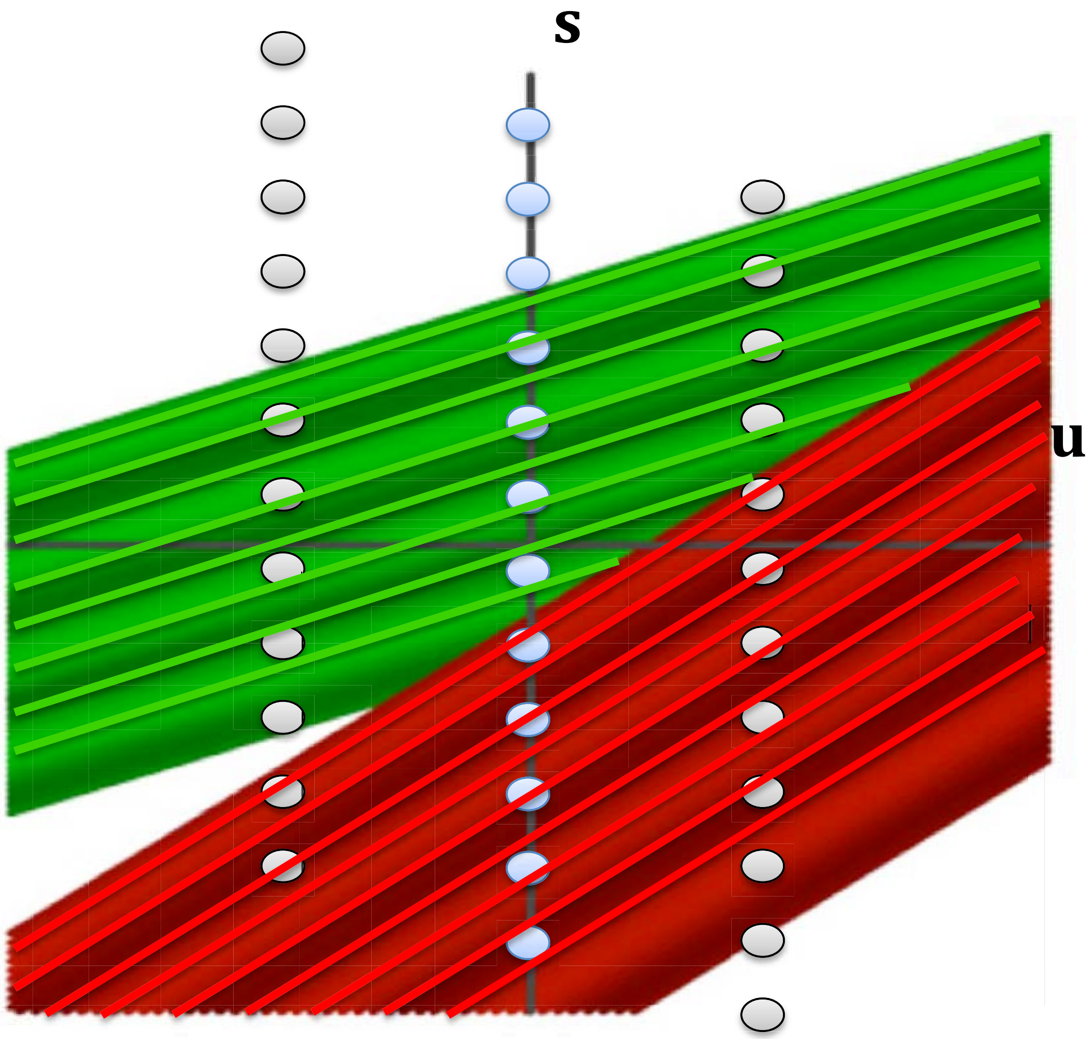


Projection Mapping

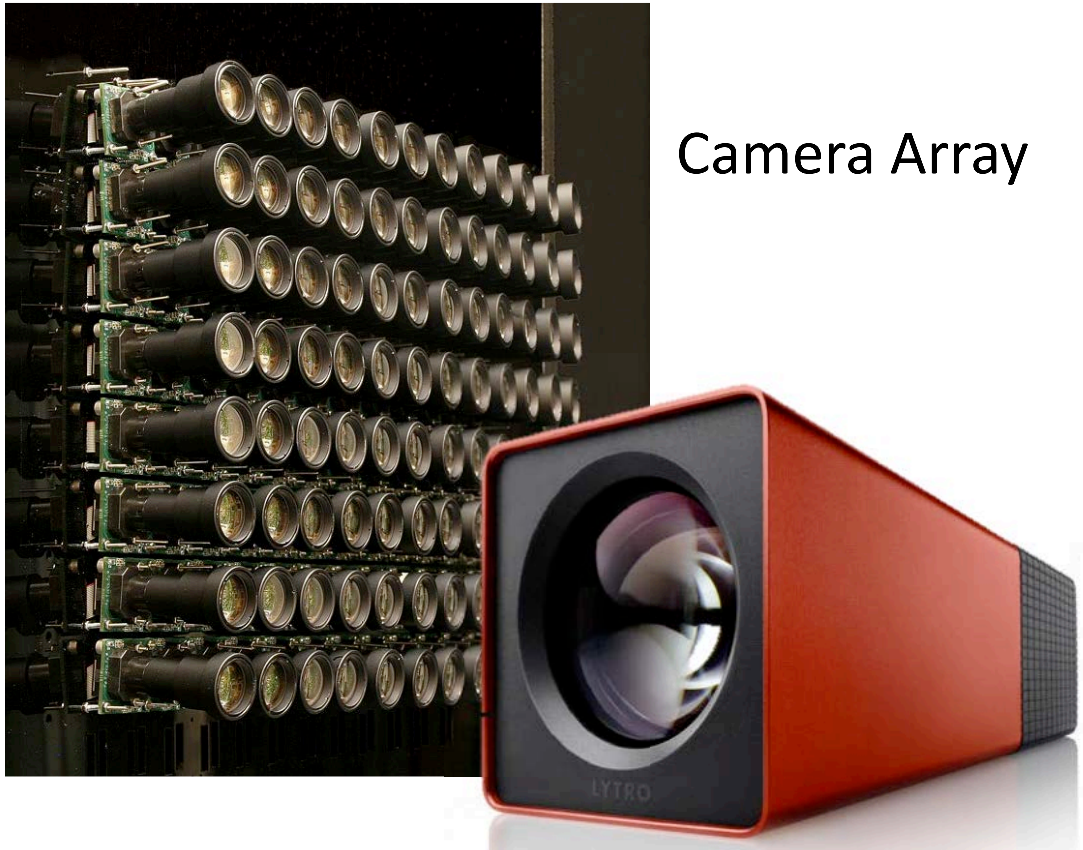


Ray Space

Stereo

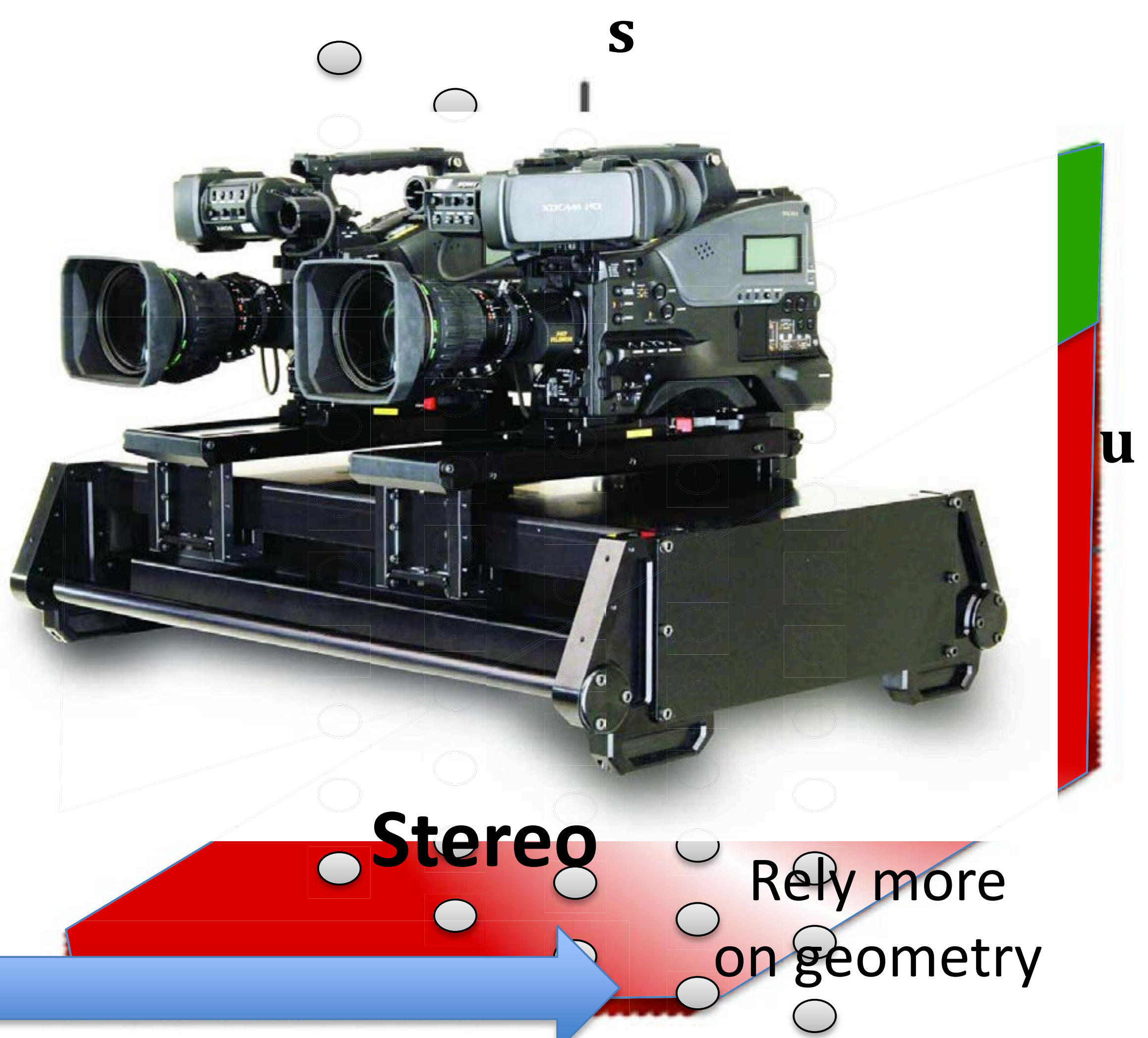


Capture Strategies



Camera Array

Lytro



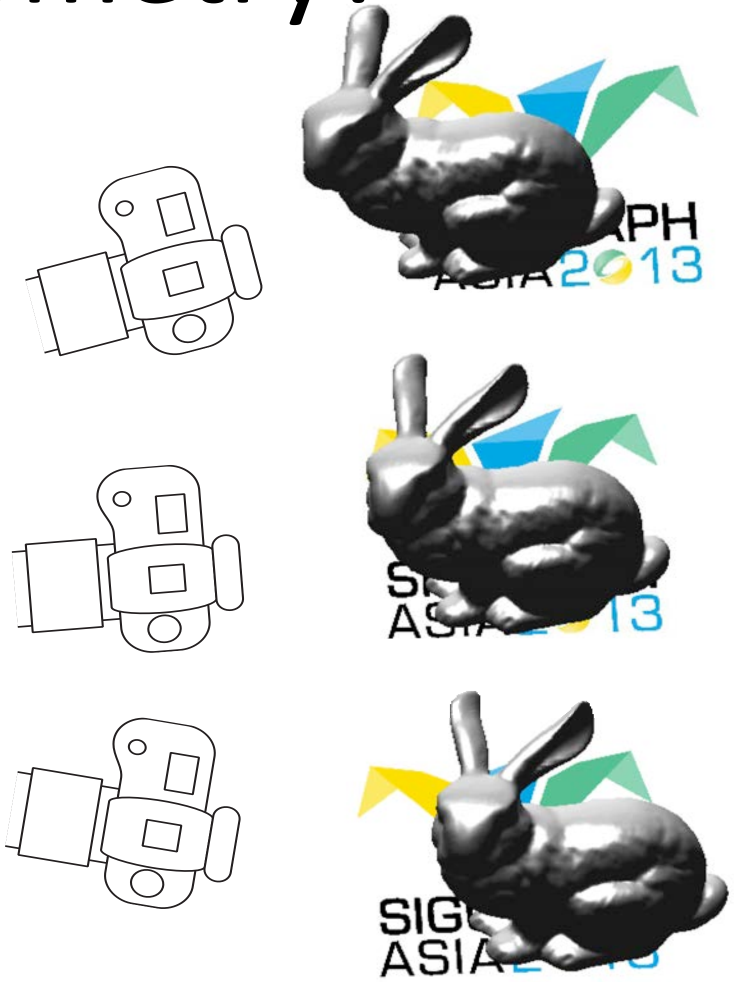
Stereo

Rely more
on sampling

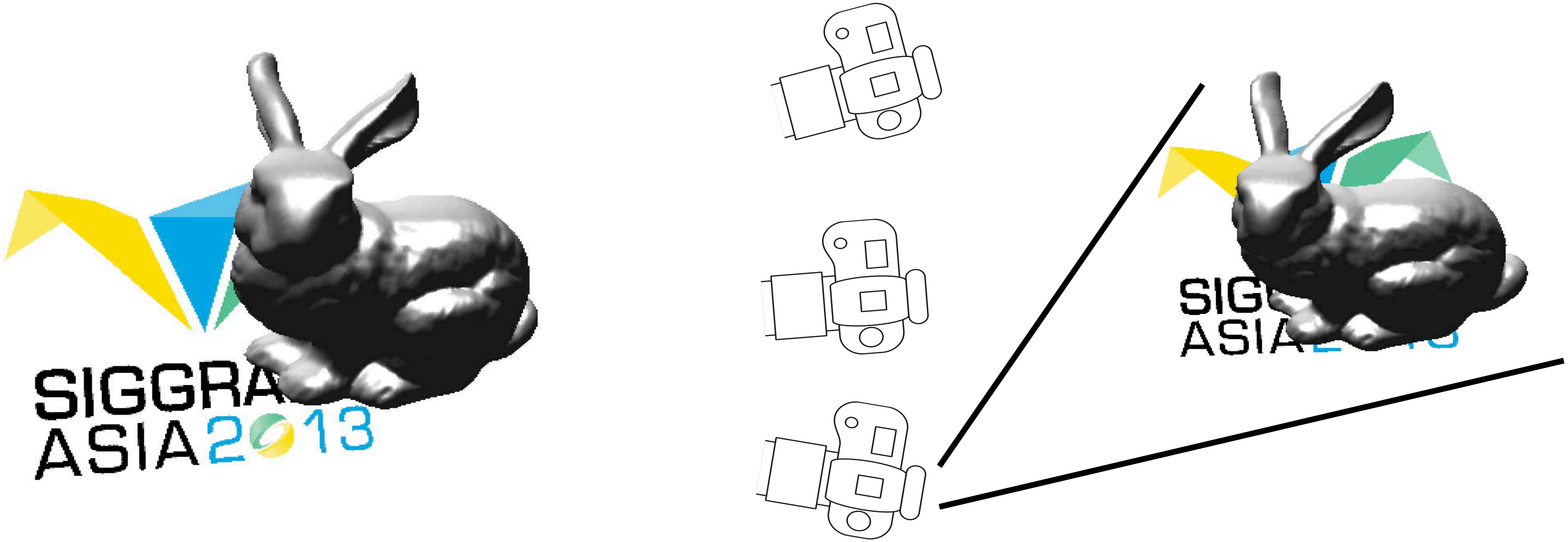
Rely more
on geometry

What happens when we don’t know geometry?

What happens when we don’t know geometry?



What happens when we don't know geometry?



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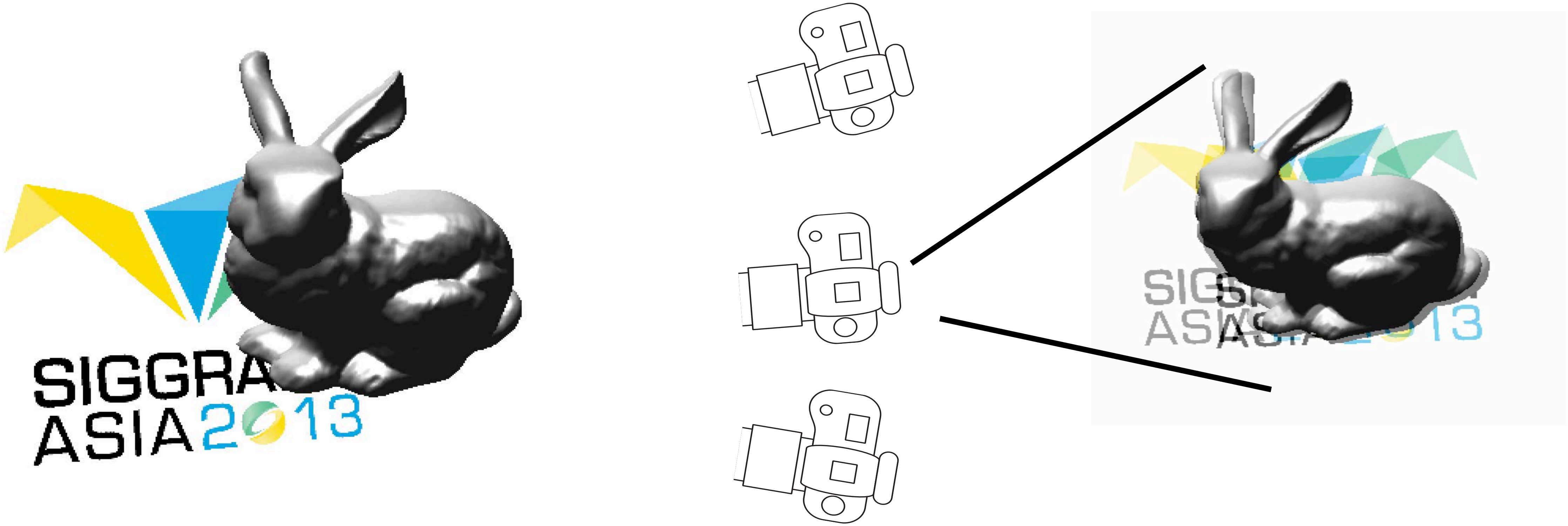

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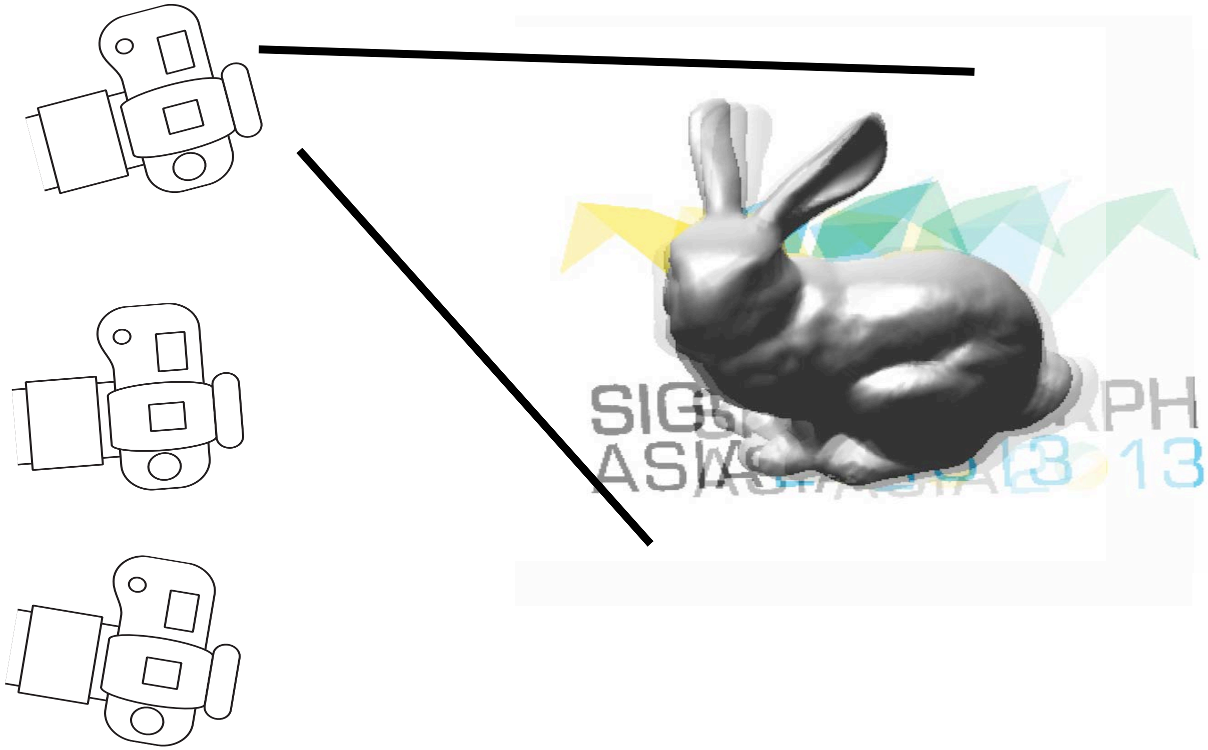
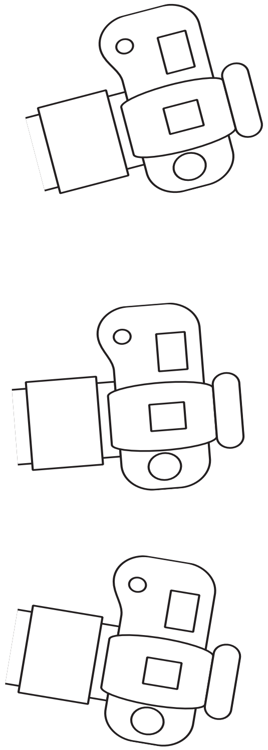
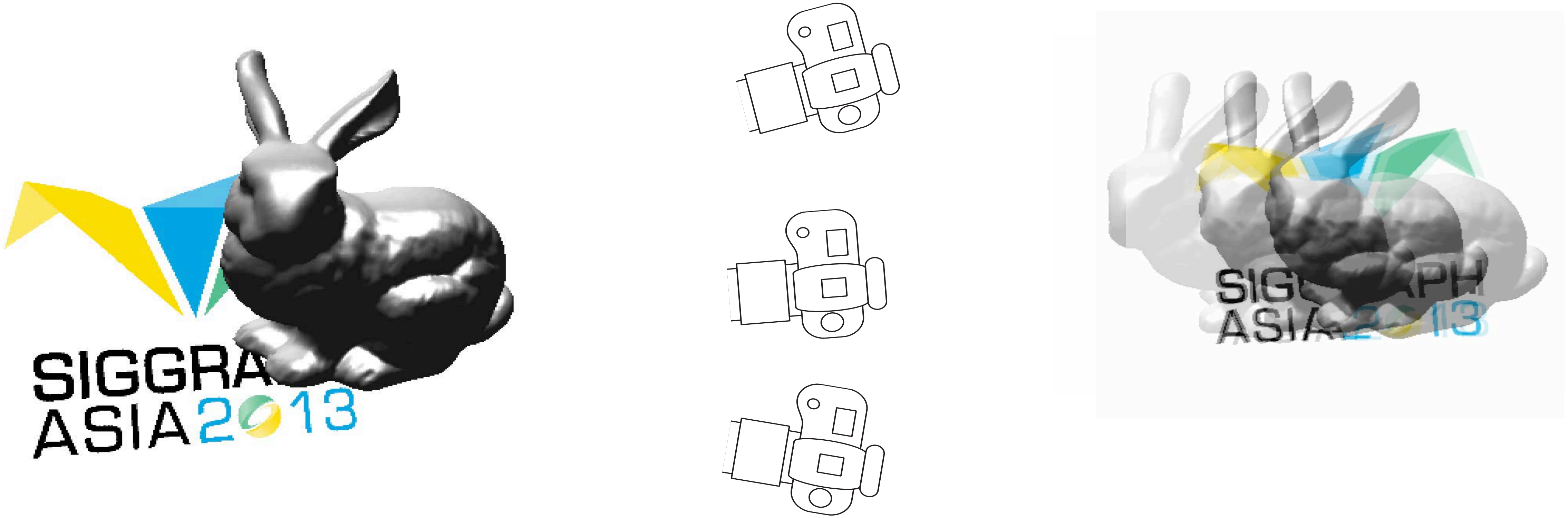
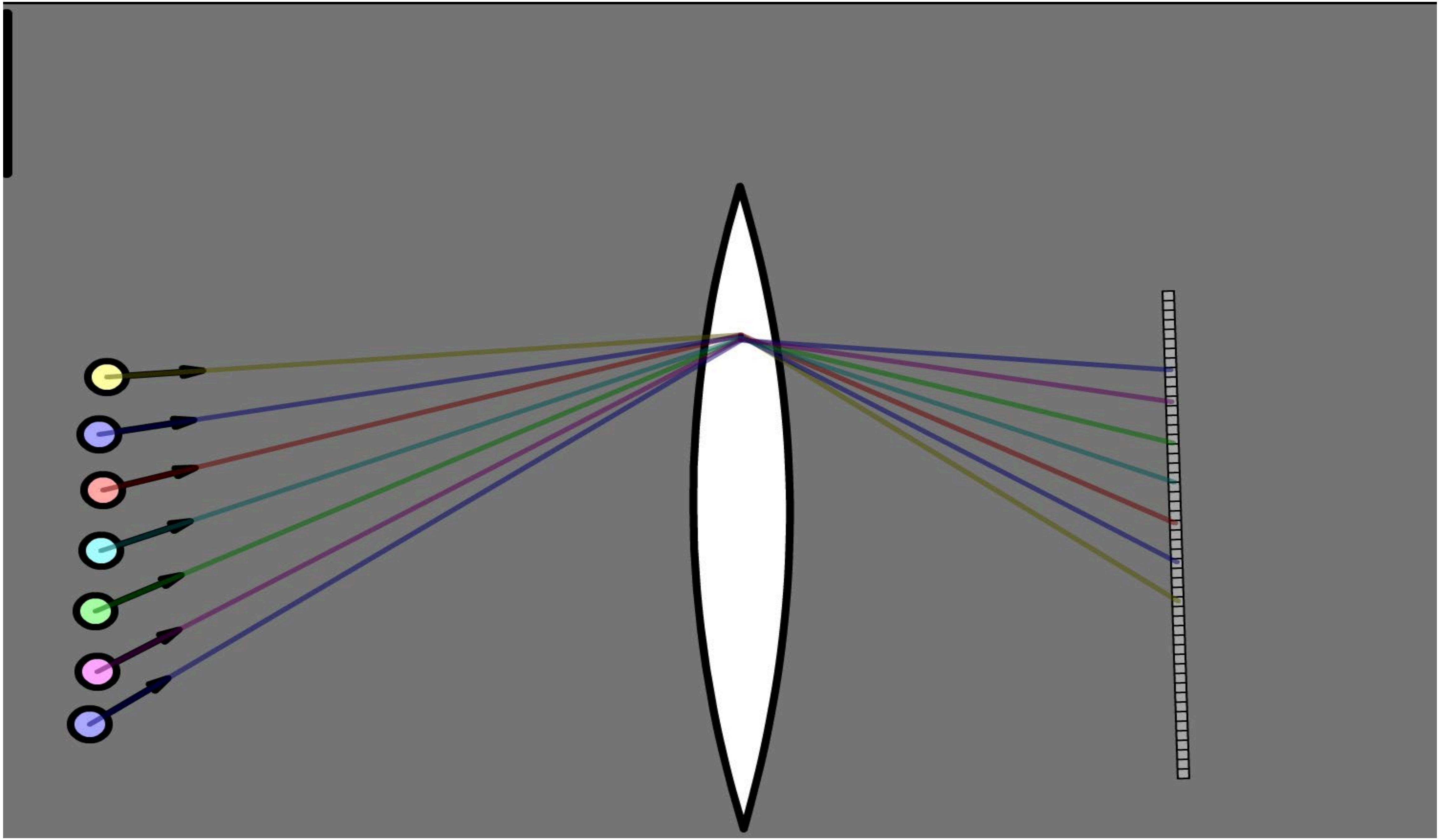
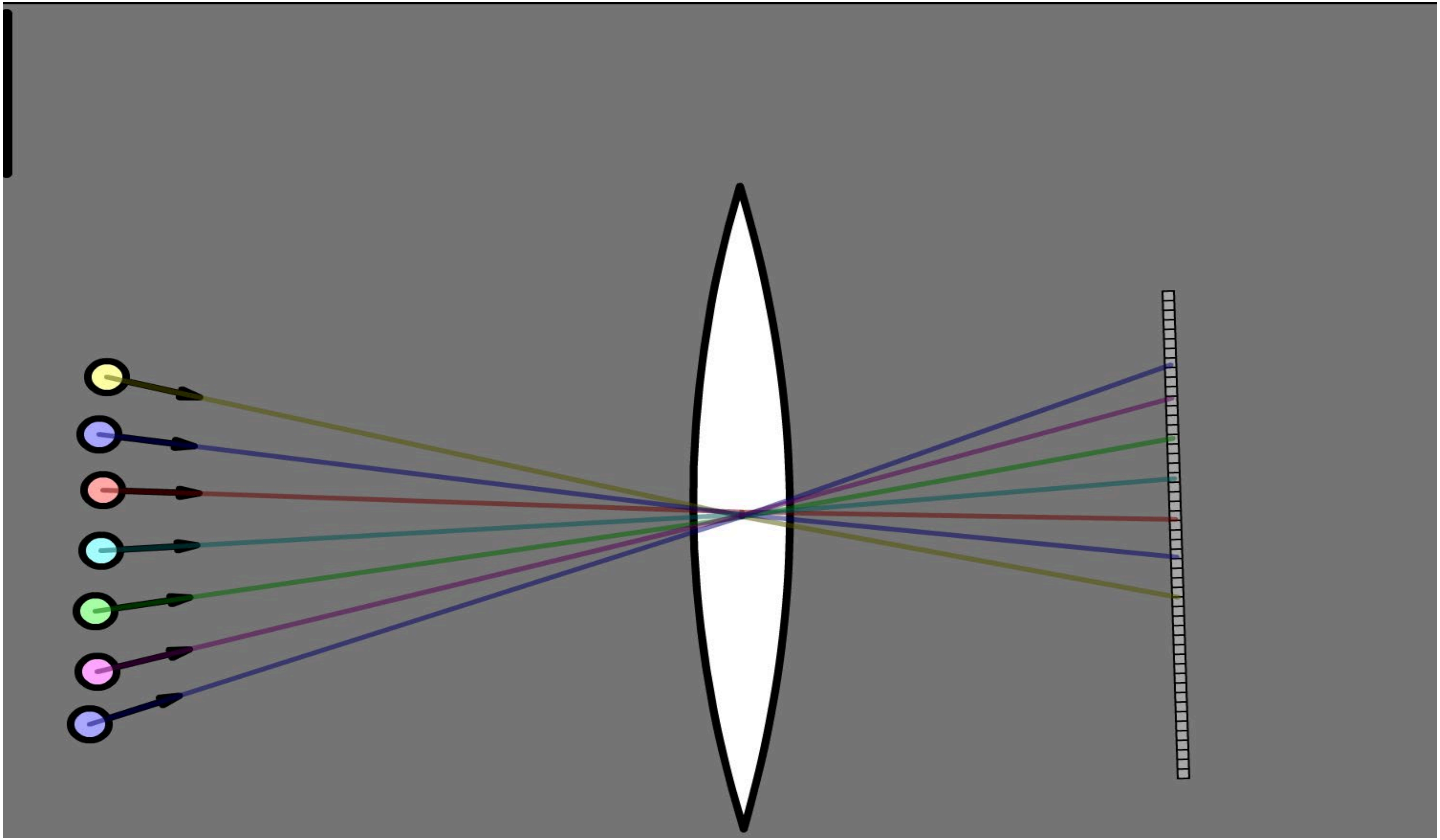


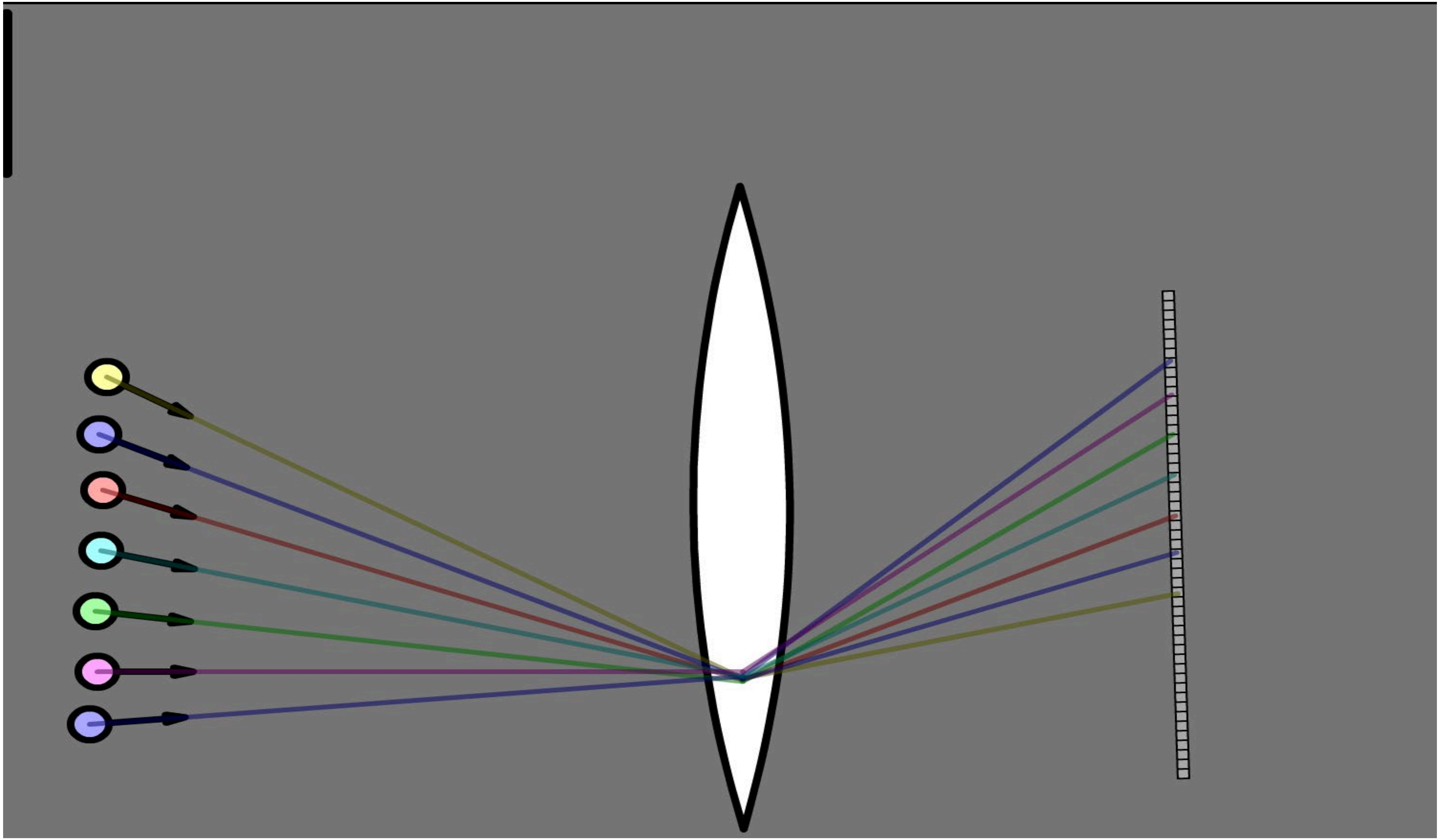
What happens when we don't know geometry?

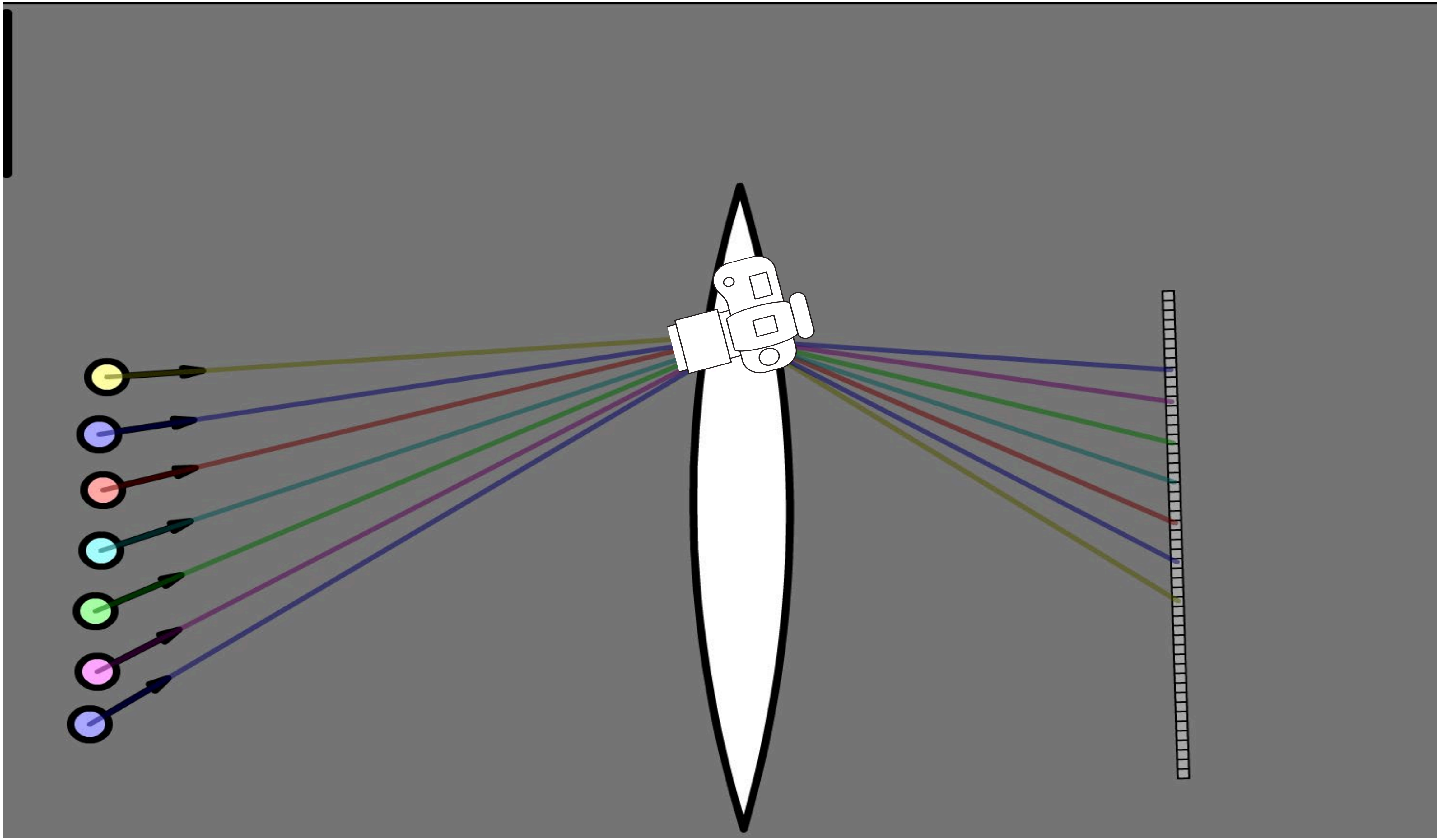


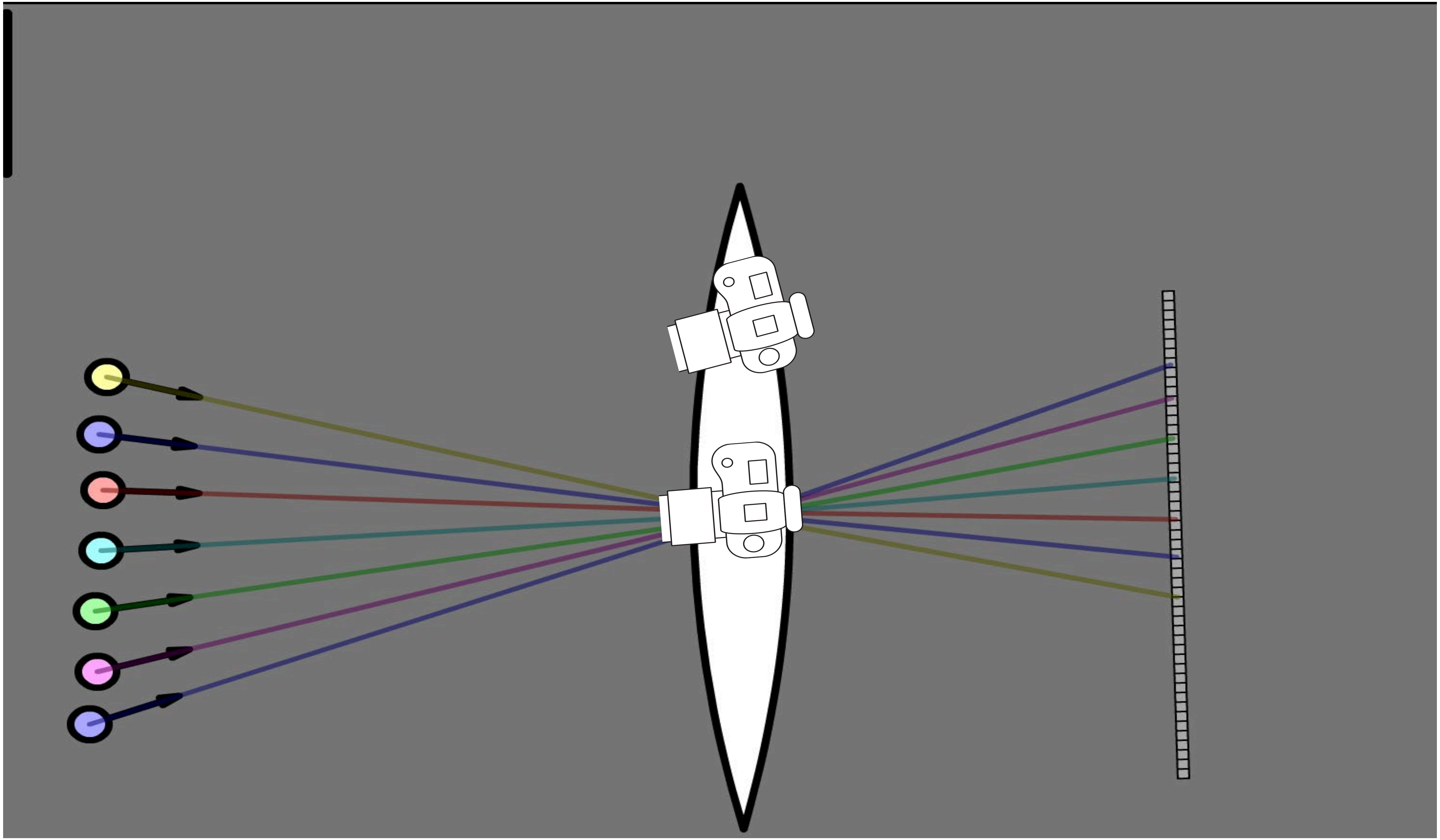

SIGGRA
ASIA

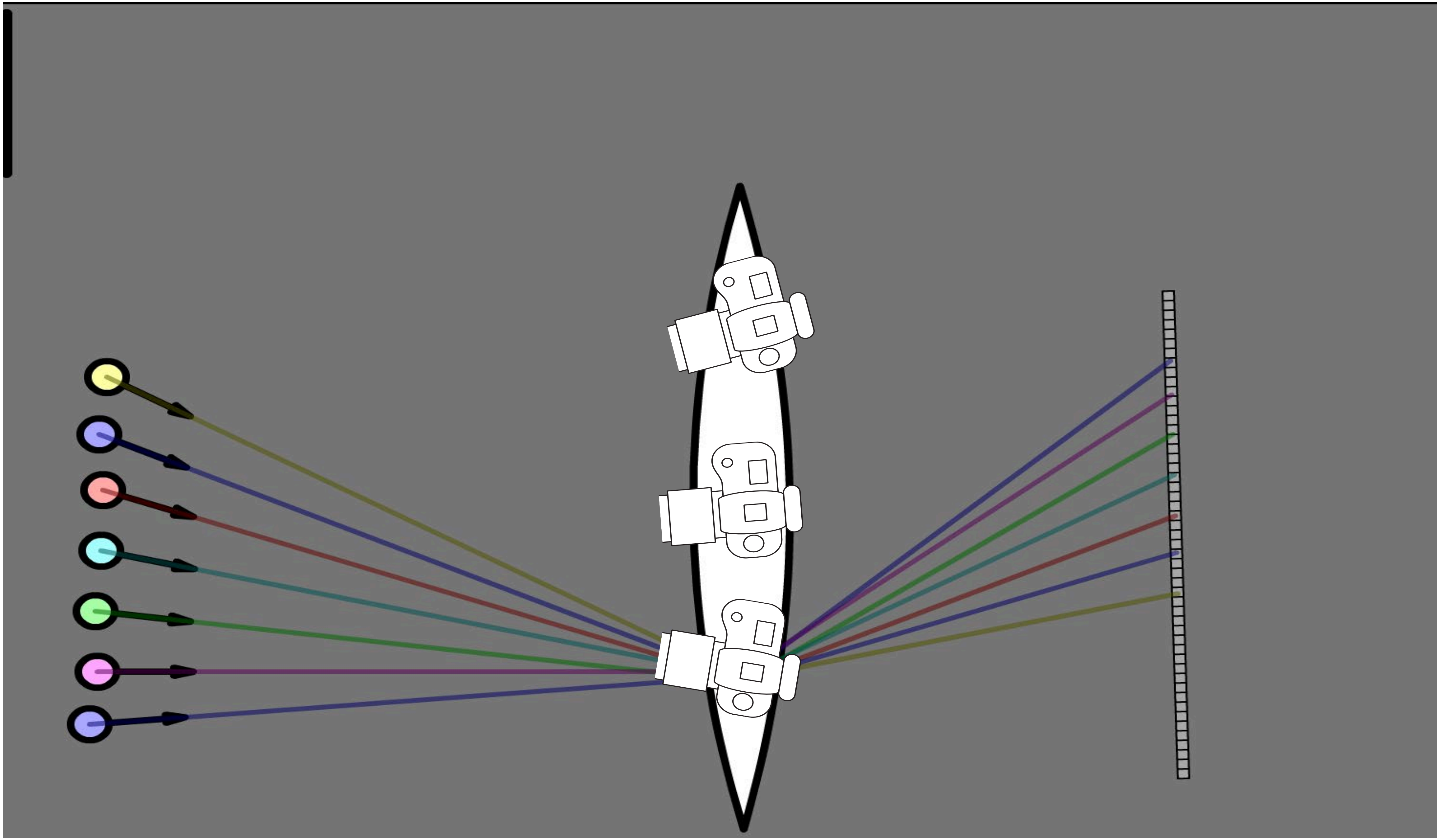


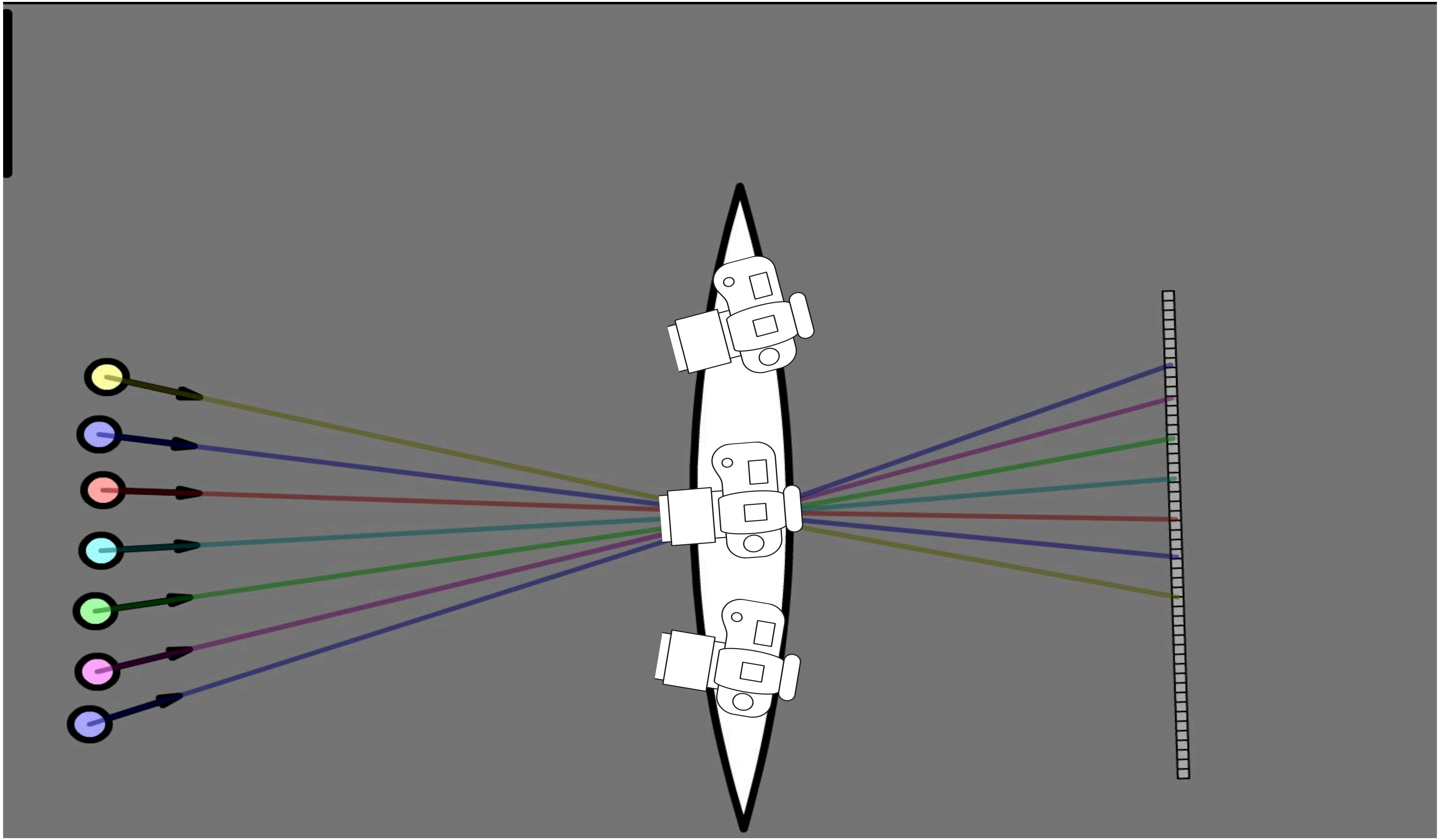


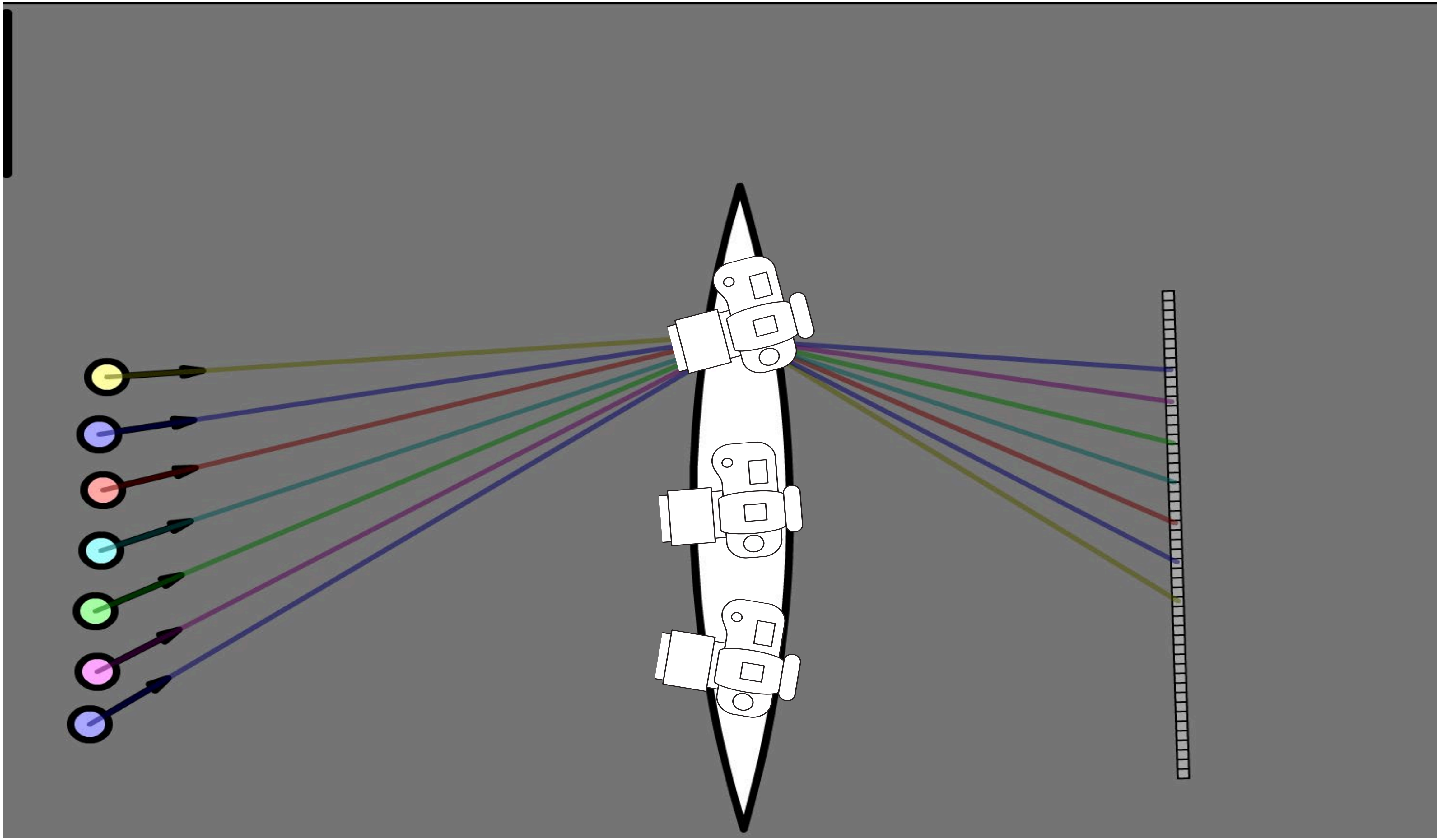


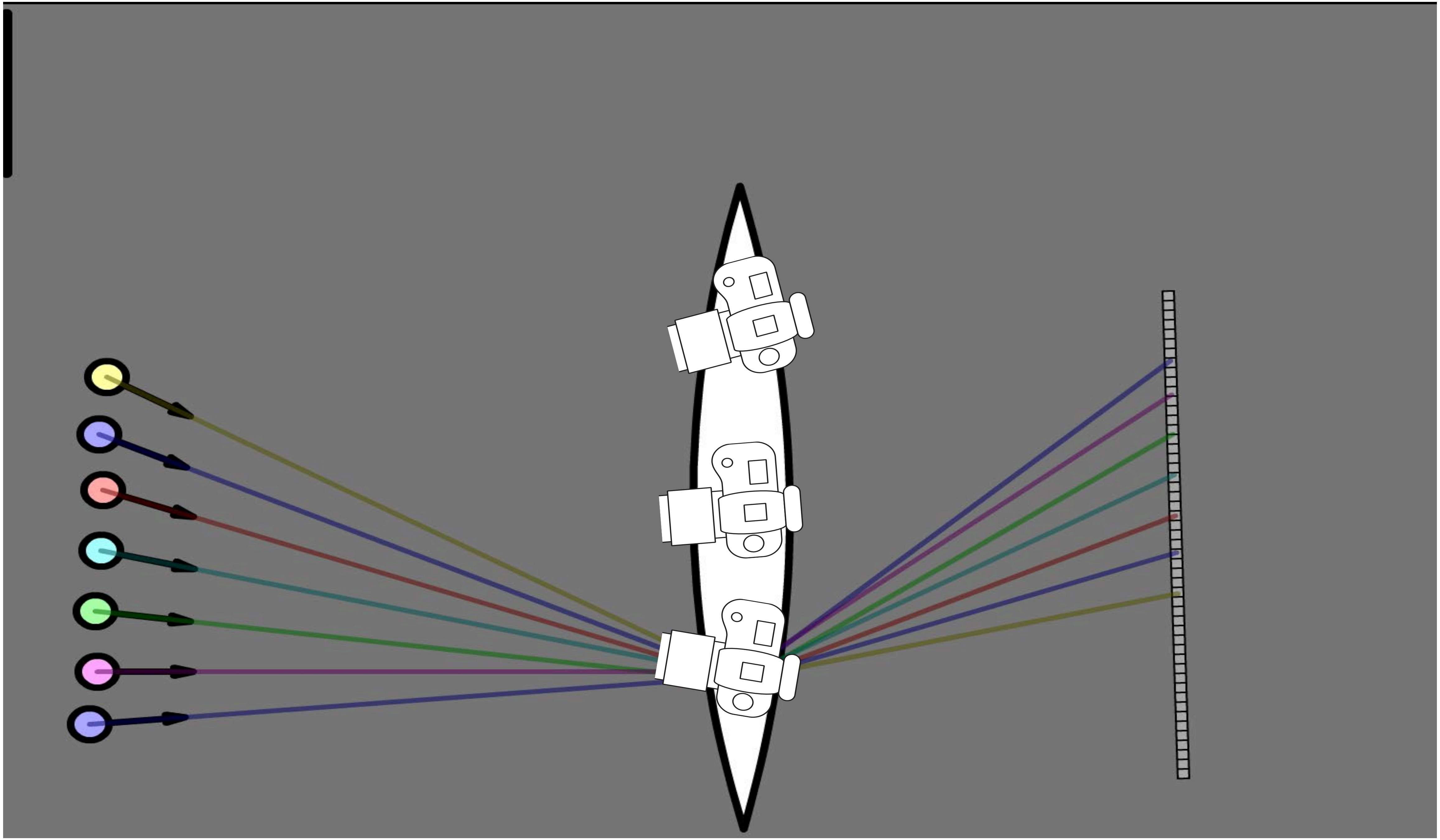


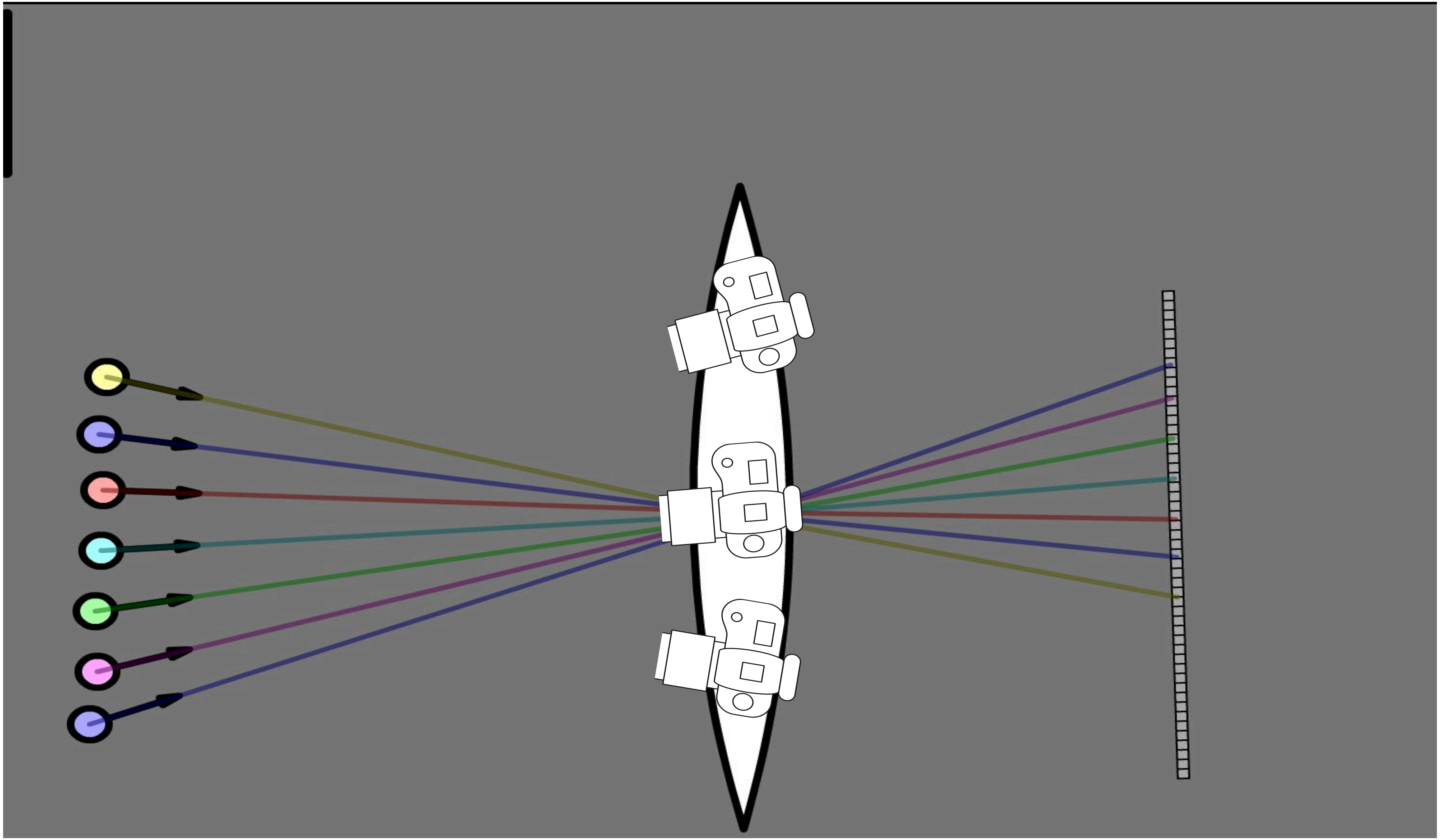


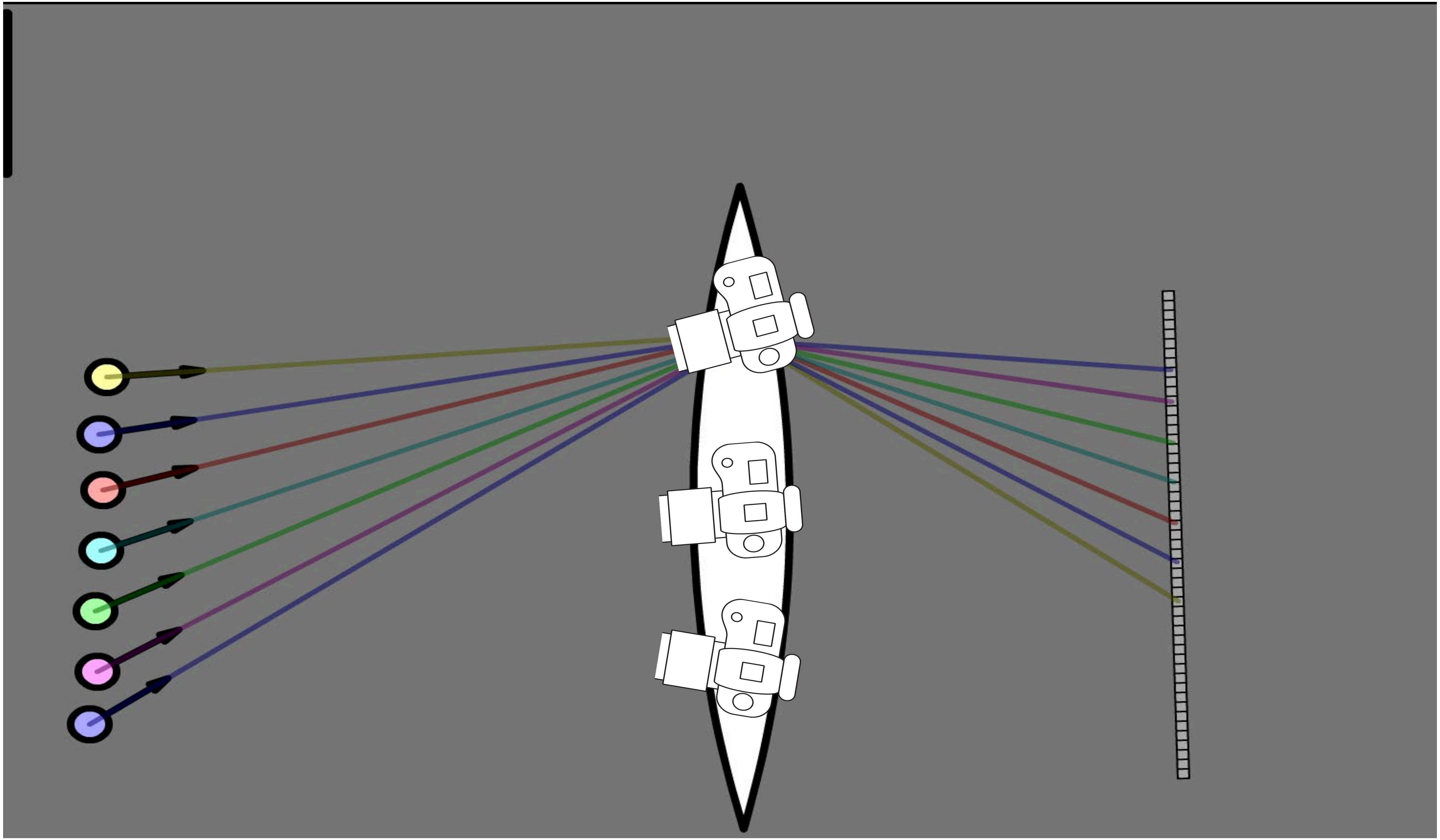


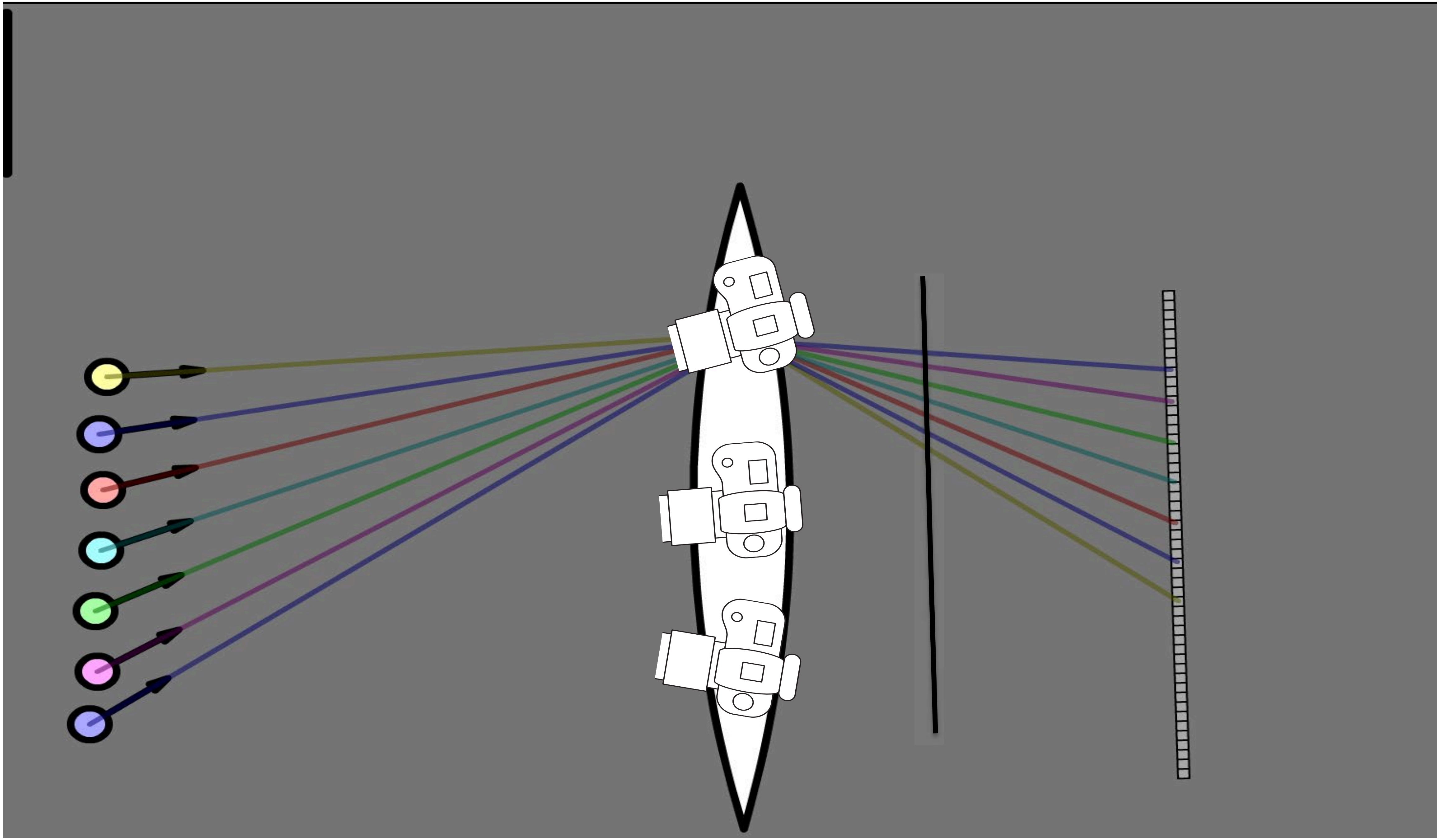


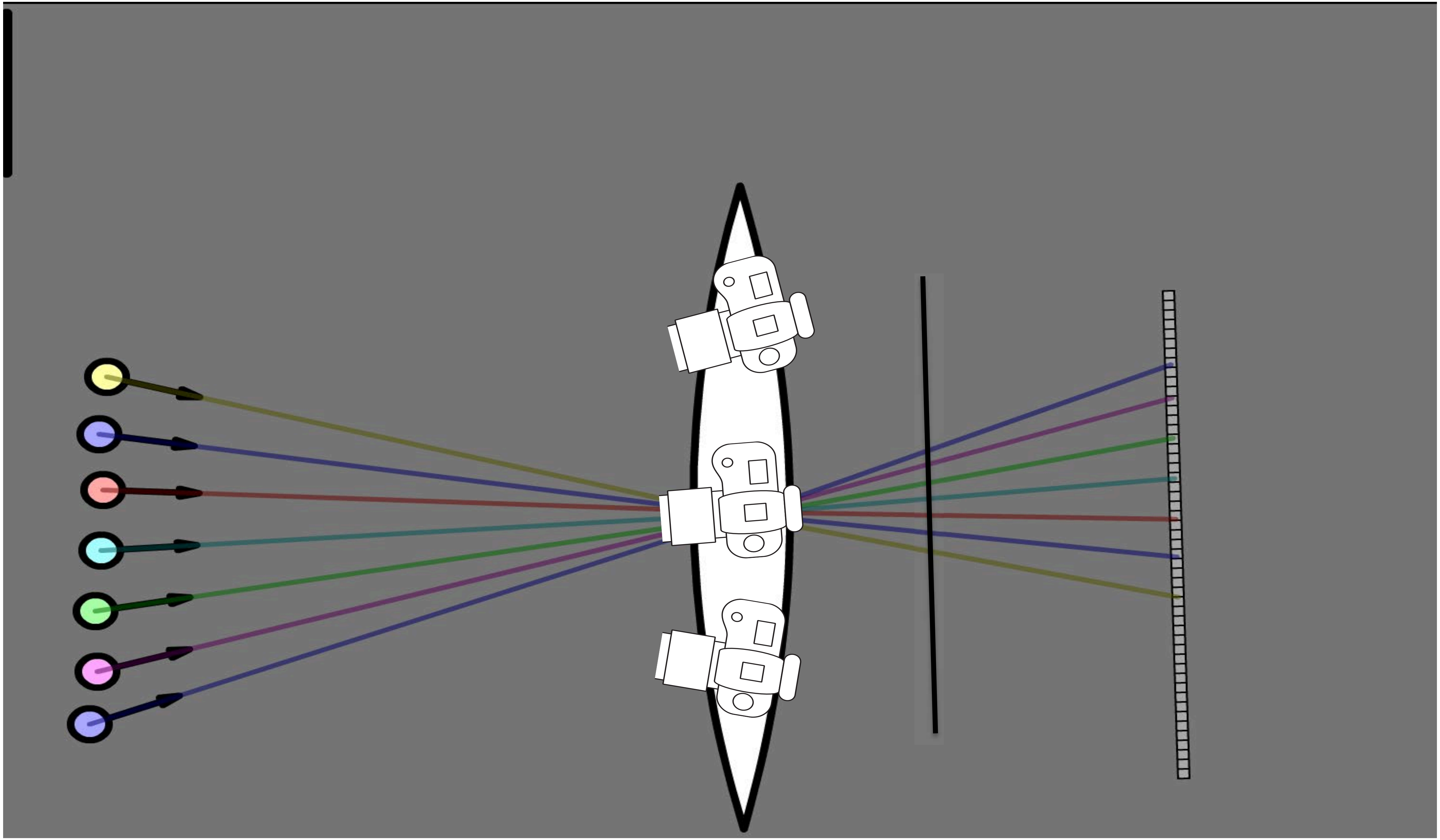


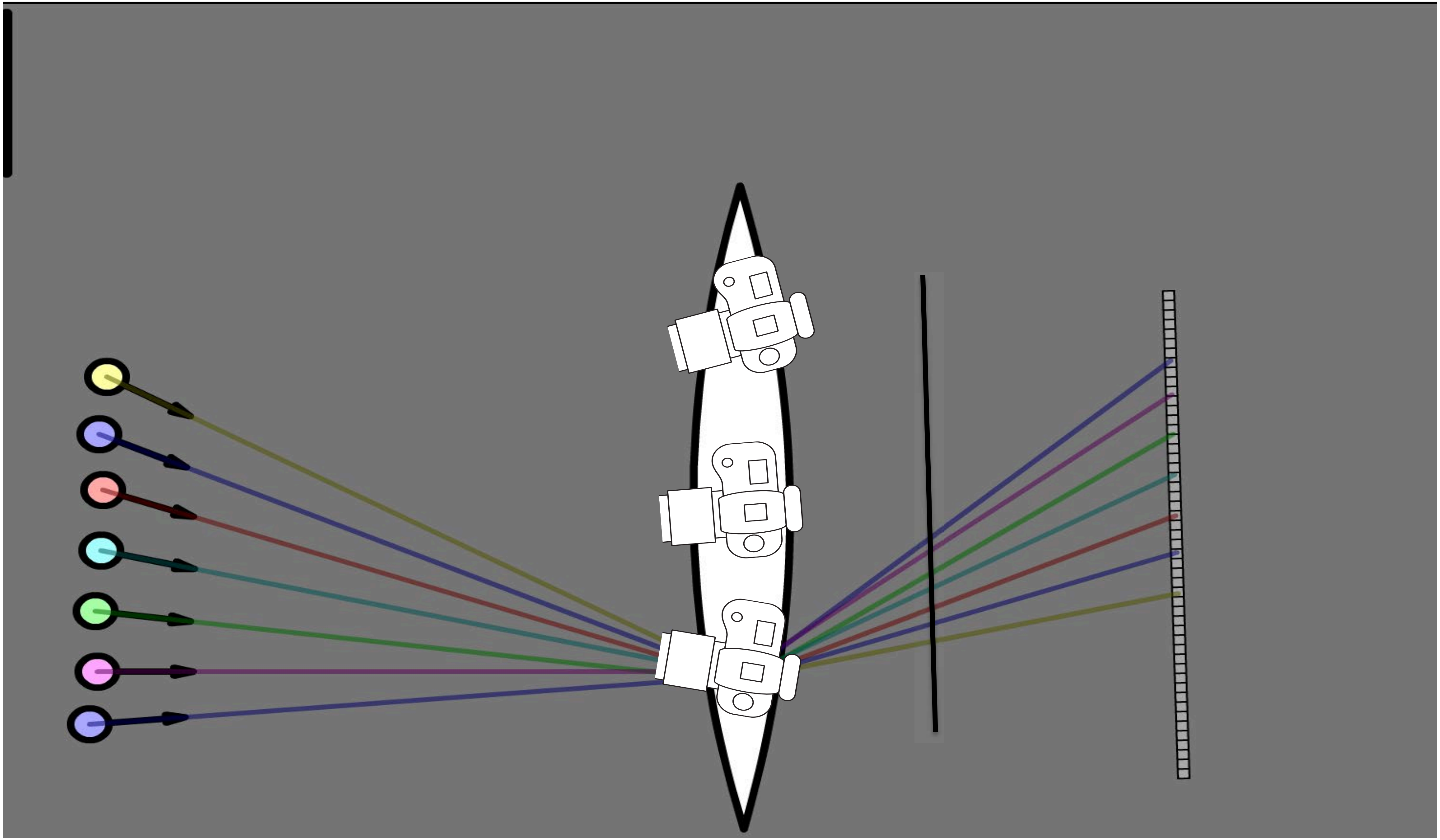




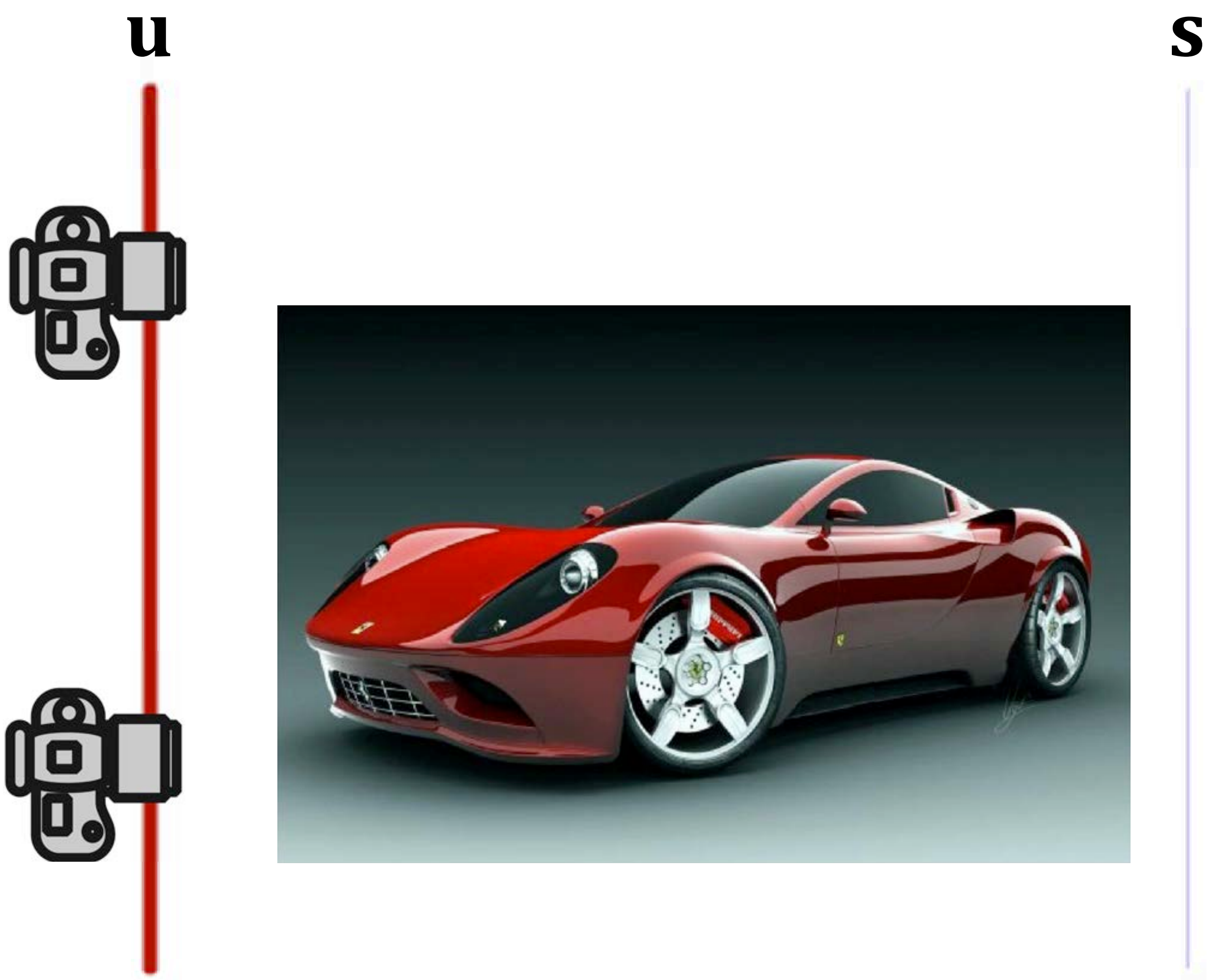


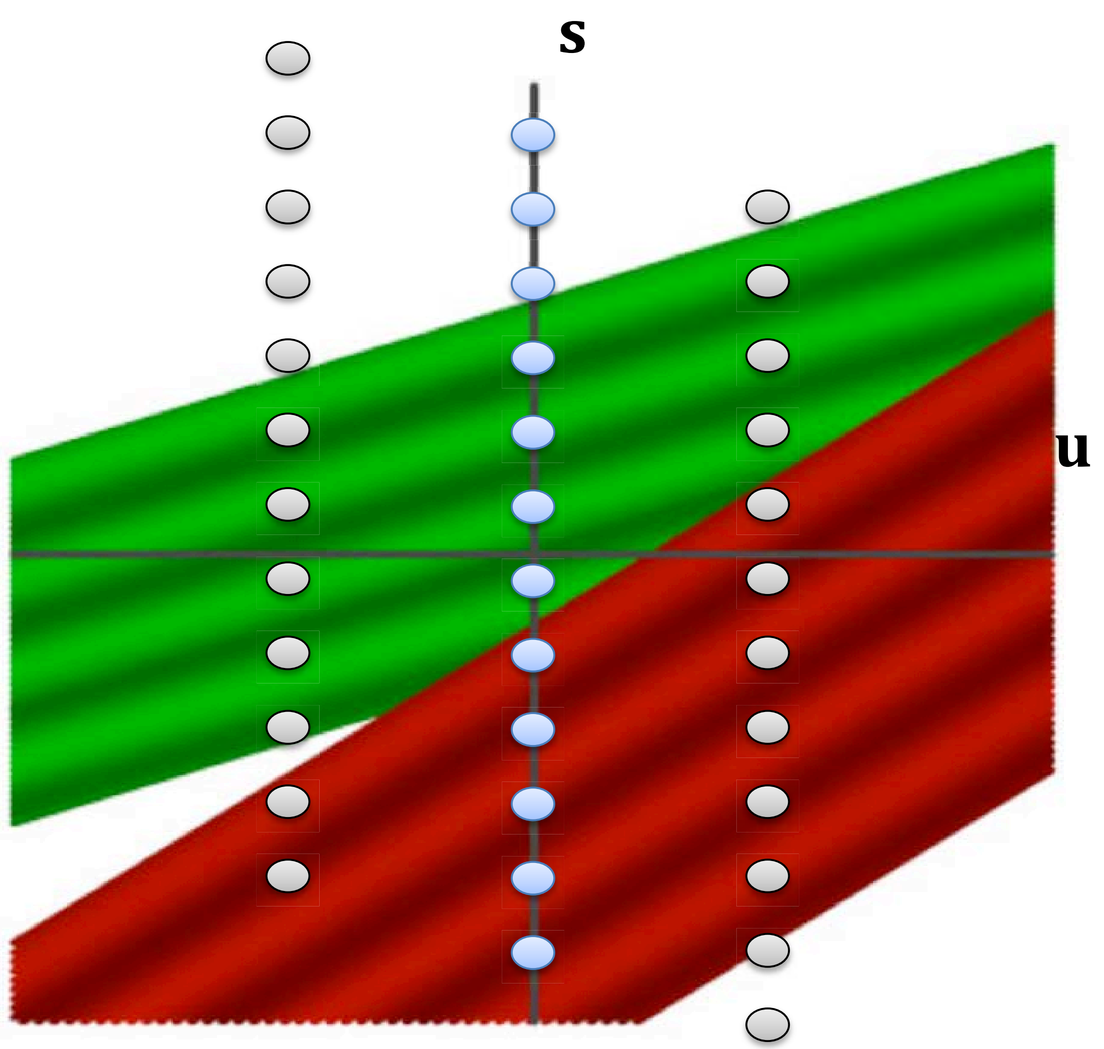




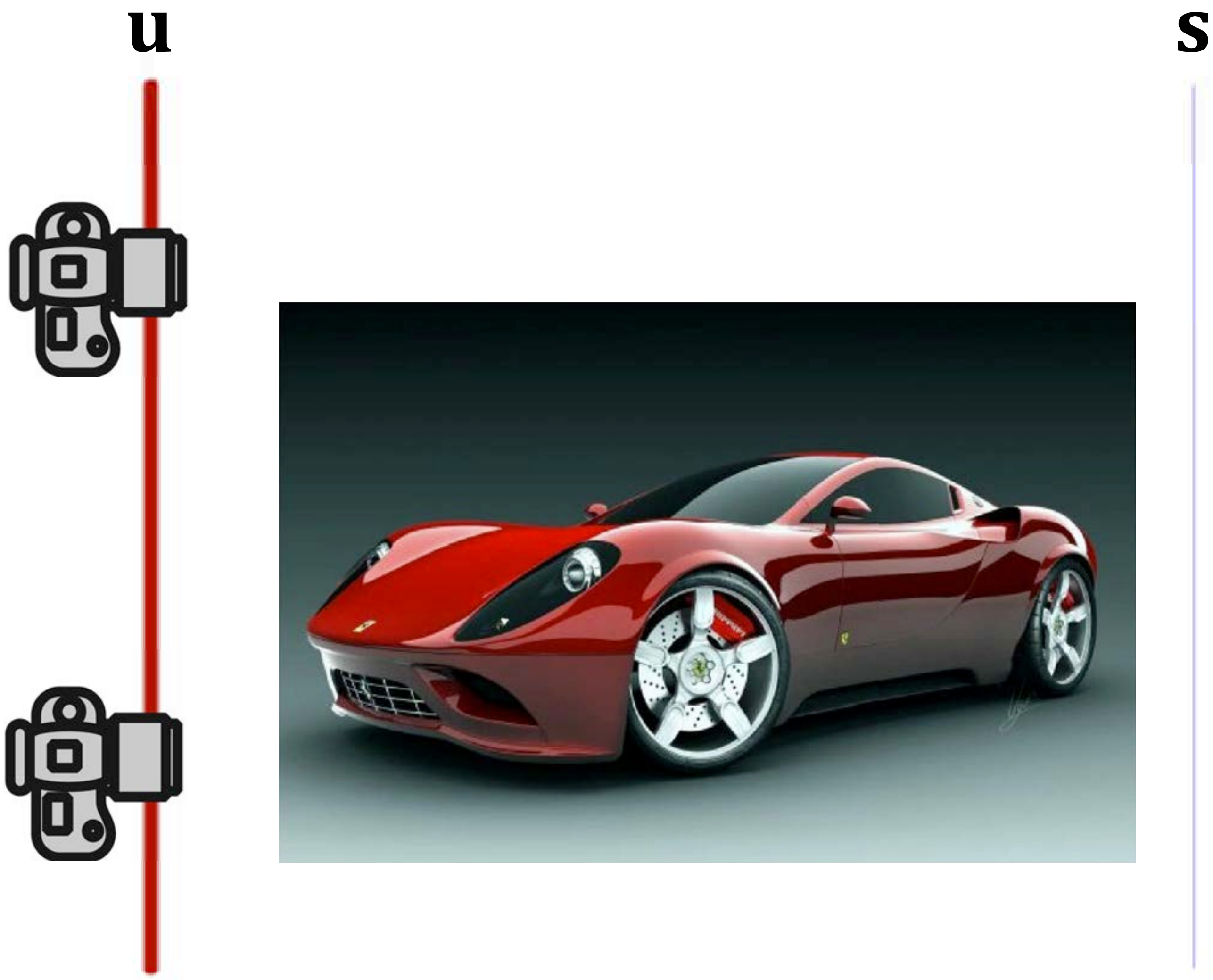


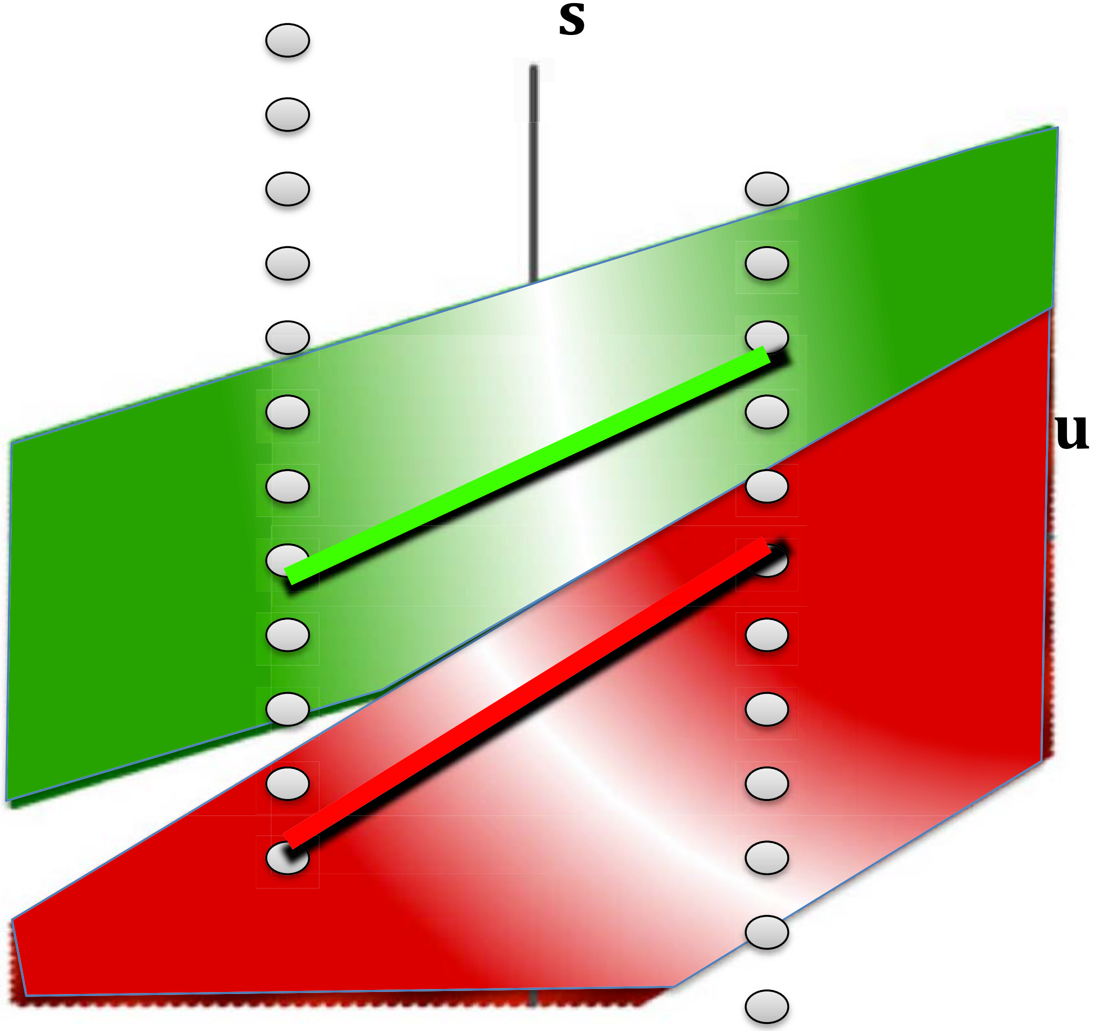
Ray Space





Ray Space





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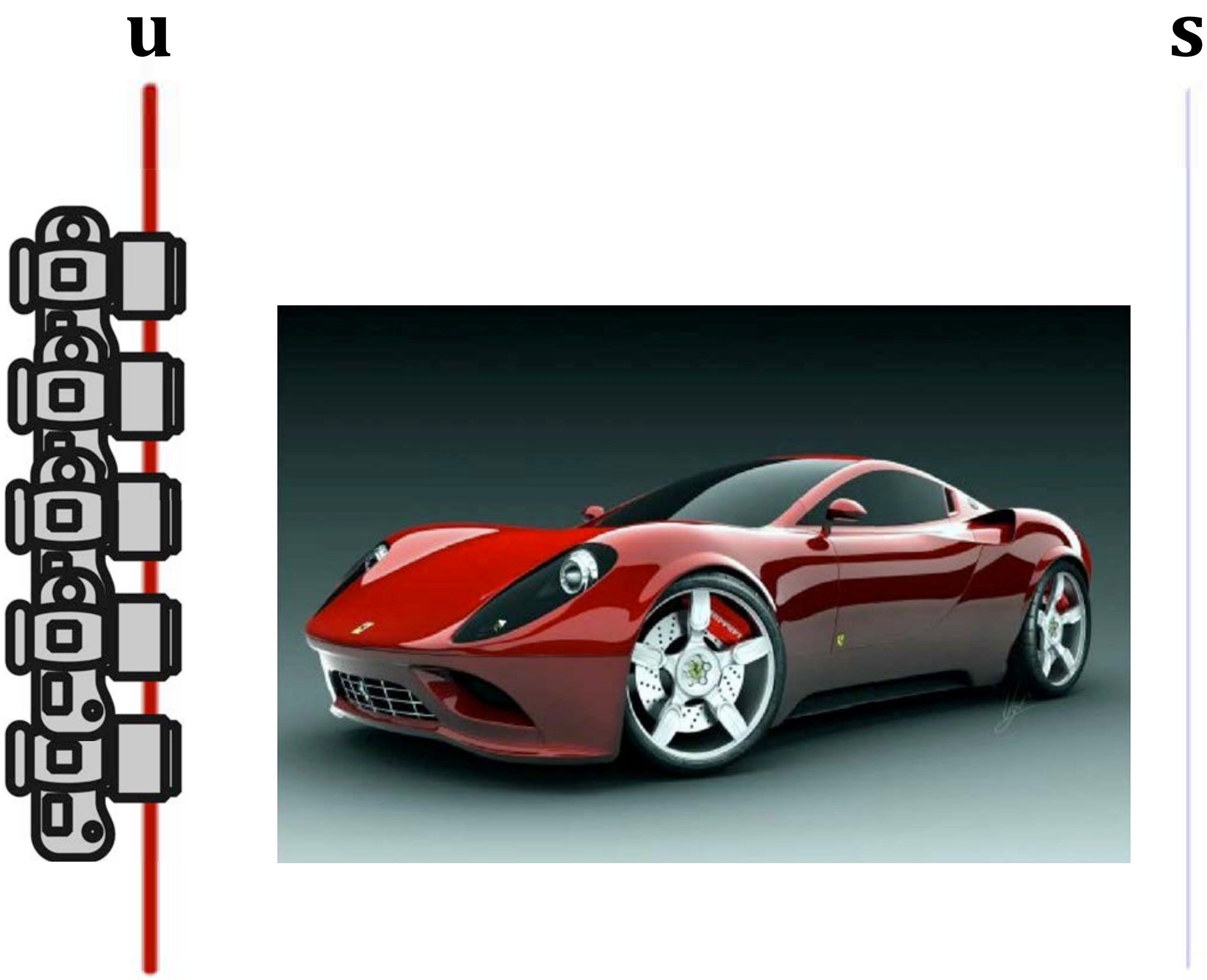
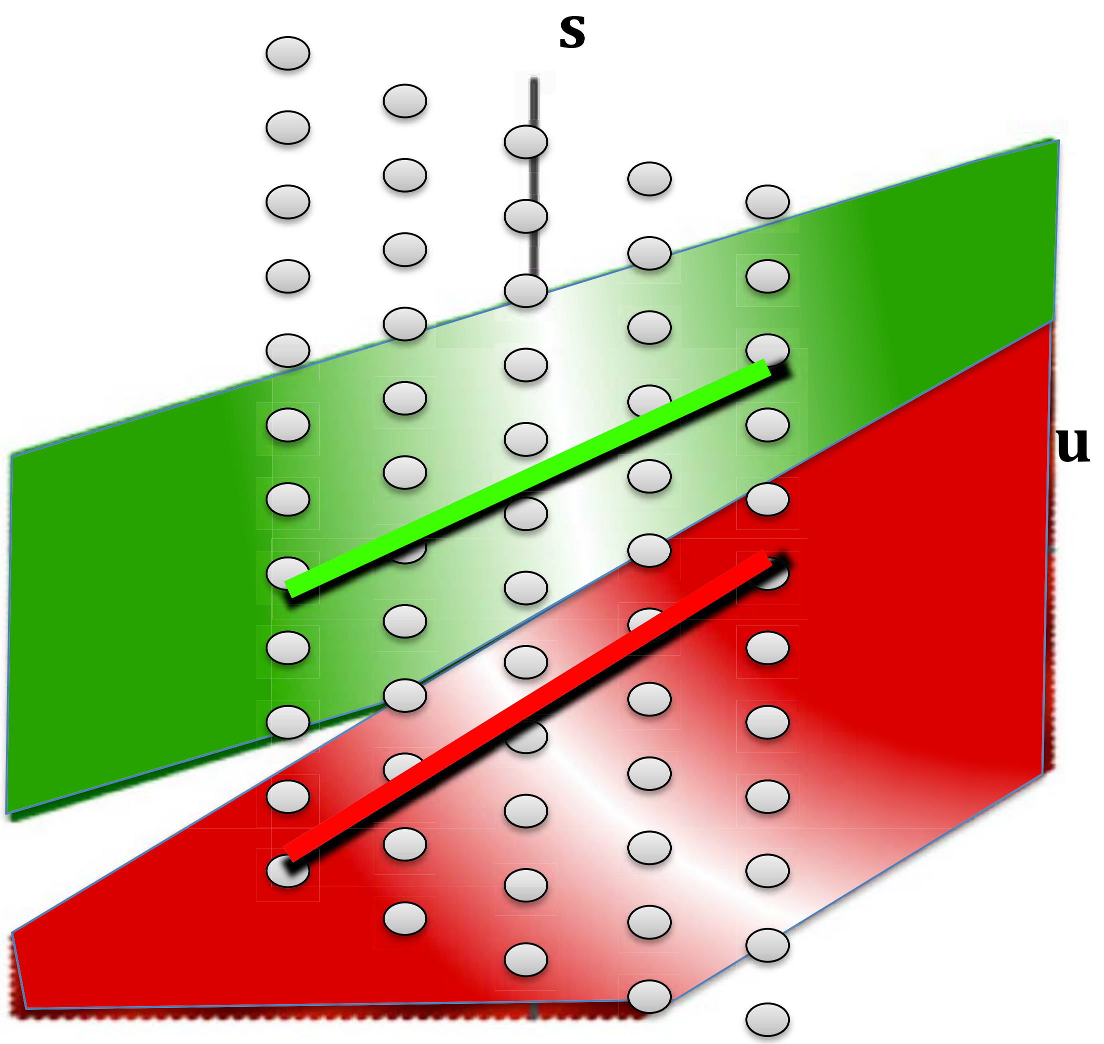
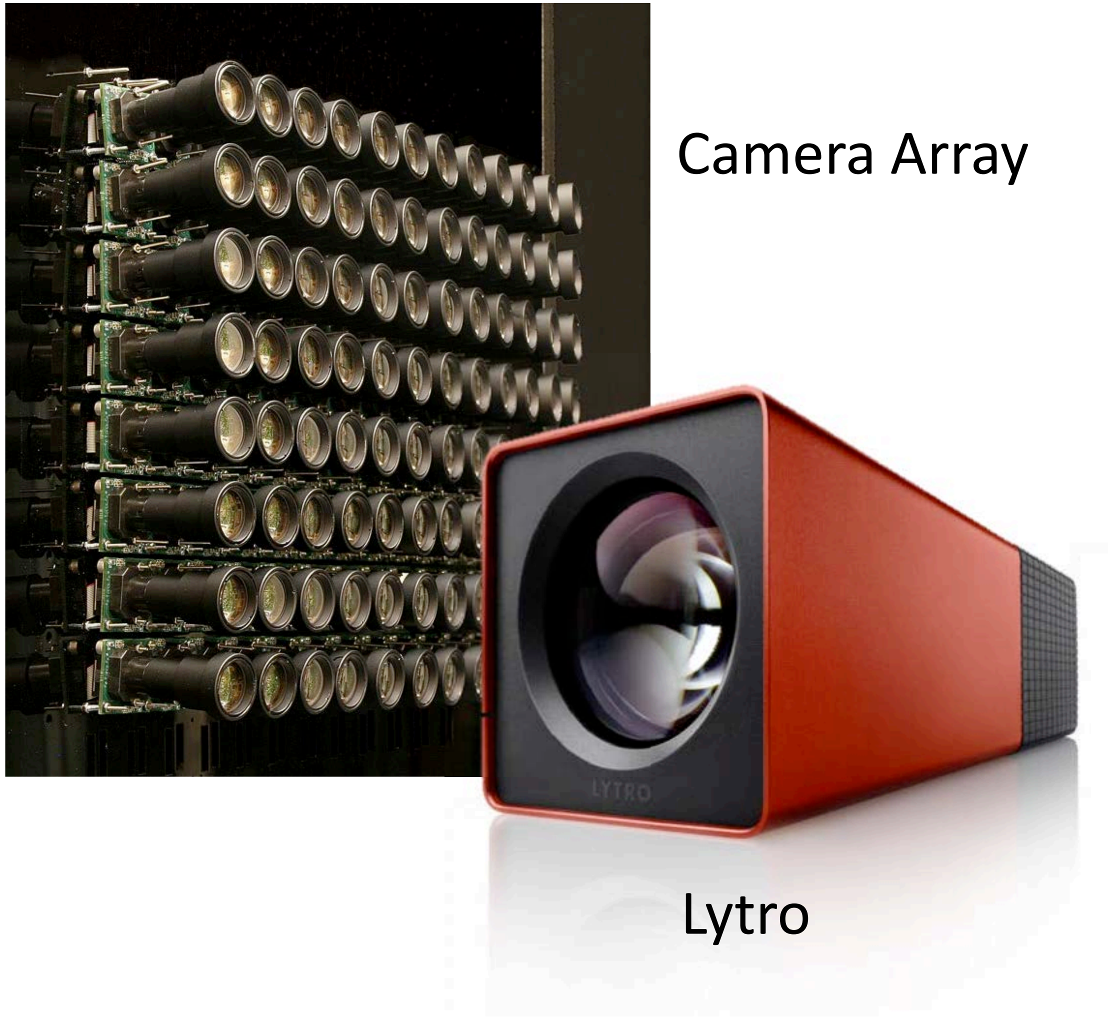


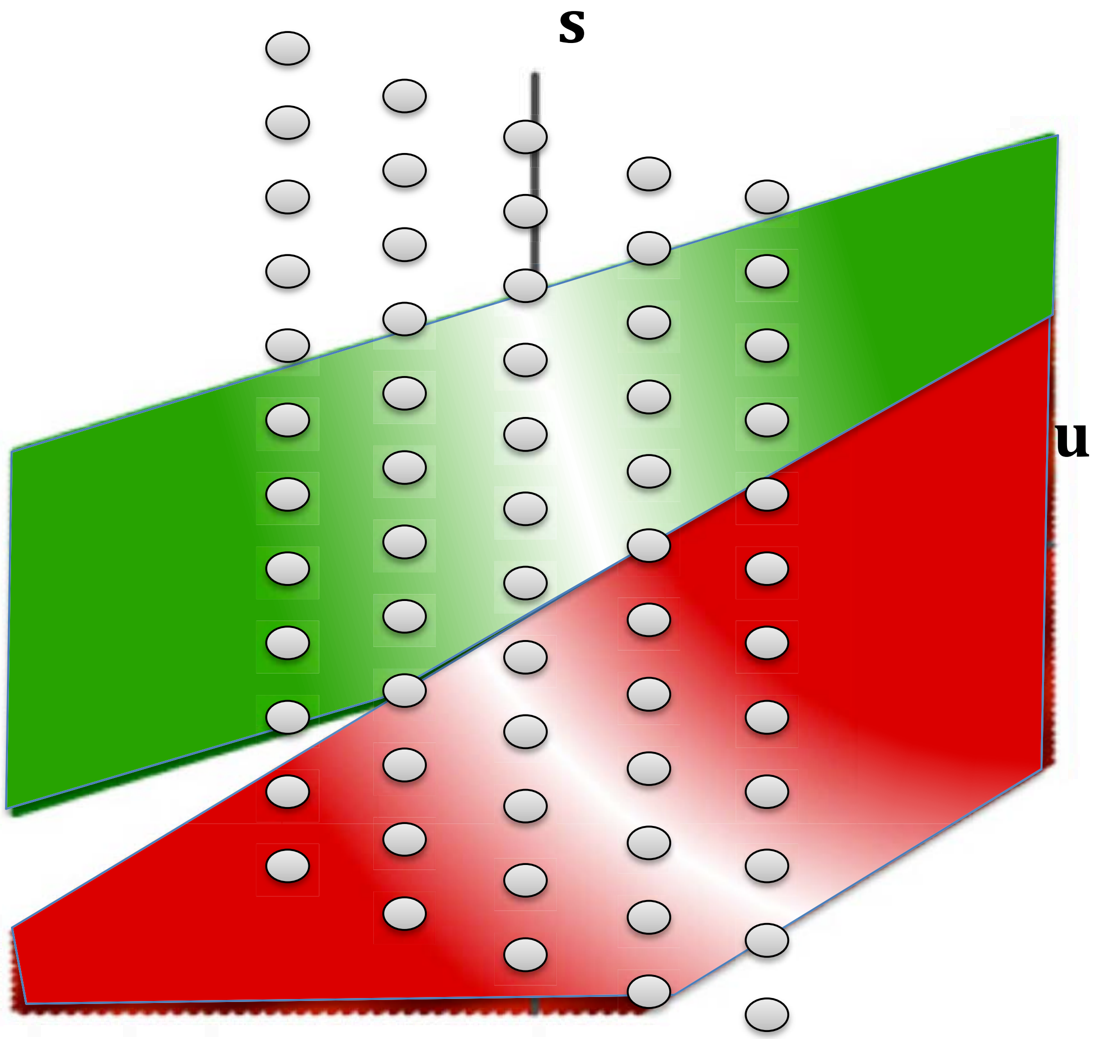


Ray Space

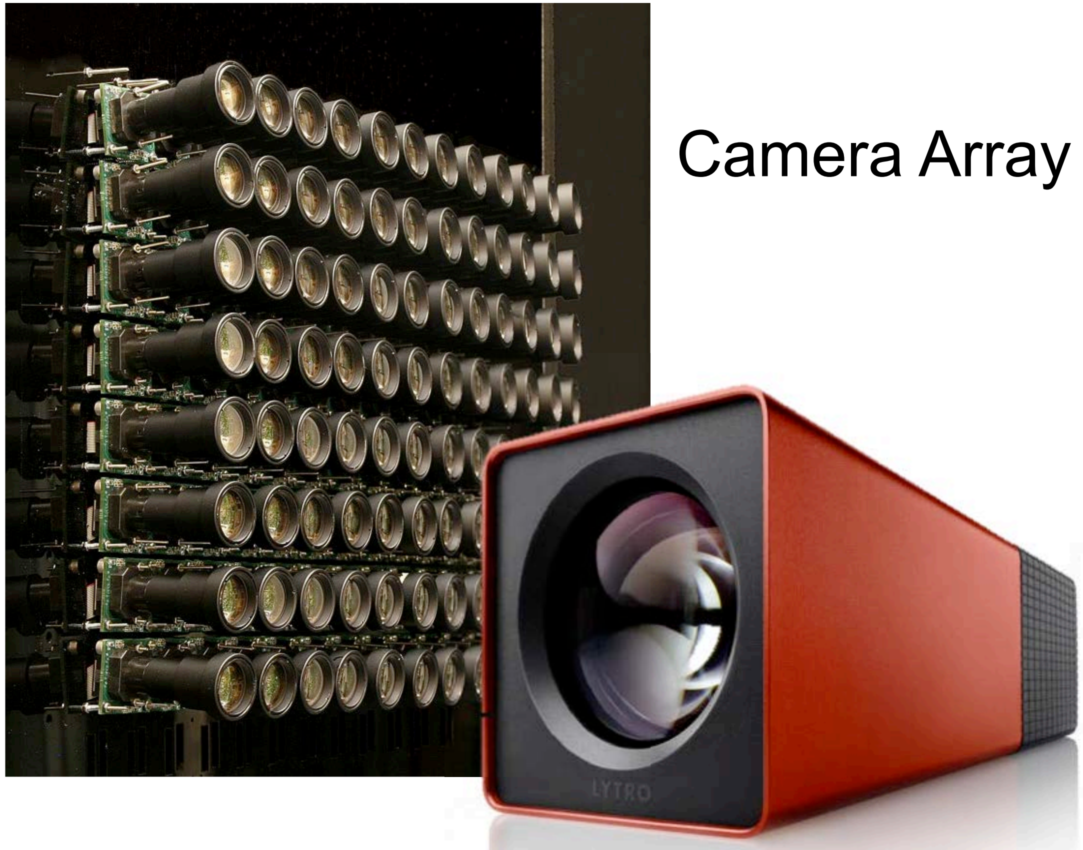


Camera Array

Lytro

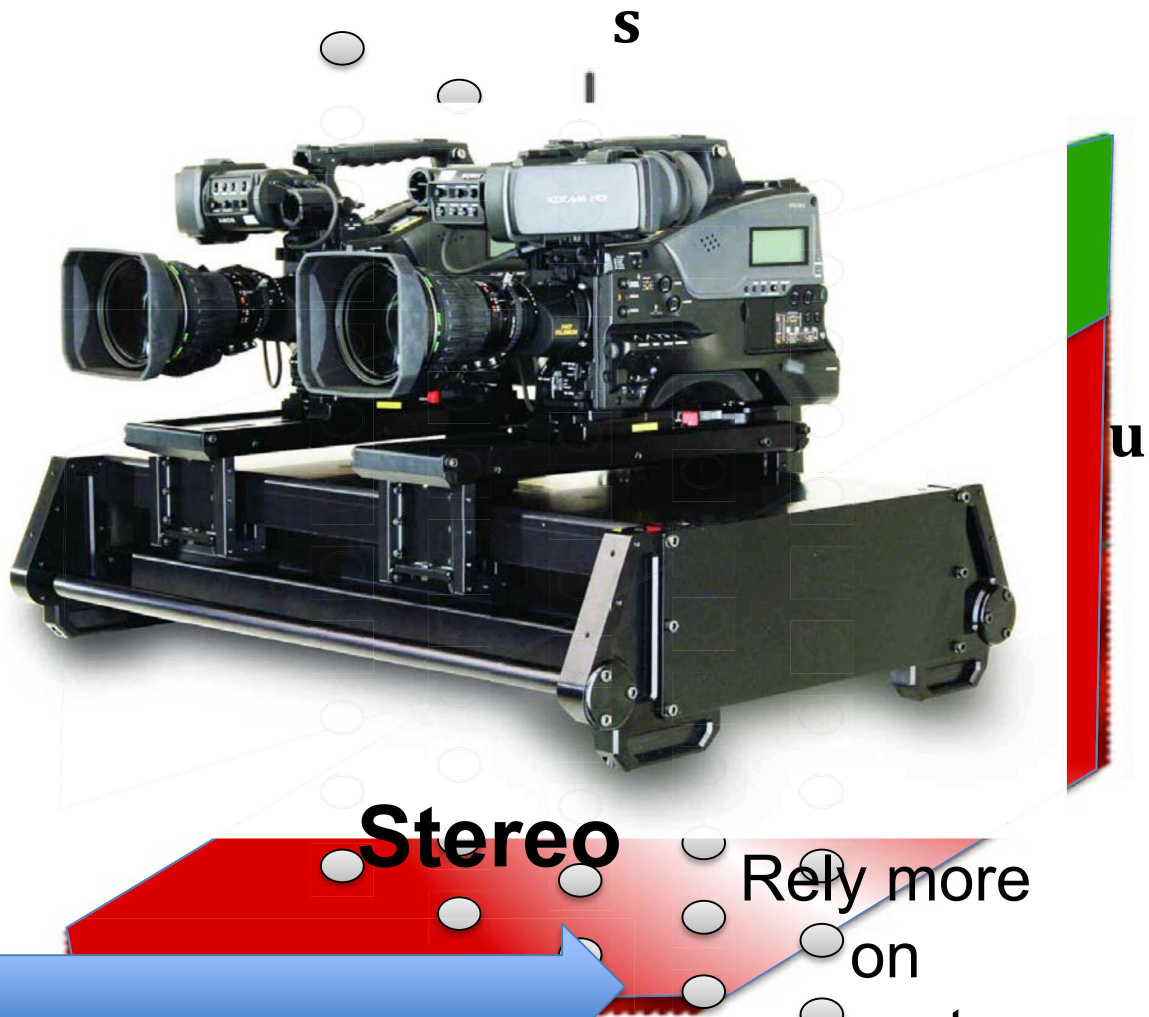


Capture Strategies



Camera Array

Lytro

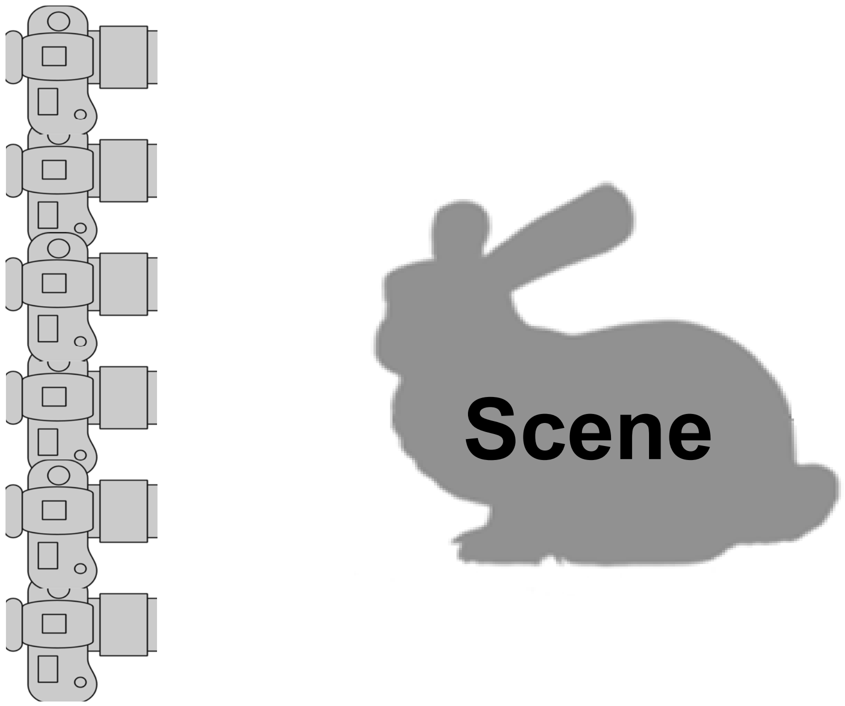


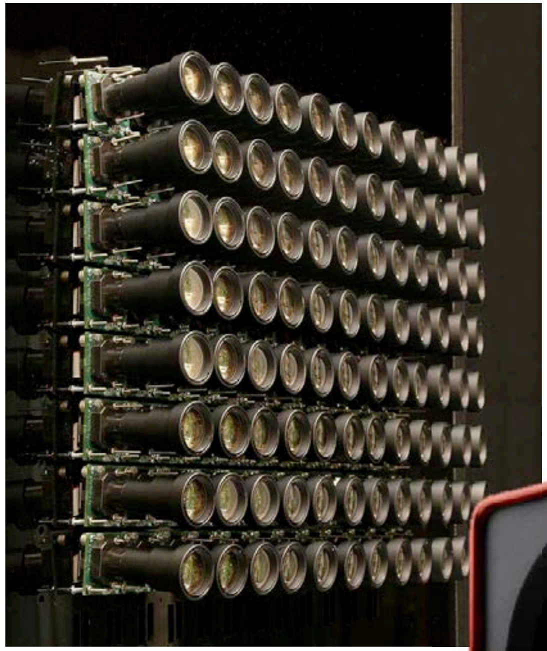
Stereo

Rely more on sampling

Rely more on geometry

Specialized Devices





Camera Array

Lytro