CS5670: Comput

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Course review

Class	Date	Topic/notes	Readings	Assignments, etc.
0	Jan 24	Introduction and Overview [ppt|pdf]	Szeliski 1	
1	29	Image filtering [ppt|pdf]	Szeliski 3.1	
2	31	Image filtering 2 [ppt|pdf]	Szeliski 3.2	
3	31	Image Resampling[ppt|pdf]	Szeliski 3.4, 2.3.1	
4	Feb 4	Features Detection [ppt|pdf]	Szeliski 4.1	
5	4	Features Invariance [ppt|pdf]	Szeliski 4.1	
6	7	Descriptors [ppt|pdf]	Szeliski 4.1	
7	12	Image Transformation [ppt|pdf]	Szeliski 3.6	
8	14	Alignment [ppt|pdf]	Szeliski 6.1	PA1 due
9	14	RANSAC [ppt|pdf]	Szeliski 6.1	
10	21	Cameras [ppt|pdf]	Szeliski 2.1.3-2.1.6	
11	28	Panoramas [ppt|pdf]	Szeliski 9	

Topics – image processing

- Filtering
- Edge detection
- Image resampling / aliasing / interpolation
- Feature detection
  - Harris corners
  - SIFT
  - Invariant features
- Feature matching

Topics – 2D geometry

- Image transformations
- Image alignment / least squares
- RANSAC
- Panoramas

Topics – 3D geometry

- Cameras
- Perspective projection
- Single-view modeling (points, lines, vanishing points, etc.)
- Stereo
- Two-view geometry (F-matrices, E-matrices)
- Structure from motion
- Multi-view stereo

Topics – geometry, continued

- Light, color, perception
- Lambertian reflectance
- Photometric stereo

Topics – Recognition

- Different kinds of recognition problems
  - Classification, detection, segmentation, etc.
- Machine learning basics
  - Nearest neighbors
  - Linear classifiers
  - Hyperparameters
  - Training, test, validation datasets
- Loss functions for classification

Topics – Recognition, continued

- Neural networks
- Convolutional neural networks
  - Architectural components: convolutional layers, pooling layers, fully connected layers
- Generative methods

Questions?

Image Processing

Linear filtering

- One simple function on images: linear filtering (cross-correlation, convolution)
  - Replace each pixel by a linear combination of its neighbors
- The prescription for the linear combination is called the “kernel” (or “mask”, “filter”)

10	5	3
4	6	1
1	1	8

Local image data

0	0	0
0	0.5	0
0	1	0.5

kernel

	8	

Modified image data

Source: L. Zhang

Convolution

- Same as cross-correlation, except that the kernel is "flipped" (horizontally and vertically)

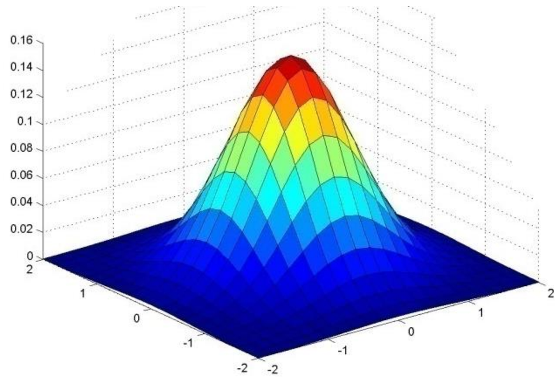
$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v]F[i-u,j-v]$$

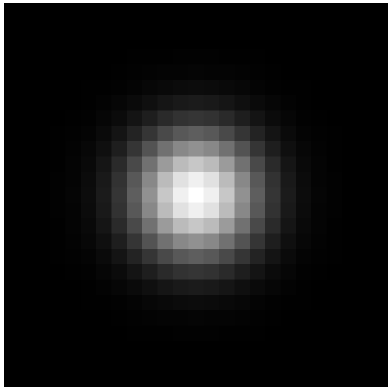
This is called a **convolution** operation:

$$G = H * F$$

- Convolution is **commutative** and **associative**

Gaussian Kernel





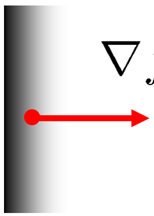
$$G_\sigma = \frac{1}{2\pi\sigma^2} e^{-\frac{(x^2+y^2)}{2\sigma^2}}$$

Source: C. Rasmussen

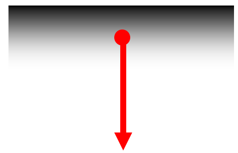
Image gradient

- The *gradient* of an image: $\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$

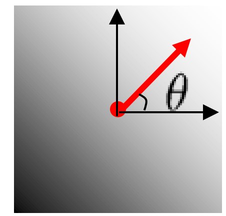
The gradient points in the direction of most rapid increase in intensity



$\nabla f = \left[\frac{\partial f}{\partial x}, 0\right]$



$\nabla f = \left[0, \frac{\partial f}{\partial y}\right]$



$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$

The *edge strength* is given by the gradient magnitude:

$$\|\nabla f\| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

The gradient direction is given by:

$$\theta = \tan^{-1}\left(\frac{\partial f}{\partial y} \Big/ \frac{\partial f}{\partial x}\right)$$

- how does this relate to the direction of the edge?

Source: Steve Seitz

Finding edges

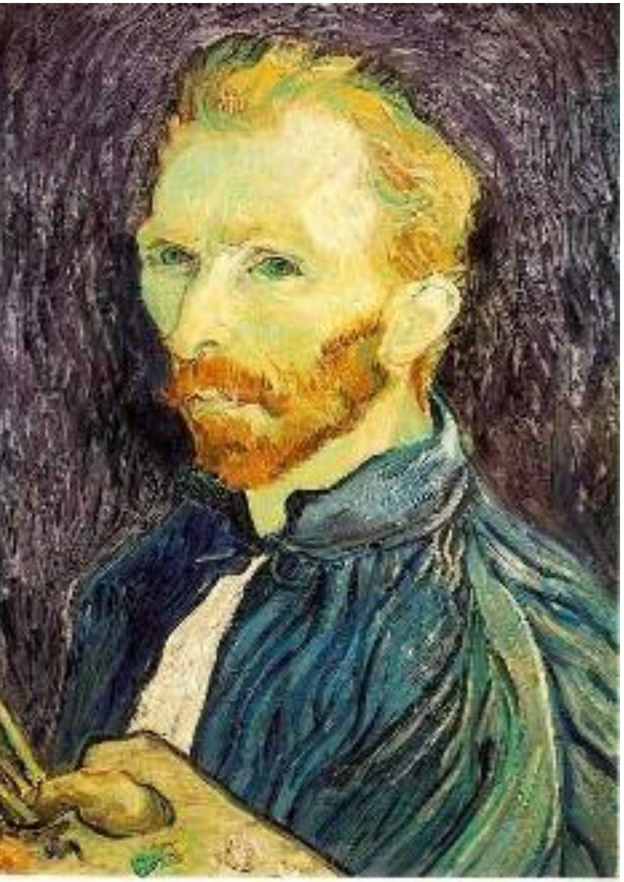
gradient magnitude

Finding edges

thinning

(non-maximum suppression)

Image sub-sampling



1/2

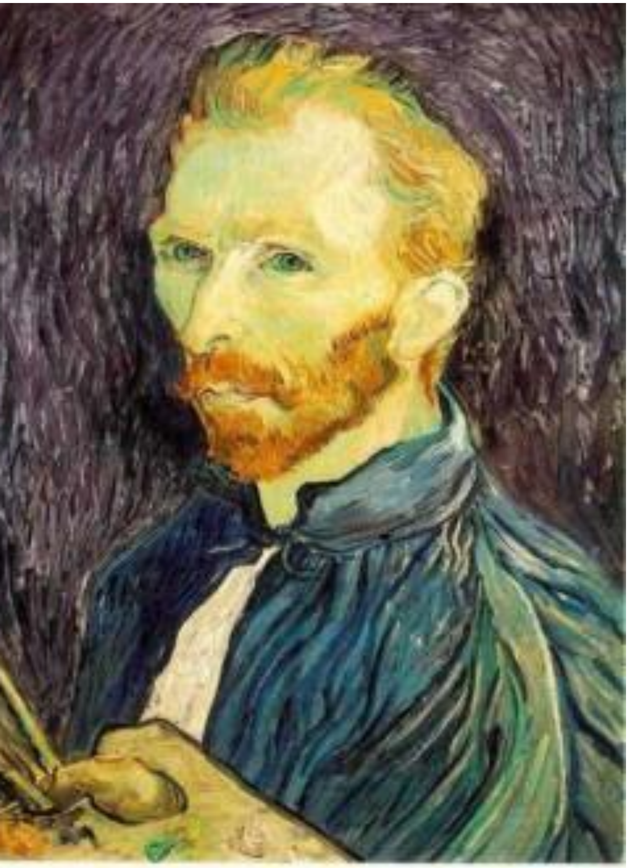
1/4 (2x zoom)

1/8 (4x zoom)

Why does this look so crufty?

Source: S. Seitz

Subsampling with Gaussian pre-filtering



Gaussian 1/2

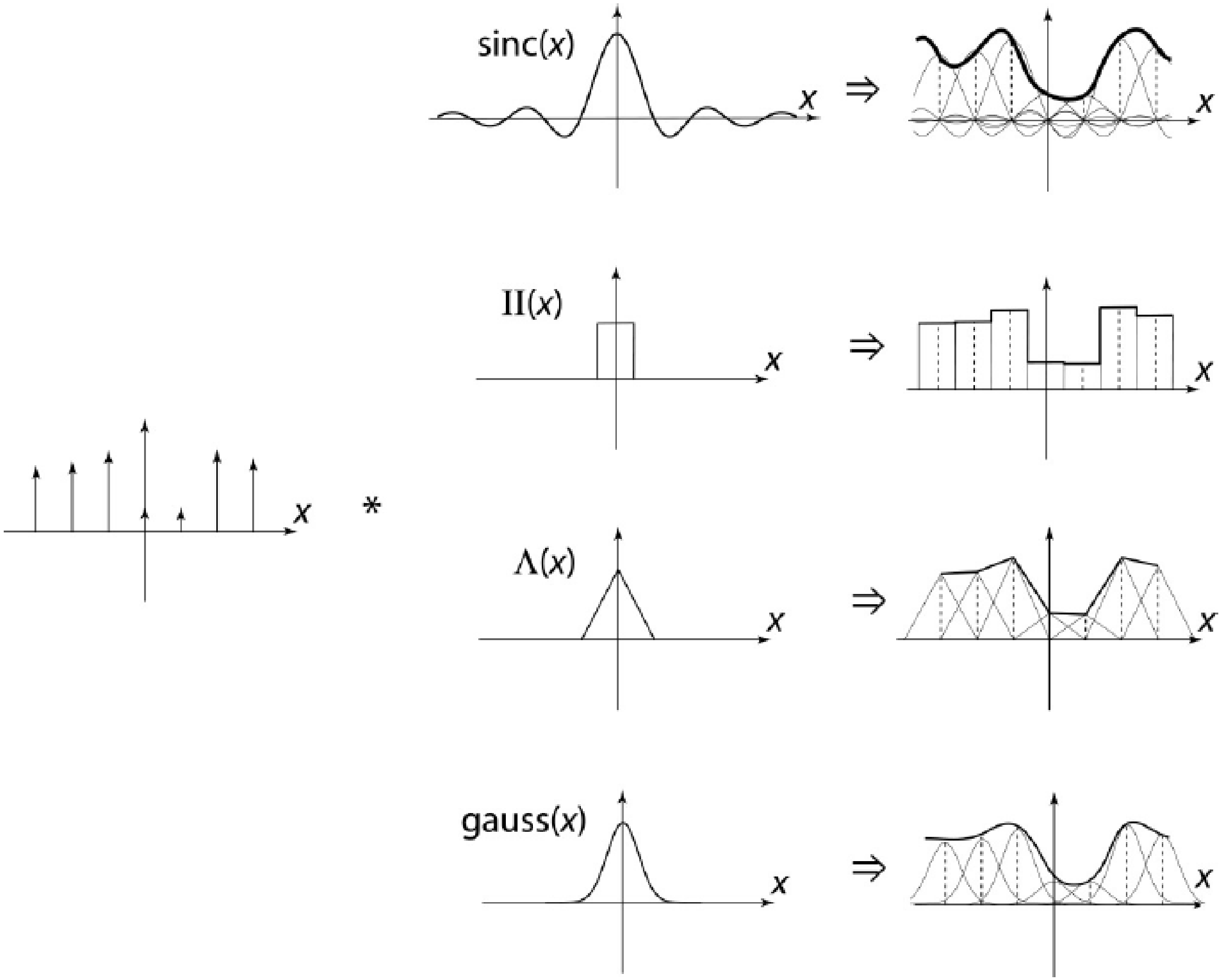
G 1/4

G 1/8

- Solution: filter the image, *then* subsample

Source: S. Seitz

Image interpolation



"Ideal" reconstruction

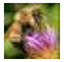
Nearest-neighbor interpolation

Linear interpolation

Gaussian reconstruction

Source: B. Curless

Image interpolation

Original image:  x 10

Nearest-neighbor interpolation

Bilinear interpolation

Bicubic interpolation

The second moment matrix

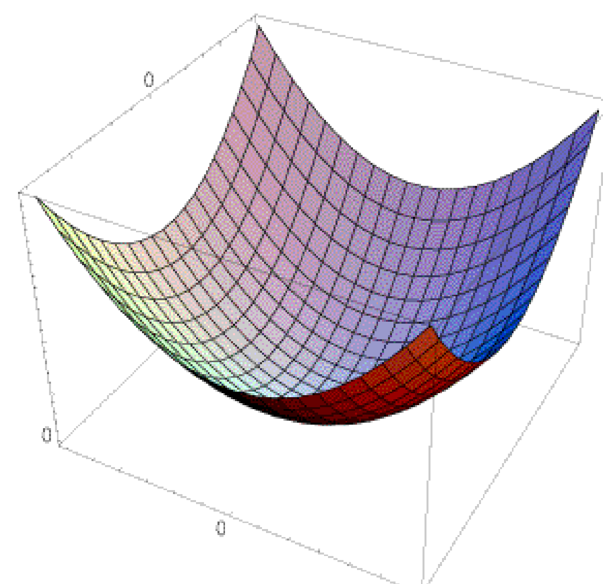
The surface *E*(*u*,*v*) is locally approximated by a quadratic form.

$$
\begin{aligned}
E(u,v) &\approx Au^2 + 2Buv + Cv^2 \\
&\approx \begin{bmatrix} u & v \end{bmatrix} \underbrace{\begin{bmatrix} A & B \\ B & C \end{bmatrix}}_{H} \begin{bmatrix} u \\ v \end{bmatrix}
\end{aligned}
$$

$$A = \sum_{(x,y)\in W} I_x^2$$

$$B = \sum_{(x,y)\in W} I_x I_y$$

$$C = \sum_{(x,y)\in W} I_y^2$$



The Harris operator

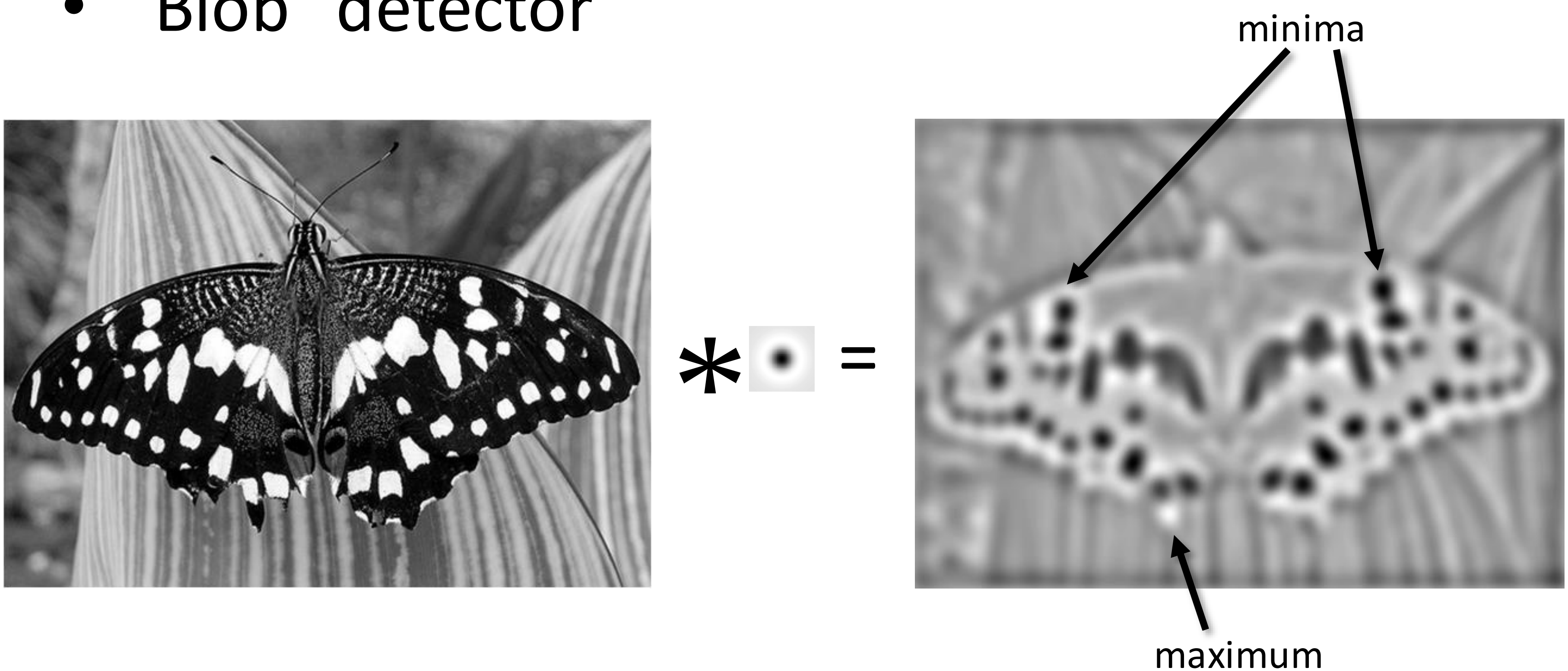
λ_{min} is a variant of the "Harris operator" for feature detection

$$f = \frac{\lambda_1 \lambda_2}{\lambda_1 + \lambda_2}$$
$$= \frac{determinant(H)}{trace(H)}$$

- The *trace* is the sum of the diagonals, i.e., $trace(H) = h_{11} + h_{22}$
- Very similar to λ_{min} but less expensive (no square root)
- Called the "Harris Corner Detector" or "Harris Operator"
- Lots of other detectors, this is one of the most popular

Laplacian of Gaussian

- “Blob” detector



- Find maxima *and minima* of LoG operator in space and scale

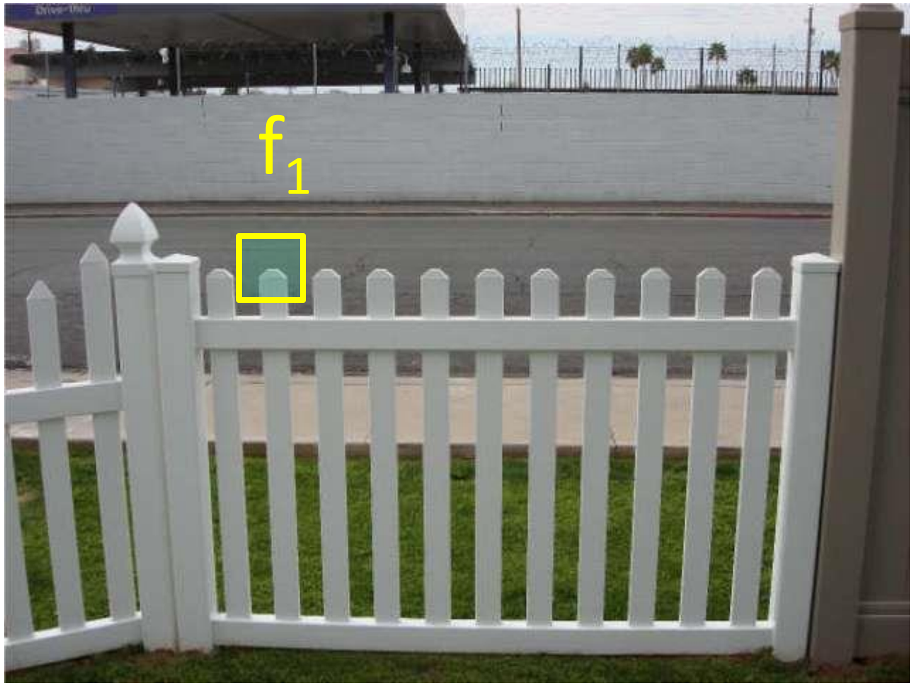
Scale-space blob detector: Example

sigma = 11.9912

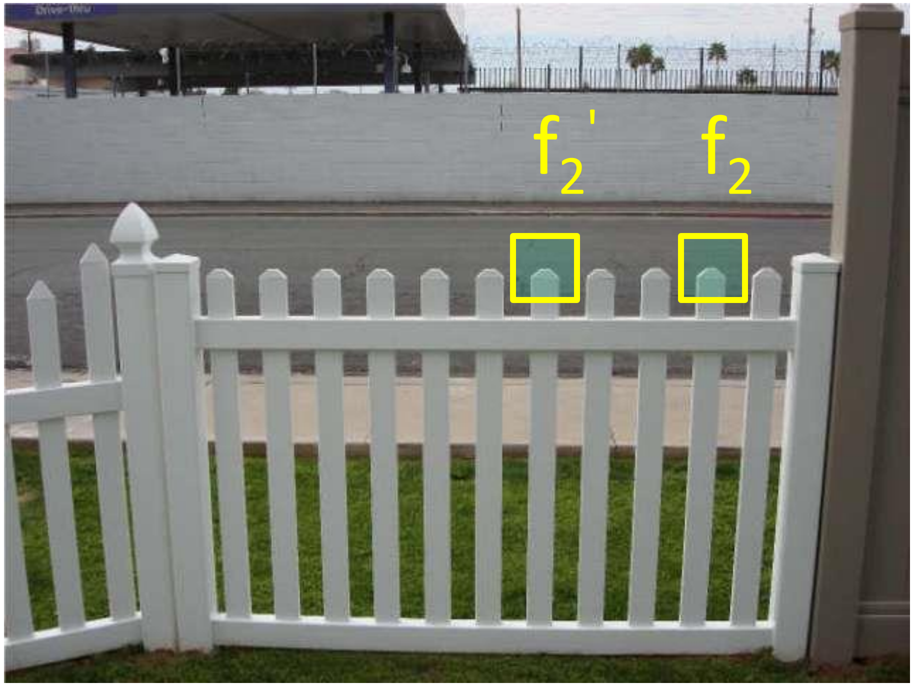
Feature distance

How to define the difference between two features f_1, f_2?

- Better approach: ratio distance = $||f_1 - f_2|| / || f_1 - f_2' ||$
  - f_2 is best SSD match to f_1 in I_2
  - f_2' is 2^{nd} best SSD match to f_1 in I_2
  - gives large values for ambiguous matches



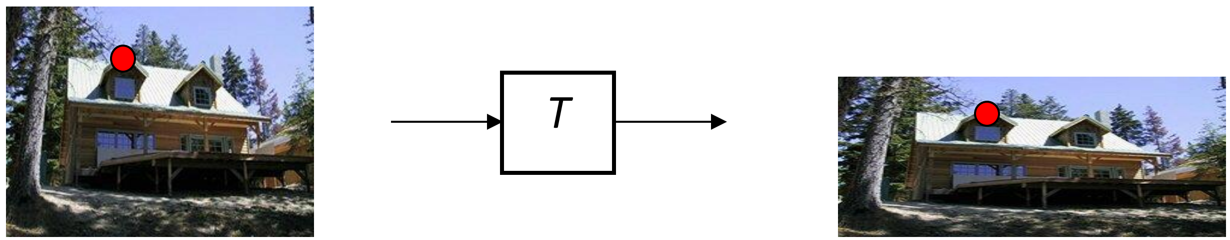
I_1



I_2

2D Geometry

Parametric (global) warping



p = (x,y) **p'** = (x',y')

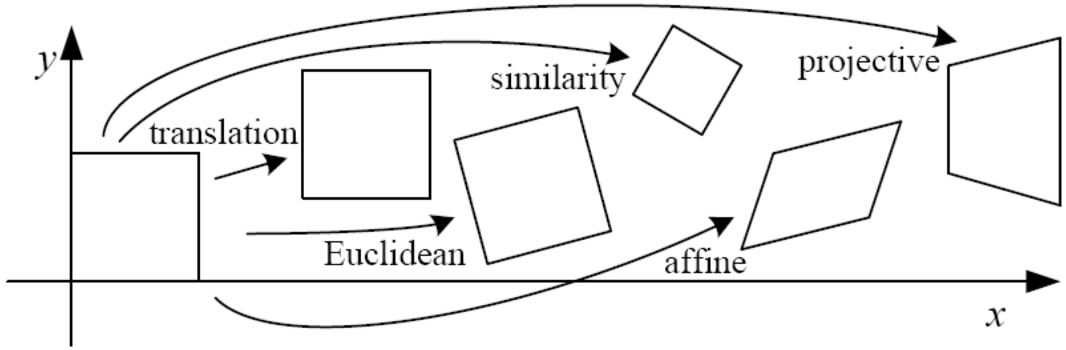
- Transformation T is a coordinate-changing machine:

  p' = *T*(p)

- What does it mean that *T* is global?
  - Is the same for any point p
  - can be described by just a few numbers (parameters)
- Let's consider *linear* xforms (can be represented by a 2D matrix):

$$\mathbf{p}' = \mathbf{T}\mathbf{p} \qquad \left[\begin{array}{c} x' \\ y' \end{array}\right] = \mathbf{T}\left[\begin{array}{c} x \\ y \end{array}\right]$$

2D image transformations



Name	Matrix	# D.O.F.	Preserves:	Icon
translation	$\begin{bmatrix} \boldsymbol{I} \mid \boldsymbol{t} \end{bmatrix}_{2\times 3}$	2	orientation $+\cdots$	
rigid (Euclidean)	$\begin{bmatrix} \boldsymbol{R} \mid \boldsymbol{t} \end{bmatrix}_{2\times 3}$	3	lengths $+\cdots$	
similarity	$\begin{bmatrix} s\boldsymbol{R} \mid \boldsymbol{t} \end{bmatrix}_{2\times 3}$	4	angles $+\cdots$	
affine	$\begin{bmatrix} \boldsymbol{A} \end{bmatrix}_{2\times 3}$	6	parallelism $+\cdots$	
projective	$\begin{bmatrix} \tilde{\boldsymbol{H}} \end{bmatrix}_{3\times 3}$	8	straight lines	

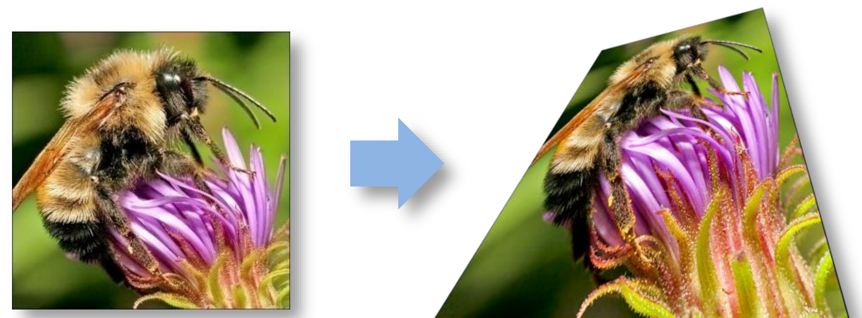
These transformations are a nested set of groups

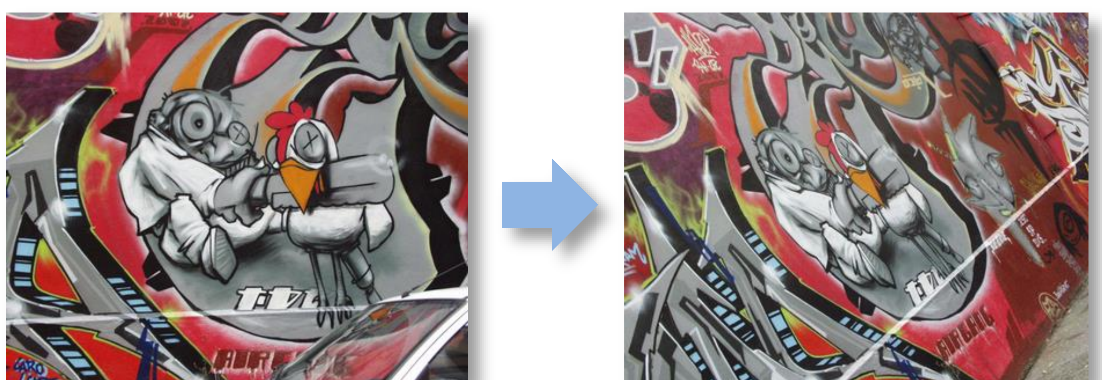
- Closed under composition and inverse is a member

Projective Transformations aka Homographies aka Planar Perspective Maps

$$\mathbf{H} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & 1 \end{bmatrix}$$

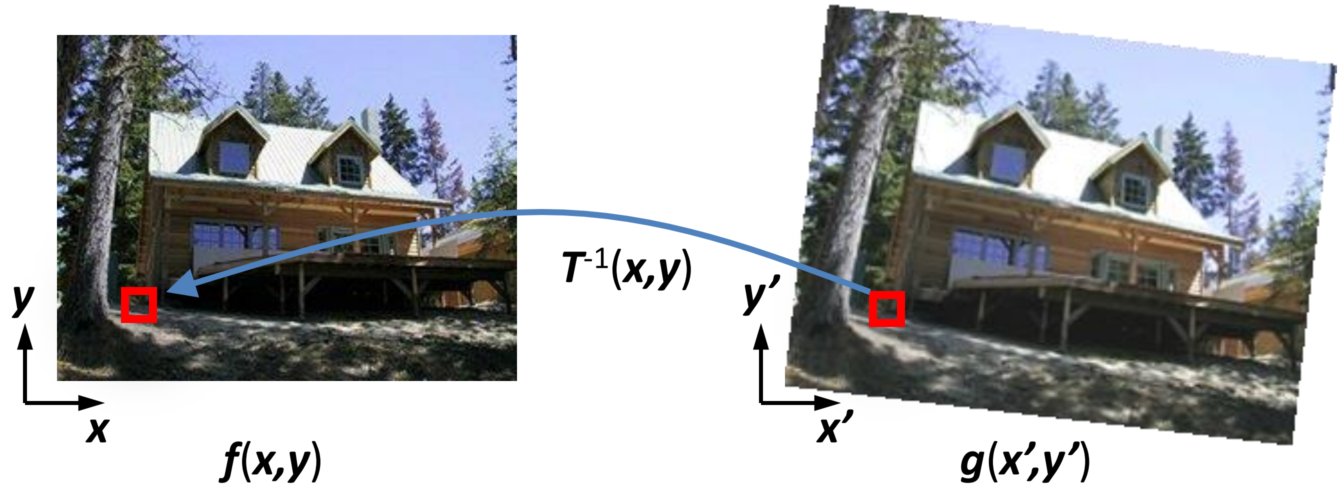
Called a *homography*
(or *planar perspective map*)





Inverse Warping

- Get each pixel $\boldsymbol{g}(\boldsymbol{x}',\boldsymbol{y}')$ from its corresponding location $(\boldsymbol{x},\boldsymbol{y}) = \boldsymbol{T}^{-1}(\boldsymbol{x},\boldsymbol{y})$ in $\boldsymbol{f}(\boldsymbol{x},\boldsymbol{y})$
  - Requires taking the inverse of the transform



Affine transformations

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} ax + by + c \\ dx + ey + f \\ 1 \end{bmatrix}$$

Solving for affine transformations

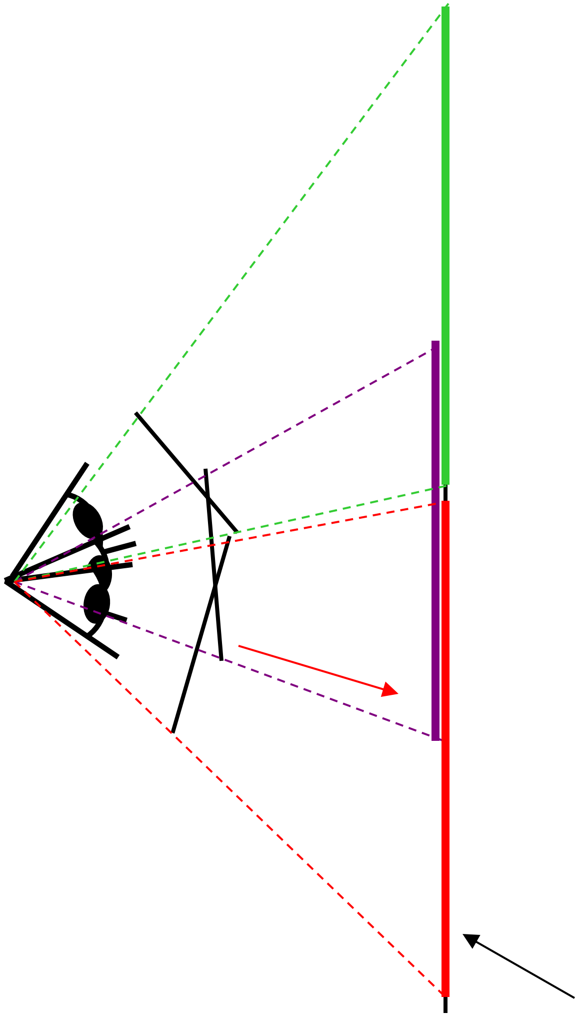
- Matrix form

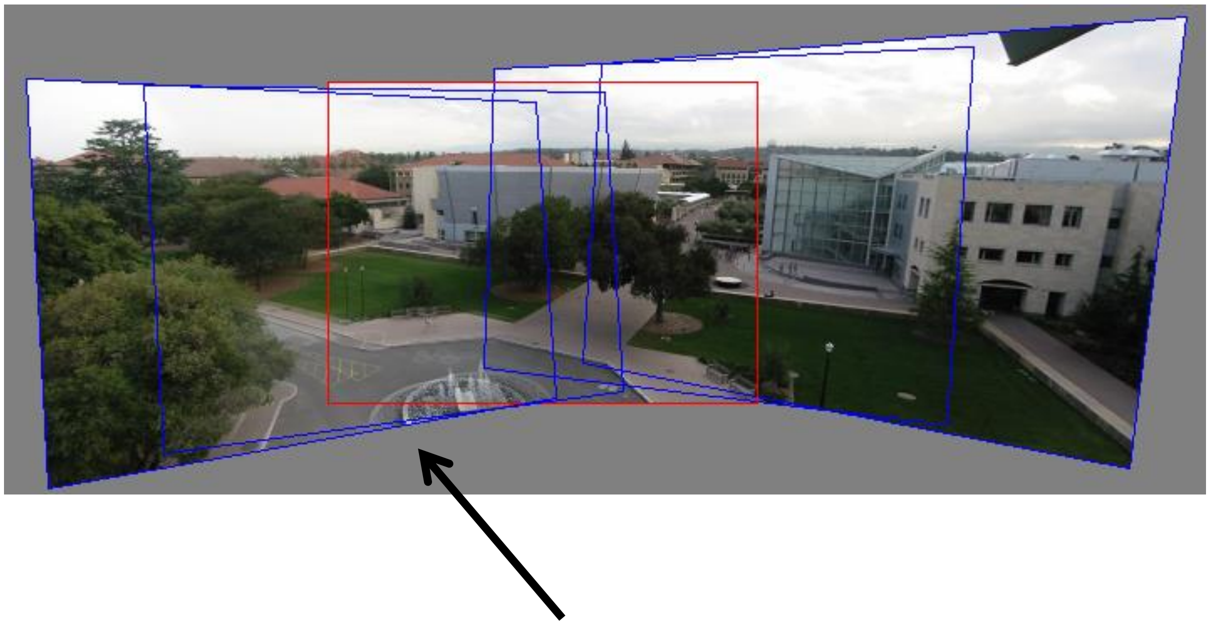
$$
\underbrace{\begin{bmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_1 & y_1 & 1 \\ x_2 & y_2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_2 & y_2 & 1 \\ & & & \vdots & & \\ x_n & y_n & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_n & y_n & 1 \end{bmatrix}}_{\substack{\mathbf{A} \\ 2n \times 6}}
\underbrace{\begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \end{bmatrix}}_{\substack{\mathbf{t} \\ 6 \times 1}}
=
\underbrace{\begin{bmatrix} x'_1 \\ y'_1 \\ x'_2 \\ y'_2 \\ \vdots \\ x'_n \\ y'_n \end{bmatrix}}_{\substack{\mathbf{b} \\ 2n \times 1}}
$$

RANSAC

- General version:
  1. Randomly choose *s* samples
     - Typically *s* = minimum sample size that lets you fit a model
  2. Fit a model (e.g., line) to those samples
  3. Count the number of inliers that approximately fit the model
  4. Repeat *N* times
  5. Choose the model that has the largest set of inliers

Projecting images onto a common plane



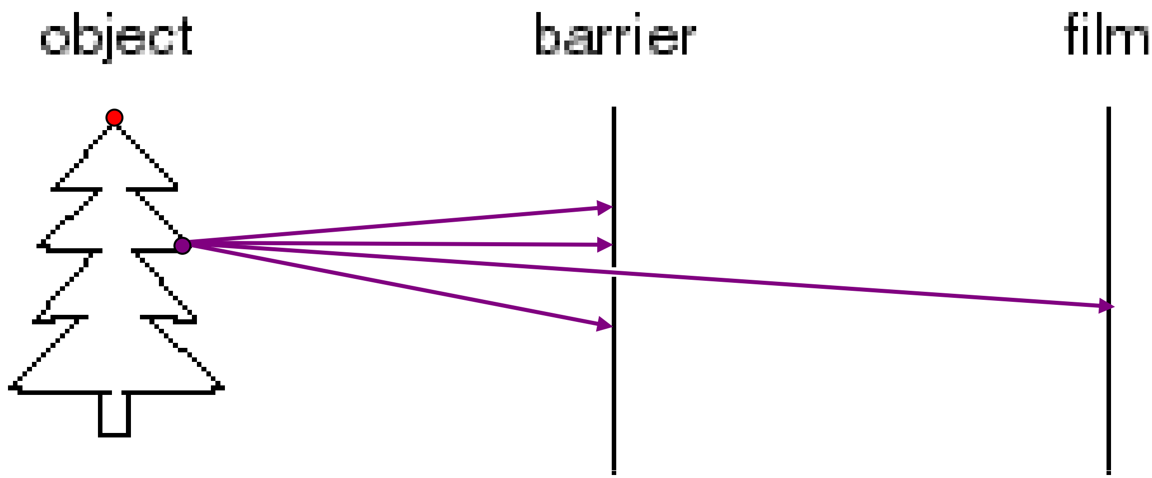


each image is warped with a homography $\mathbf{H}$

Can’t create a 360 panorama this way…

3D Geometry

Pinhole camera



- Add a barrier to block off most of the rays
  - This reduces blurring
  - The opening known as the **aperture**
  - How does this transform the image?

Perspective Projection

Projection is a matrix multiply using homogeneous coordinates:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1/d & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ -z/d \end{bmatrix} \Rightarrow (-d\frac{x}{z}, \ \ - d\frac{y}{z})$$

divide by third coordinate

This is known as **perspective projection**

- The matrix is the **projection matrix**

Projection matrix

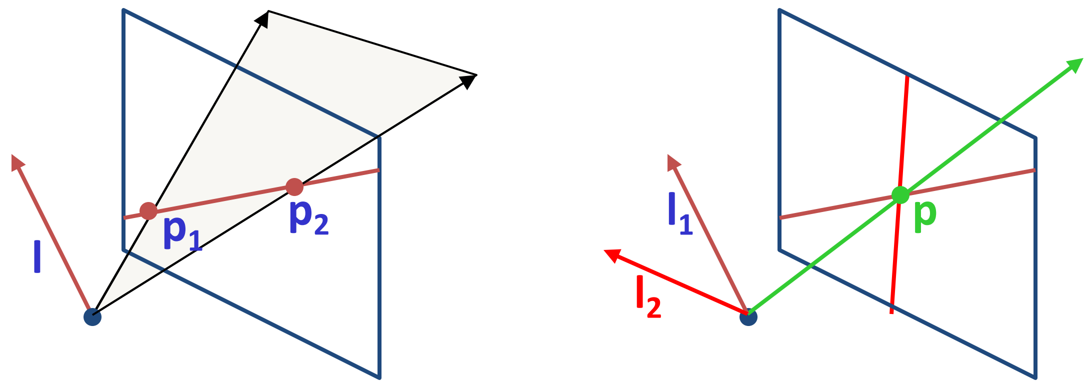
$$\Pi = \underbrace{\mathbf{K}}_{\text{intrinsics}} \underbrace{\underbrace{\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\end{bmatrix}}_{\text{projection}} \underbrace{\begin{bmatrix}\mathbf{R} & \begin{matrix}0\\0\\0\end{matrix}\\ 0\quad 0\quad 0 & 1\end{bmatrix}}_{\text{rotation}} \underbrace{\begin{bmatrix}\mathbf{I}_{3\times 3} & -\mathbf{c}\\ 0\quad 0\quad 0 & 1\end{bmatrix}}_{\text{translation}}}$$

$$\left[\ \mathbf{R}\ \middle|\ \underbrace{-\mathbf{R}\mathbf{c}}_{(\mathbf{t}\text{ in book's notation})}\ \right]$$

$$\Pi = \mathbf{K}\left[\ \mathbf{R}\ \middle|\ -\mathbf{R}\mathbf{c}\ \right]$$

Point and line duality

- A line $\mathbf{l}$ is a homogeneous 3-vector
- It is $\perp$ to every point (ray) $\mathbf{p}$ on the line: $\mathbf{l}\,\mathbf{p}=0$



What is the line $\mathbf{l}$ spanned by rays $\mathbf{p_1}$ and $\mathbf{p_2}$?

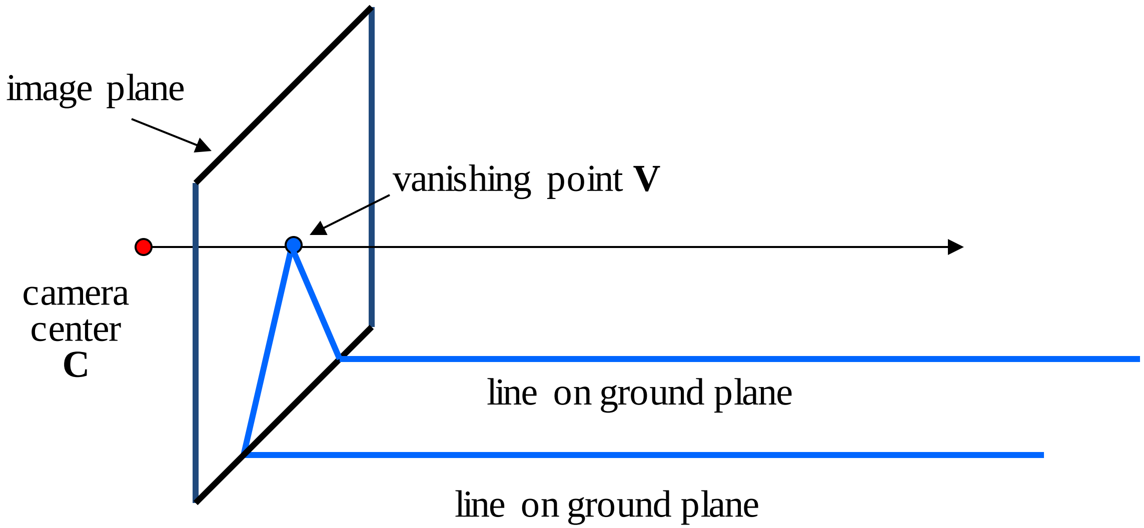
- $\mathbf{l}$ is $\perp$ to $\mathbf{p_1}$ and $\mathbf{p_2}$ $\Rightarrow$ $\mathbf{l} = \mathbf{p_1} \times \mathbf{p_2}$
- $\mathbf{l}$ can be interpreted as a *plane normal*

What is the intersection of two lines $\mathbf{l_1}$ and $\mathbf{l_2}$?

- $\mathbf{p}$ is $\perp$ to $\mathbf{l_1}$ and $\mathbf{l_2}$ $\Rightarrow$ $\mathbf{p} = \mathbf{l_1} \times \mathbf{l_2}$

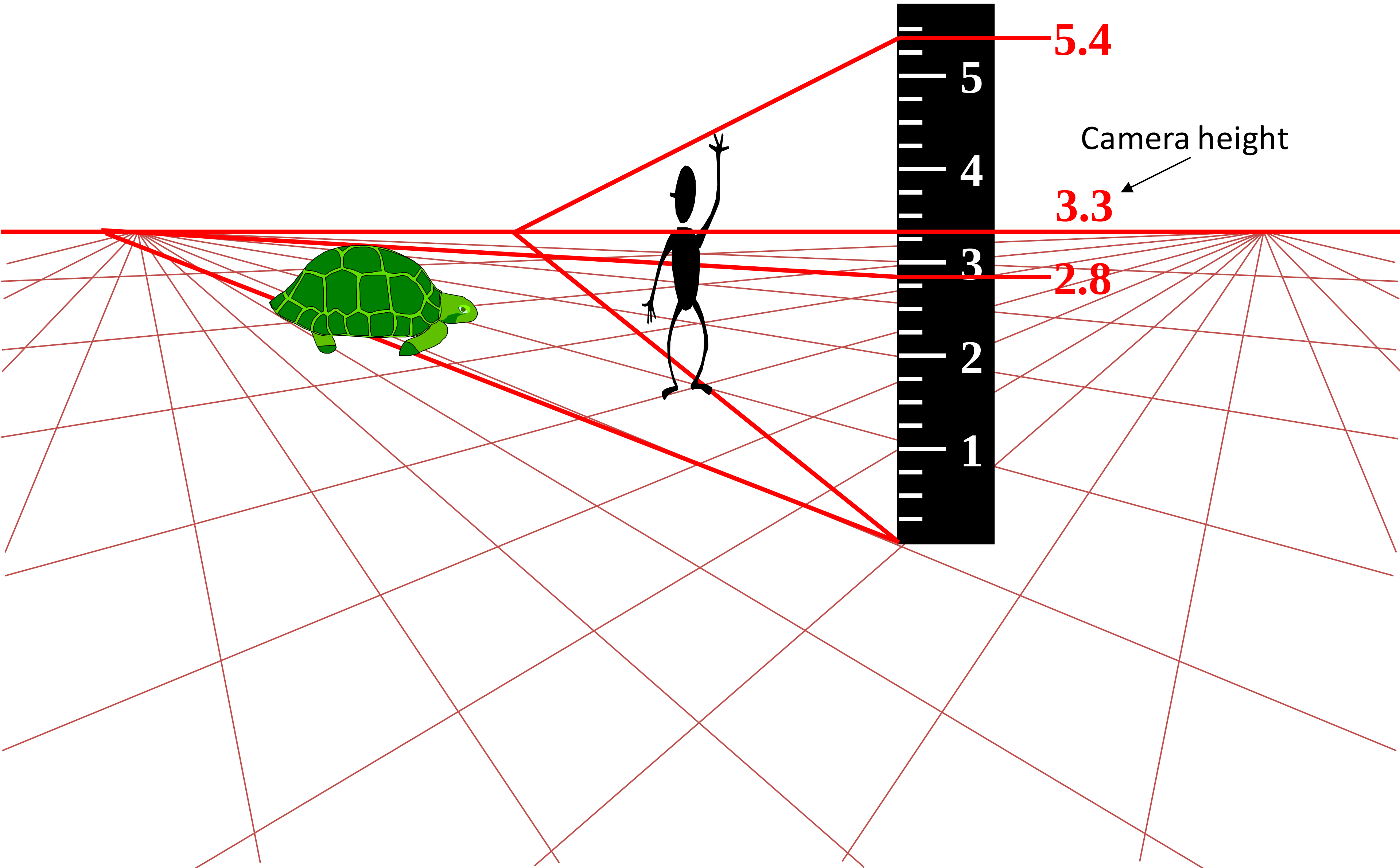
Points and lines are *dual* in projective space

Vanishing points

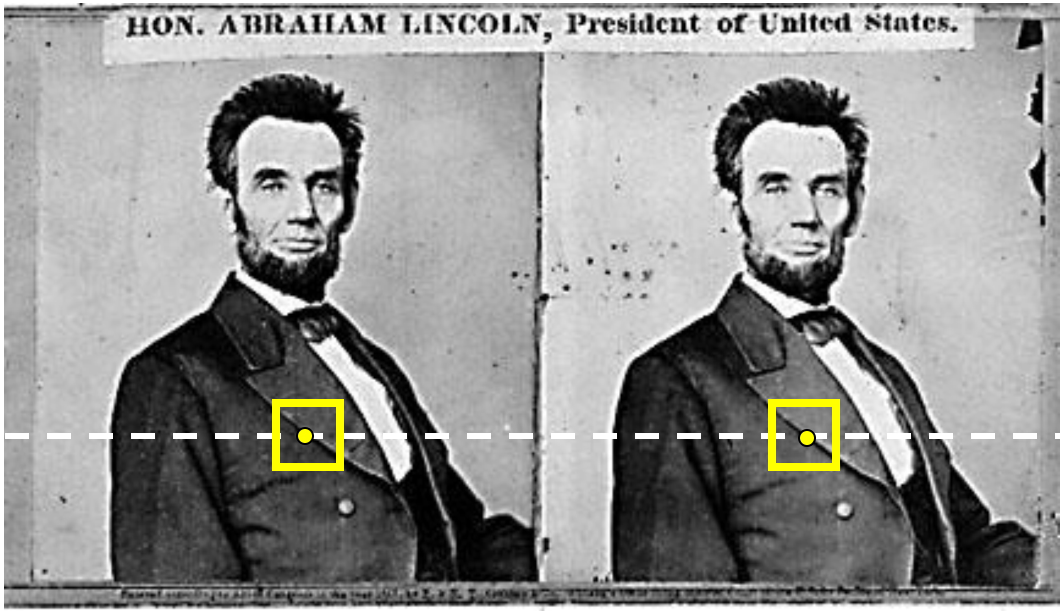


- Properties
  - Any two parallel lines (in 3D) have the same vanishing point **v**
  - The ray from **C** through **v** is parallel to the lines
  - An image may have more than one vanishing point
    - in fact, every image point is a potential vanishing point

Measuring height



Your basic stereo algorithm



For each epipolar line

For each pixel in the left image

- compare with every pixel on same epipolar line in right image
- pick pixel with minimum match cost

Improvement: match ***windows***

Stereo as energy minimization

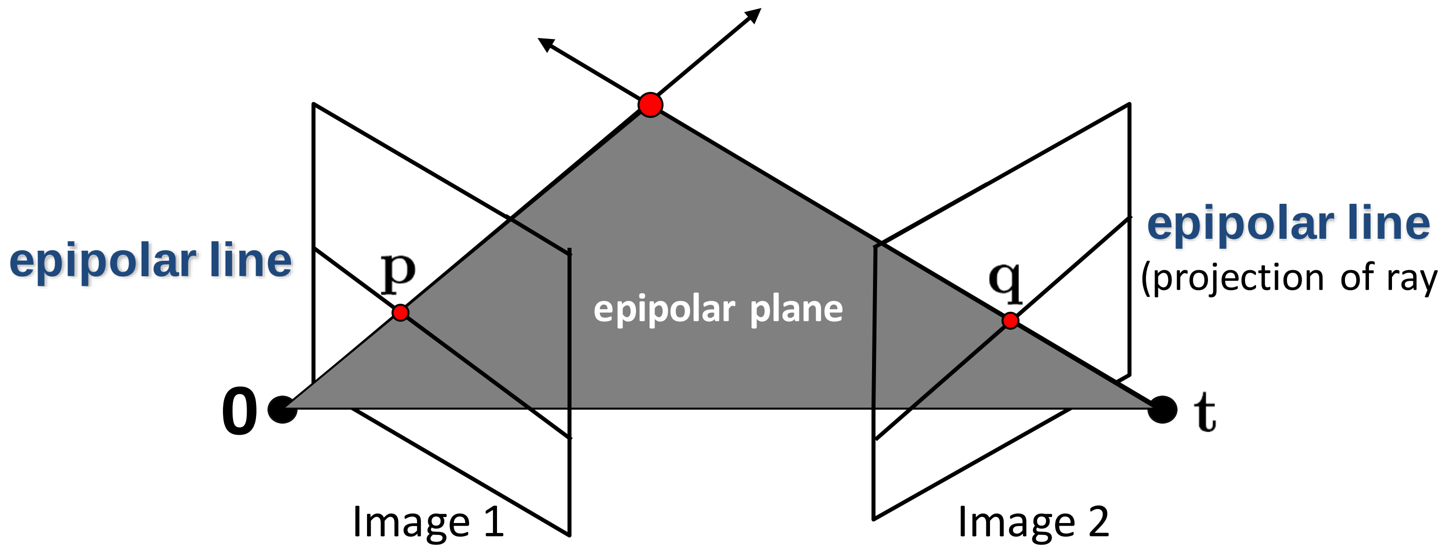
- Better objective function

$$E(d) = \underbrace{E_d(d)}_{\text{match cost}} + \lambda \underbrace{E_s(d)}_{\text{smoothness cost}}$$

Want each pixel to find a good match in the other image

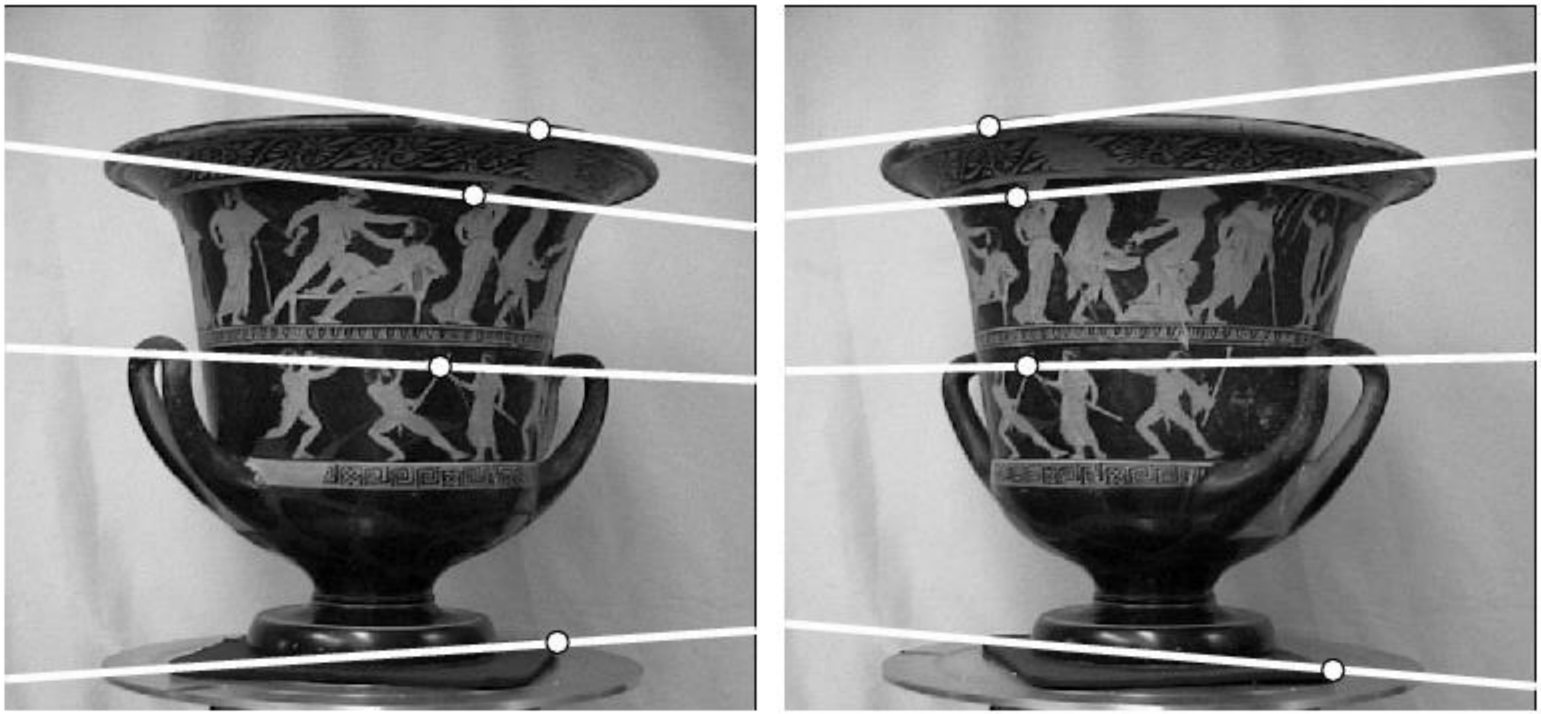
Adjacent pixels should (usually) move about the same amount

Fundamental matrix



- This *epipolar geometry* of two views is described by a Very Special 3x3 matrix $\mathbf{F}$, called the *F`undamental matrix*
- $\mathbf{F}$ maps (homogeneous) *points* in image 1 to *lines* in image 2!
- The epipolar line (in image 2) of point **p** is: $\mathbf{F}\mathbf{p}$
- *Epipolar constraint* on corresponding points: $\mathbf{q}^T\mathbf{F}\mathbf{p} = 0$

Epipolar geometry demo

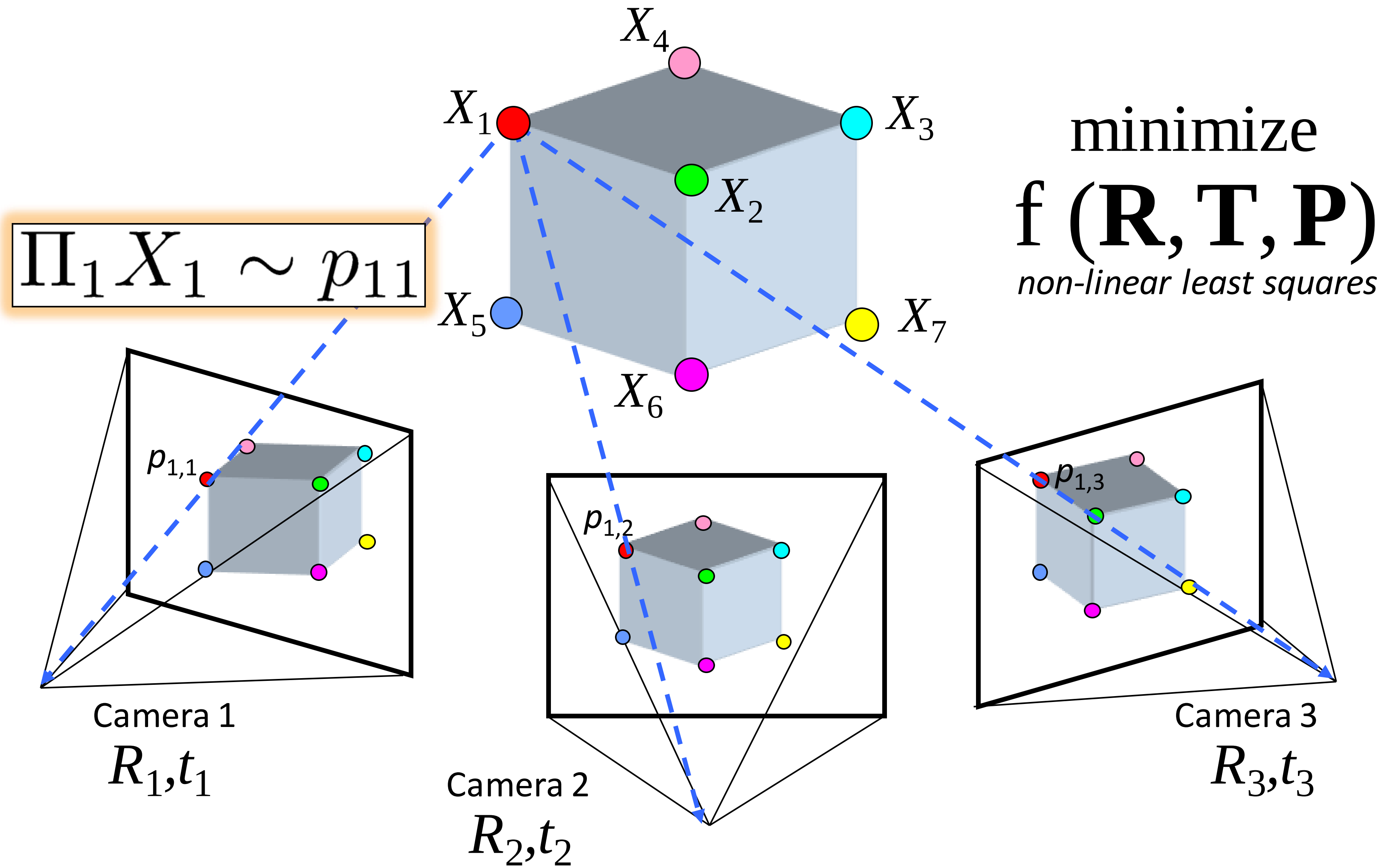


8-point algorithm

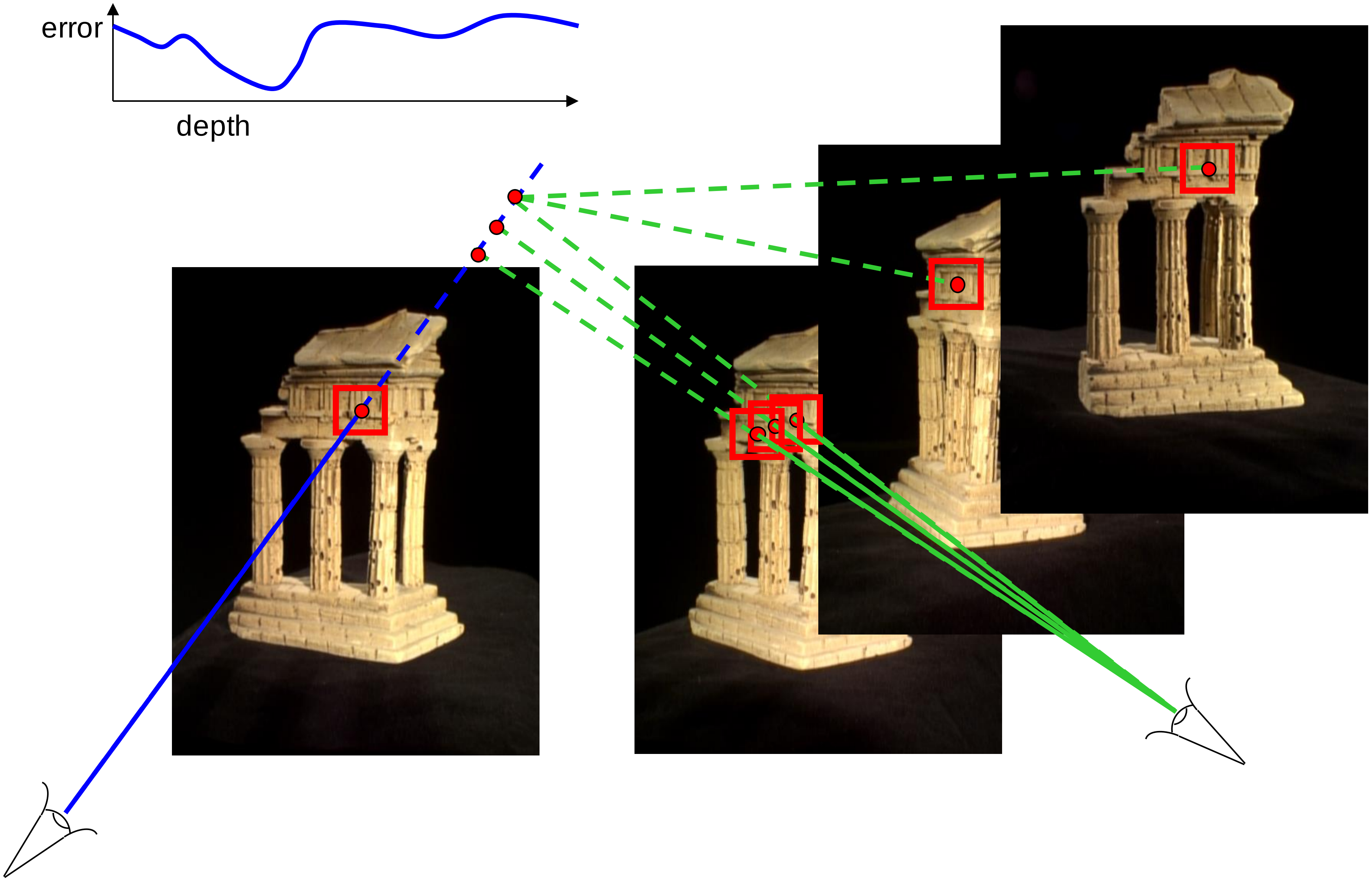
$$\begin{bmatrix} u_1u_1' & v_1u_1' & u_1' & u_1v_1' & v_1v_1' & v_1' & u_1 & v_1 & 1 \\ u_2u_2' & v_2u_2' & u_2' & u_2v_2' & v_2v_2' & v_2' & u_2 & v_2 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ u_nu_n' & v_nu_n' & u_n' & u_nv_n' & v_nv_n' & v_n' & u_n & v_n & 1 \end{bmatrix} \begin{bmatrix} f_{11} \\ f_{12} \\ f_{13} \\ f_{21} \\ f_{22} \\ f_{23} \\ f_{31} \\ f_{32} \\ f_{33} \end{bmatrix} = 0$$

- In reality, instead of solving $\mathbf{Af} = 0$, we seek $\mathbf{f}$ to minimize $\|\mathbf{Af}\|$, least eigenvector of $\mathbf{A}^{\mathrm{T}}\mathbf{A}$.

Structure from motion



Stereo: another view



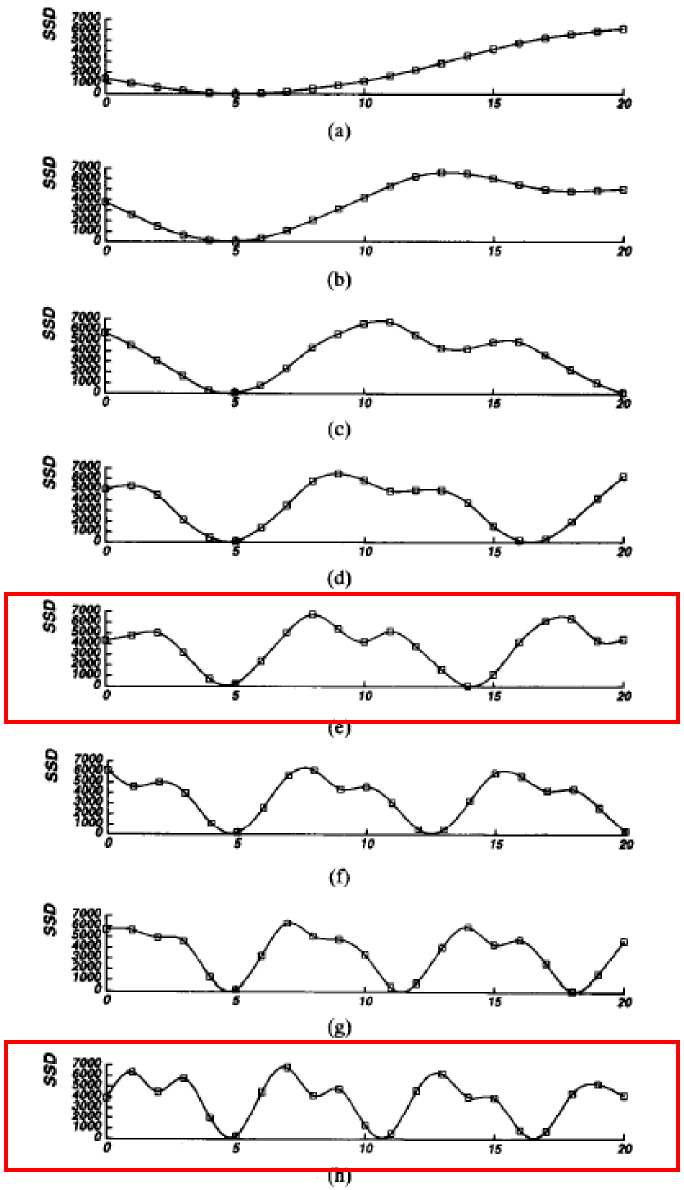


Fig. 5. SSD values versus inverse distance: (a) $B = b$; (b) $B = 2b$; (c) $B = 3b$; (d) $B = 4b$; (e) $B = 5b$; (f) $B = 6b$; (g) $B = 7b$; (h) $B = 8b$. The horizontal axis is normalized such that $8bF = 1$.

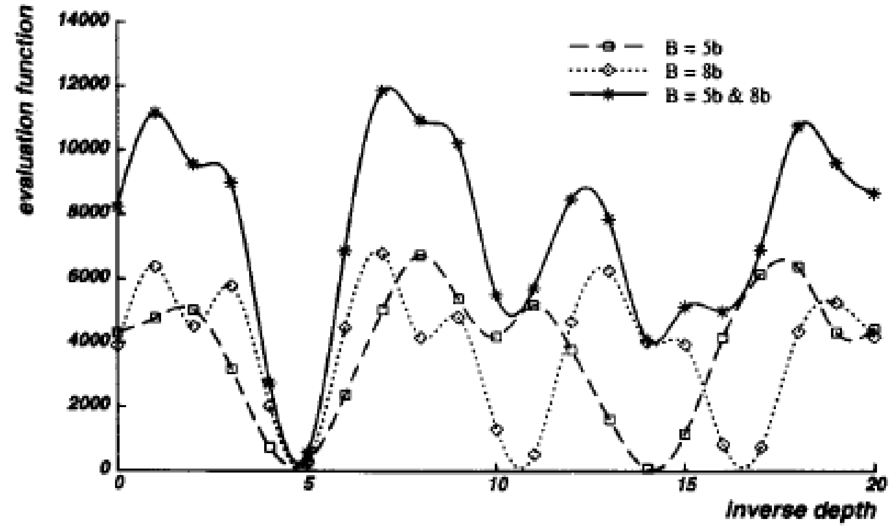


Fig. 6. Combining two stereo pairs with different baselines.

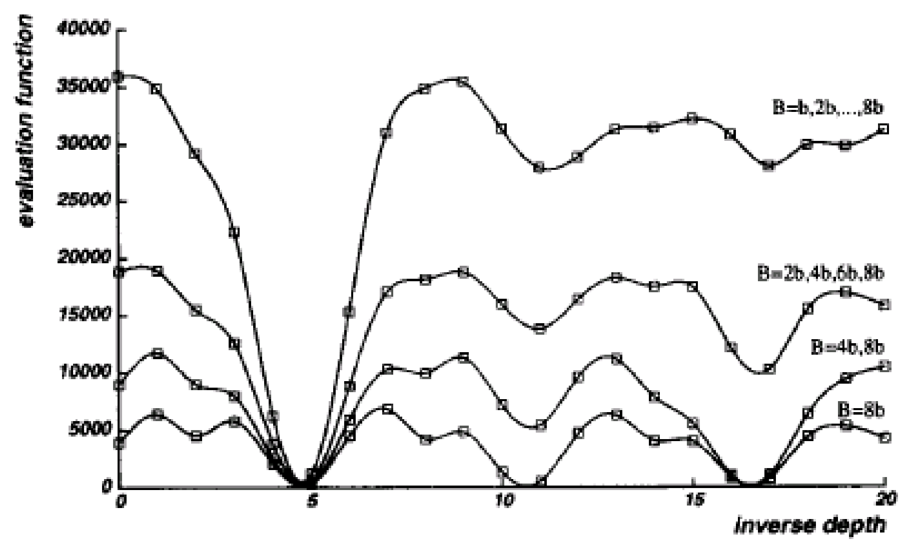
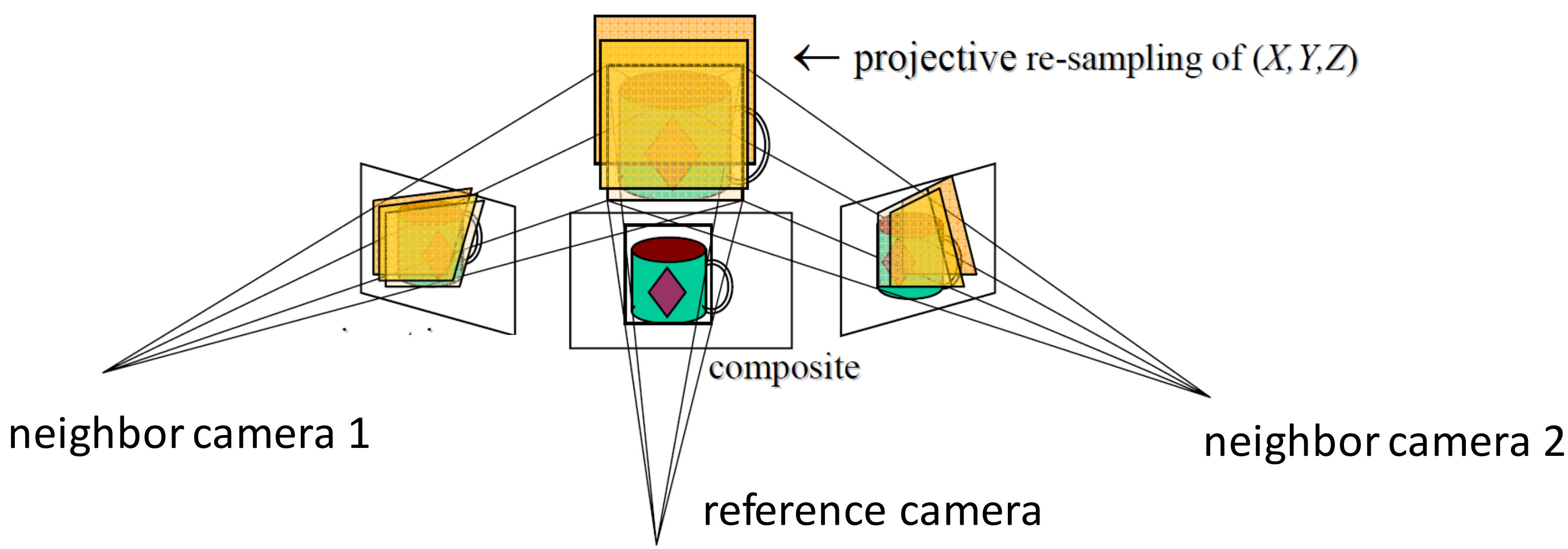


Fig. 7. Combining multiple baseline stereo pairs.

Plane-Sweep Stereo

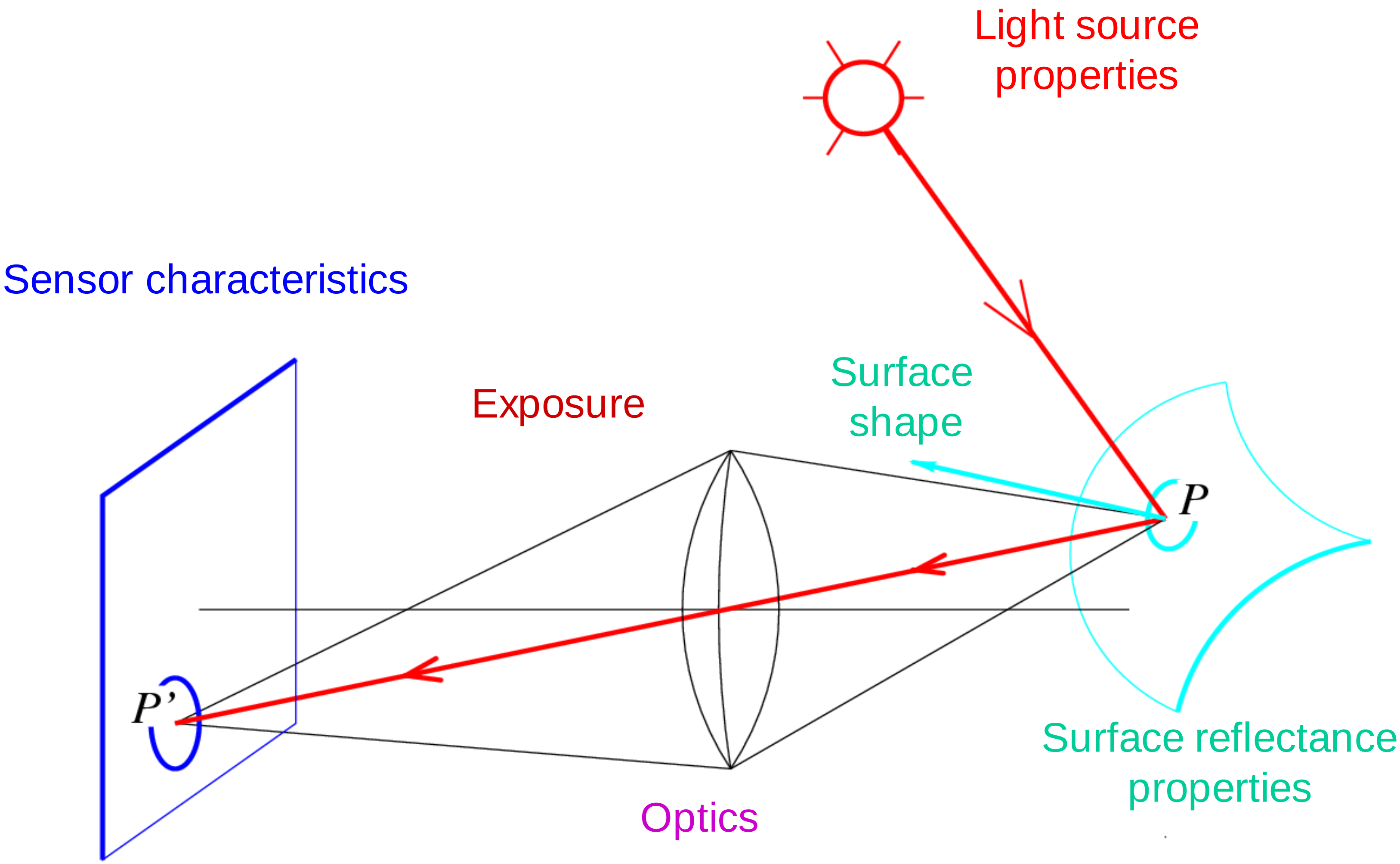
- Sweep family of planes parallel to the reference camera image plane
- Reproject neighbors onto each plane (via homography) and compare reprojections



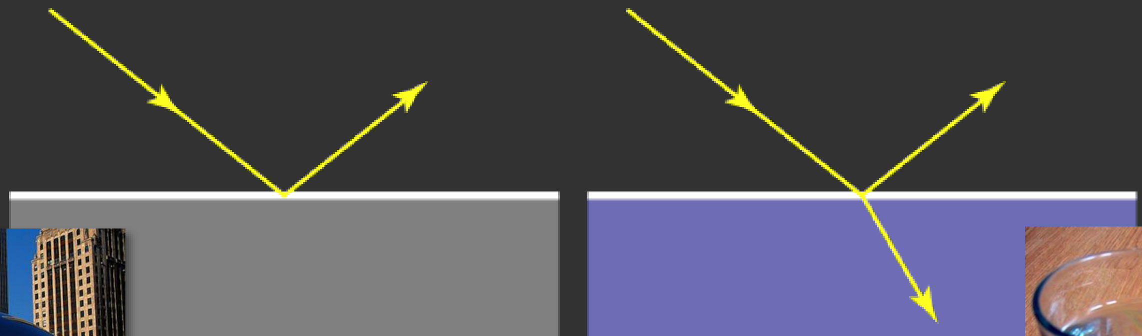
Light, reflectance, cameras

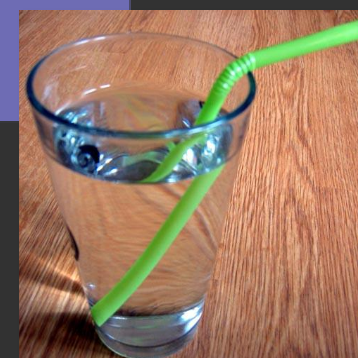
Radiometry

What determines the brightness of an image pixel?

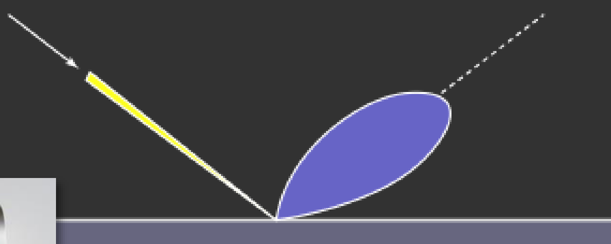


Classic reflection behavior

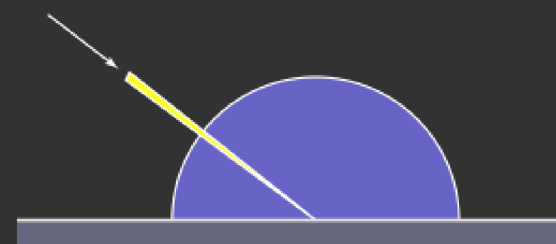


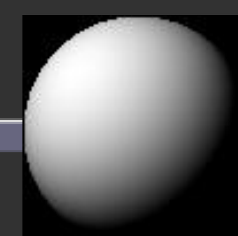


ideal specular



rough specular

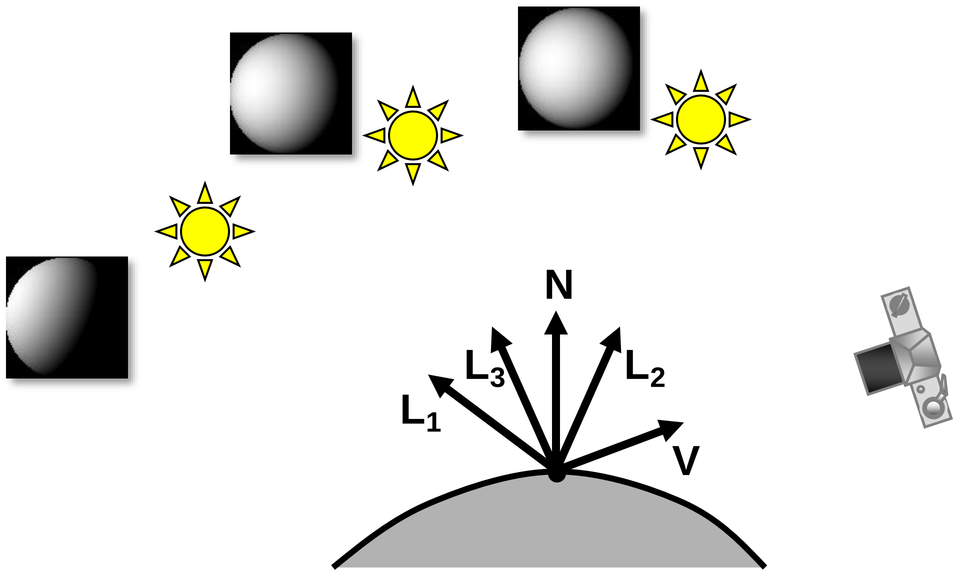




Lambertian

from Steve Marschner

Photometric stereo

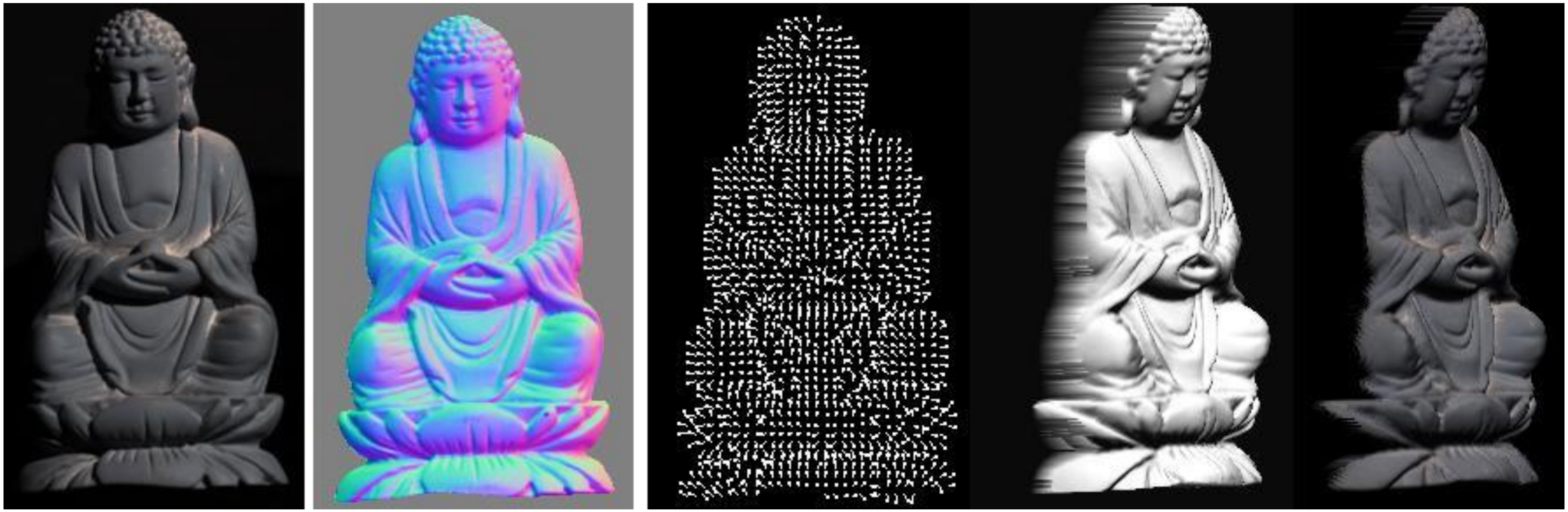


$$I_1 = k_d \mathbf{N} \cdot \mathbf{L_1}$$
$$I_2 = k_d \mathbf{N} \cdot \mathbf{L_2}$$
$$I_3 = k_d \mathbf{N} \cdot \mathbf{L_3}$$

Can write this as a matrix equation:

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = k_d \begin{bmatrix} \mathbf{L_1}^T \\ \mathbf{L_2}^T \\ \mathbf{L_3}^T \end{bmatrix} \mathbf{N}$$

Example



Recognition

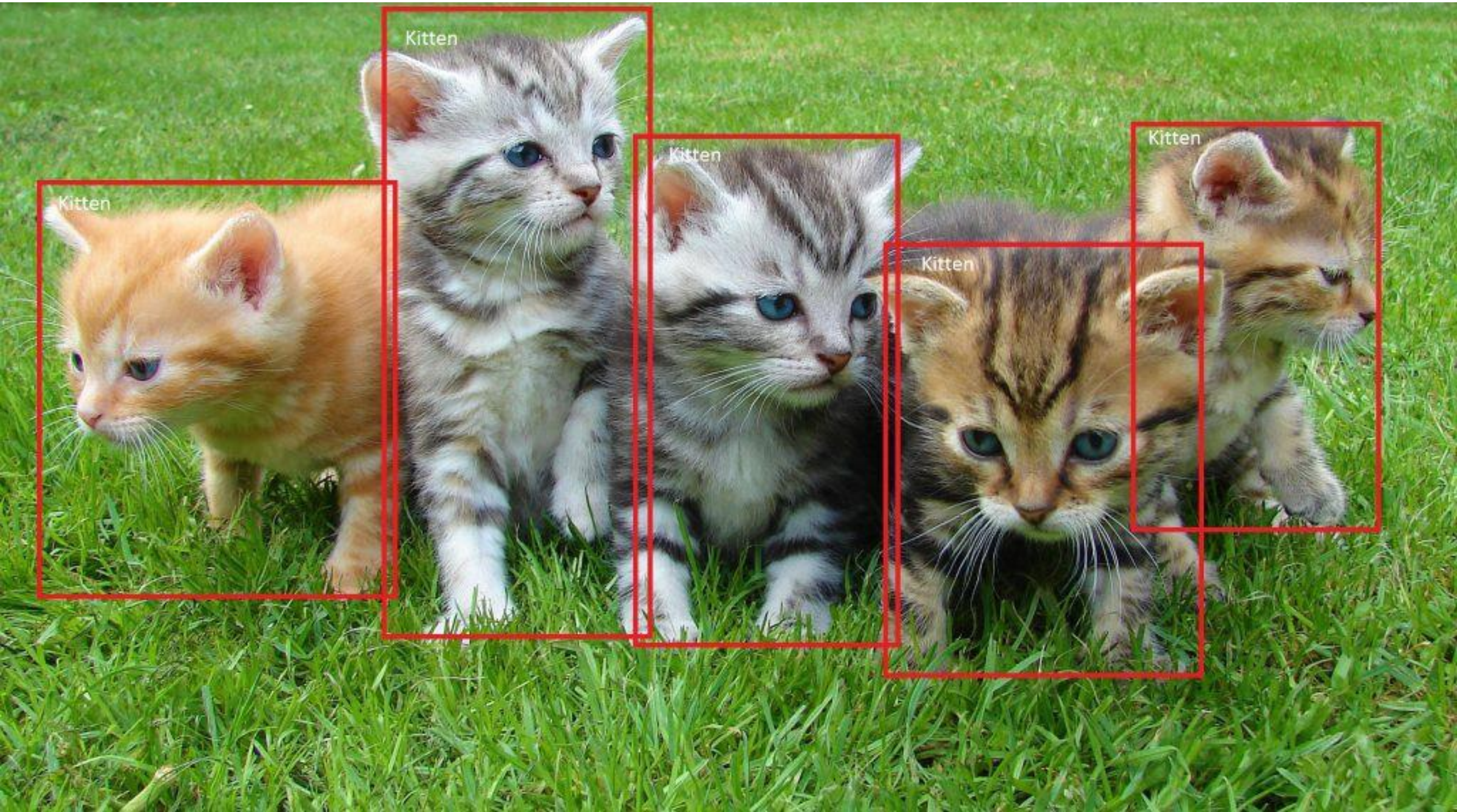
Image Classification

(assume given set of discrete labels)
{dog, cat, truck, plane, ...}

→ cat

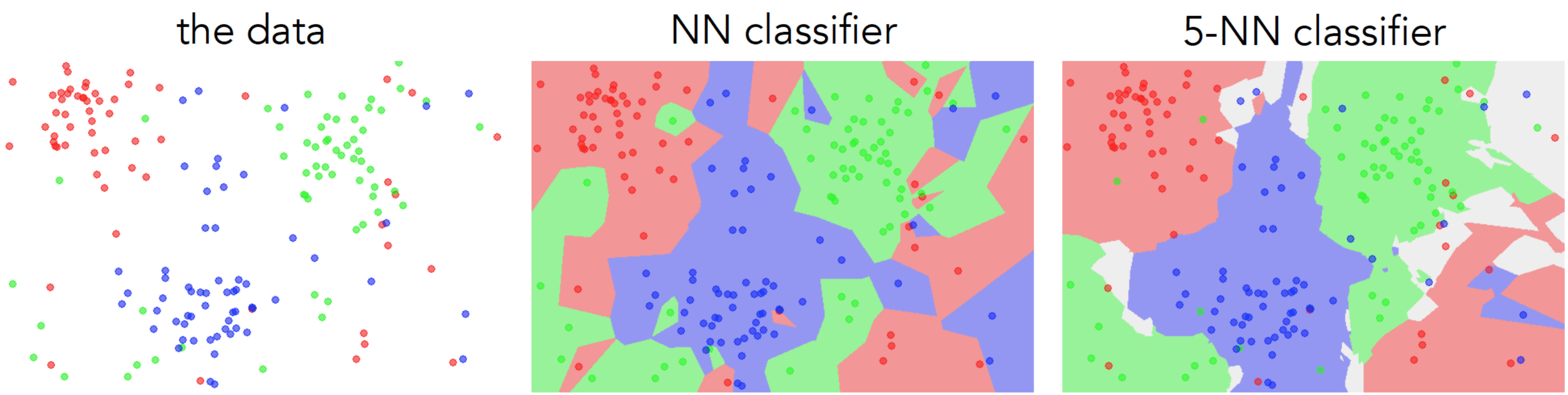
Slides from Andrej Karpathy and Fei-Fei Li
http://vision.stanford.edu/teaching/cs231n/

Object detection



k-nearest neighbor

- Find the k closest points from training data
- Take **majority vote** from K closest points



Hyperparameters

- What is the **best distance** to use?
- What is the **best value of k** to use?

- These are **hyperparameters**: choices about the algorithm that we set rather than learn

- How do we set them?
  - One option: try them all and see what works best

Setting Hyperparameters

Idea #1: Choose hyperparameters that work best on the data

BAD: K = 1 always works perfectly on training data

Idea #2: Split data into **train** and **test**, choose hyperparameters that work best on test data

BAD: No idea how algorithm will perform on new data

Idea #3: Split data into **train**, **val**, and **test**; choose hyperparameters on val and evaluate on test

Better!

Parametric approach: Linear classifier

$$\boxed{f(x,W)} = \boxed{W}\boxed{x} \quad \text{3072x1} \qquad \boxed{(+b)} \ \text{10x1}$$

10x1 **10x3072**

[32x32x3]
array of numbers 0...1

10 numbers, indicating class scores

parameters, or “weights”

Loss function, cost/objective function

- Given ground truth labels (y_i), scores $f(x_i, \mathbf{W})$
  - how unhappy are we with the scores?

- Loss function or objective/cost function measures unhappiness

- During training, **want to find the parameters W that minimizes the loss function**

Softmax classifier

$$f(x_i, W) = Wx_i$$

score function is the same

$$\boxed{\frac{e^{f_{y_i}}}{\sum_j e^{f_j}}}$$

softmax function

$$[1, -2, 0] \rightarrow [e^1, e^{-2}, e^0] = [2.71, 0.14, 1] \rightarrow [0.7, 0.04, 0.26]$$

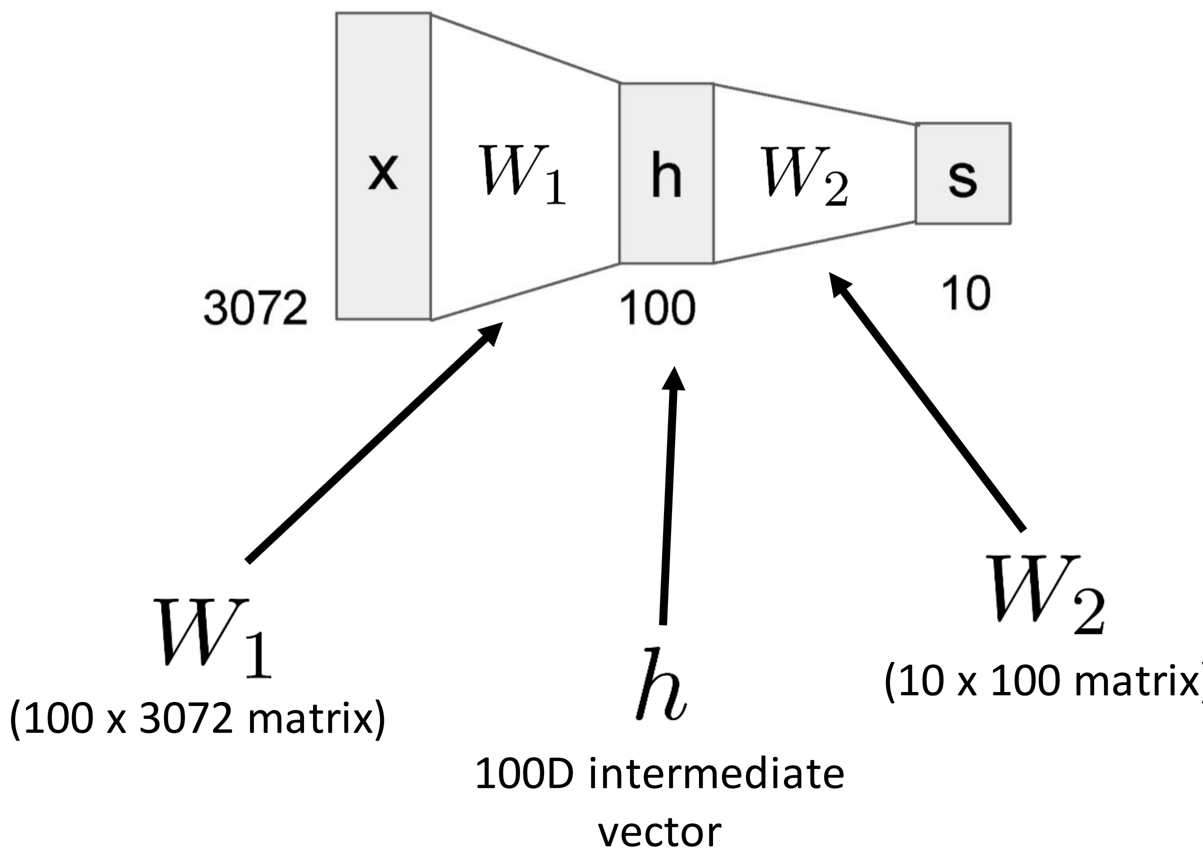
Interpretation: squashes values into range 0 to 1

$$P(y_i \mid x_i; W)$$

Neural networks

(**Before**) Linear score function: $f = Wx$

(**Now**) 2-layer Neural Network $f = W_2 \max(0, W_1 x)$



Convolutional neural networks

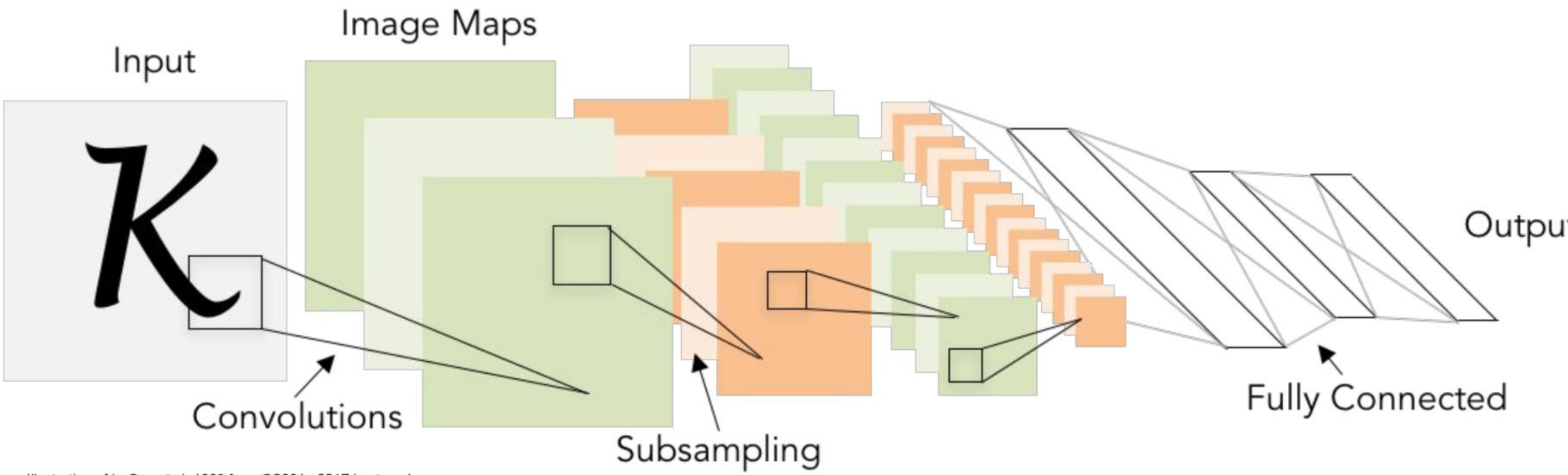
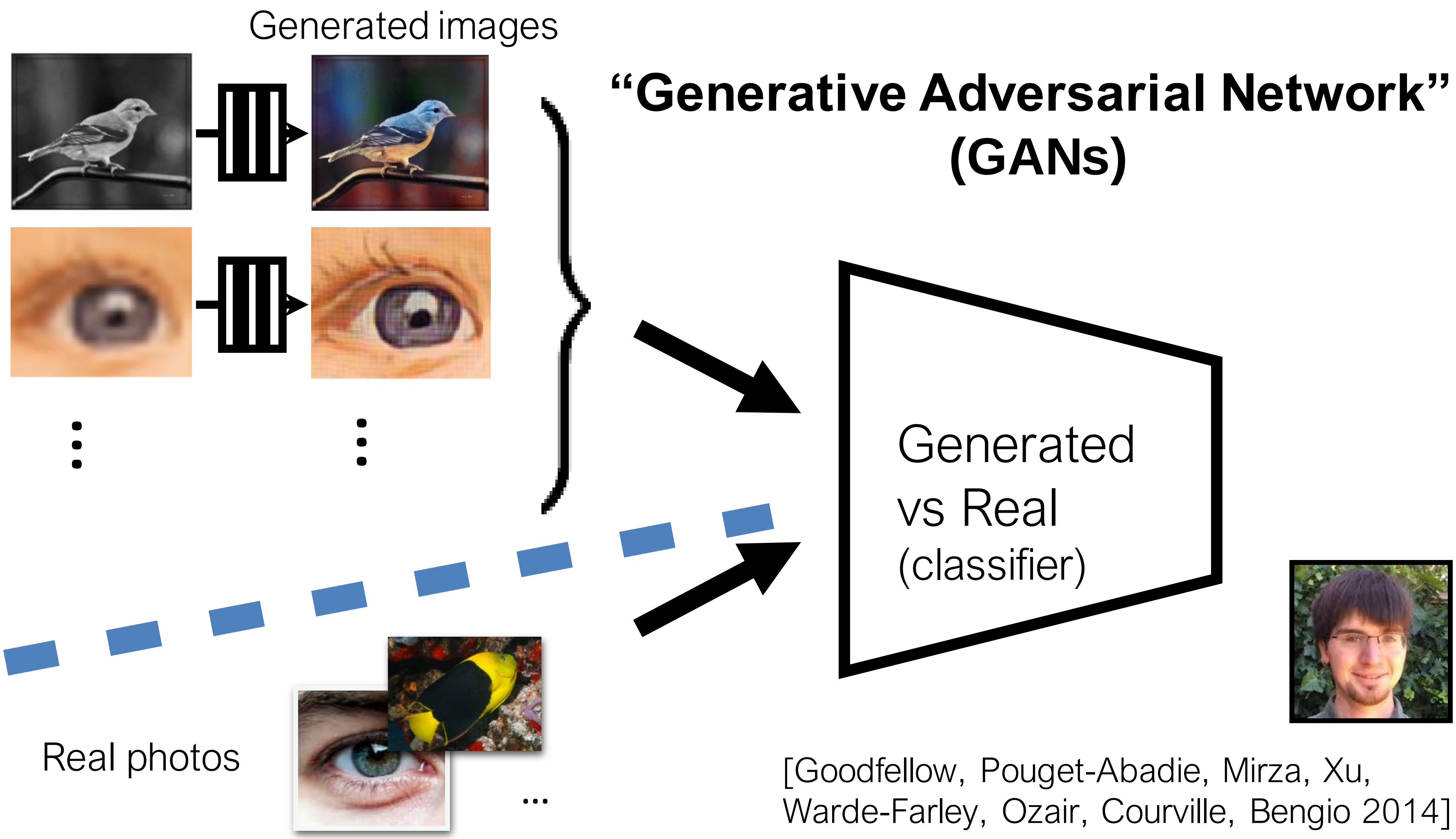


Illustration of LeCun et al. 1998 from CS231n 2017 Lecture 1


Generated images
“Generative Adversarial Network”
(GANs)
Generated
vs Real
(classifier)
Real photos
...
[Goodfellow, Pouget-Abadie, Mirza, Xu,
Warde-Farley, Ozair, Courville, Bengio 2014]

Questions?

- Good luck!