

# CS5670: Computer Vision

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## Lecture 5: Feature invariance



# Reading

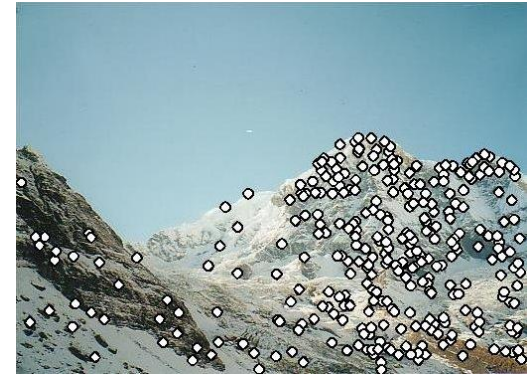
- Szeliski: 4.1

# Announcements

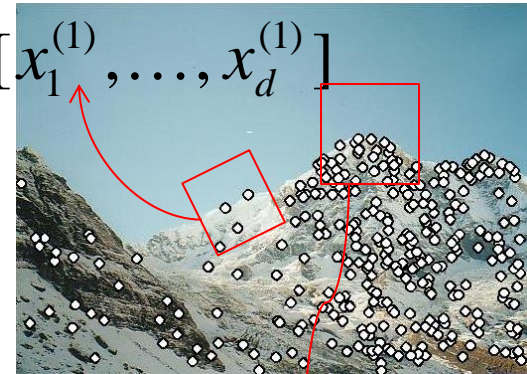
- Project 1 artifact due tonight at 11:59pm
- Project 2 will be out next week
  - To be done in groups of 2 (please start forming your teams)

# Local features: main components

1) Detection: Identify the interest points

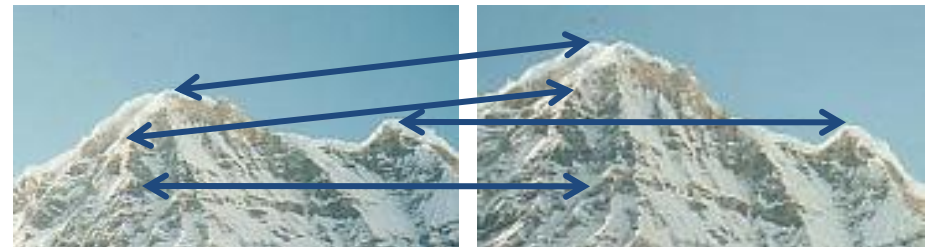


2) Description: Extract vector feature descriptor surrounding  $\mathbf{x}_1 = [x_1^{(1)}, \dots, x_d^{(1)}]$  each interest point.



$$\mathbf{x}_2 = [x_1^{(2)}, \dots, x_d^{(2)}]$$

3) Matching: Determine correspondence between descriptors in two views



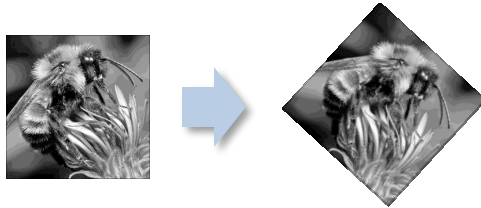
# Harris features (in red)



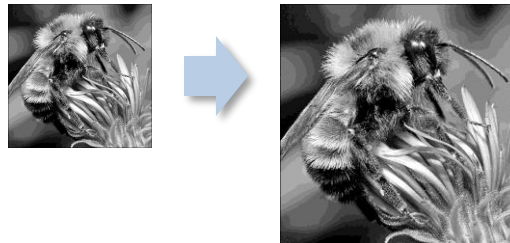
# Image transformations

- Geometric

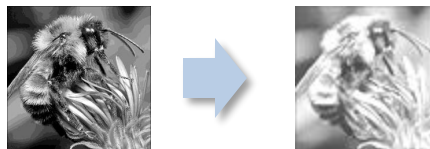
**Rotation**



**Scale**



- Photometric  
**Intensity change**



# Invariance and equivariance

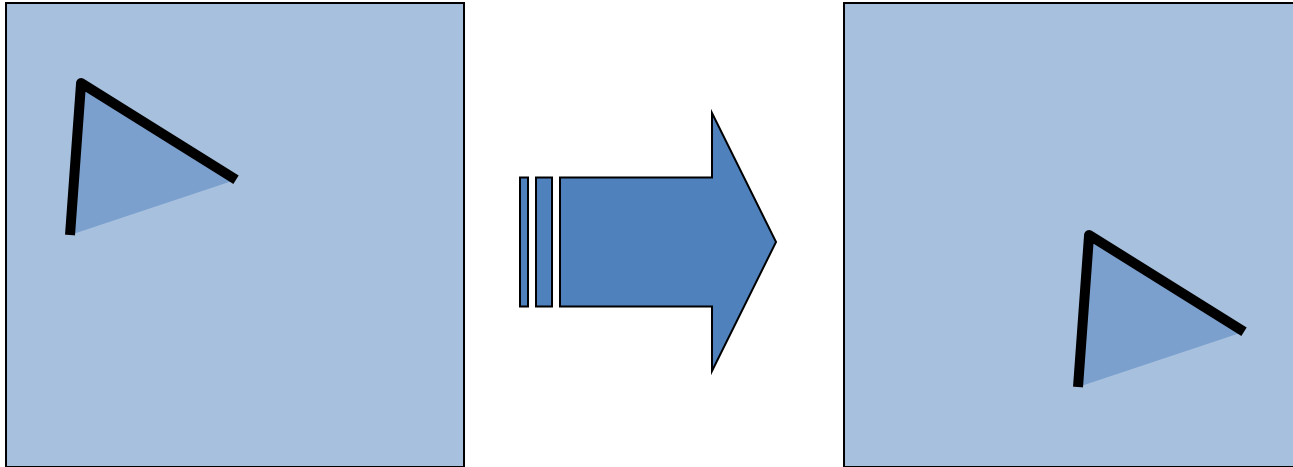
- We want corner locations to be *invariant* to photometric transformations and *equivariant* to geometric transformations
  - **Invariance:** image is transformed and corner locations do not change
  - **Equivariance:** if we have two transformed versions of the same image, features should be detected in corresponding locations
  - (Sometimes “invariant” and “equivariant” are both referred to as “invariant”)
  - (Sometimes “equivariant” is called “covariant”)





# Harris detector: Invariance properties

## -- Image translation



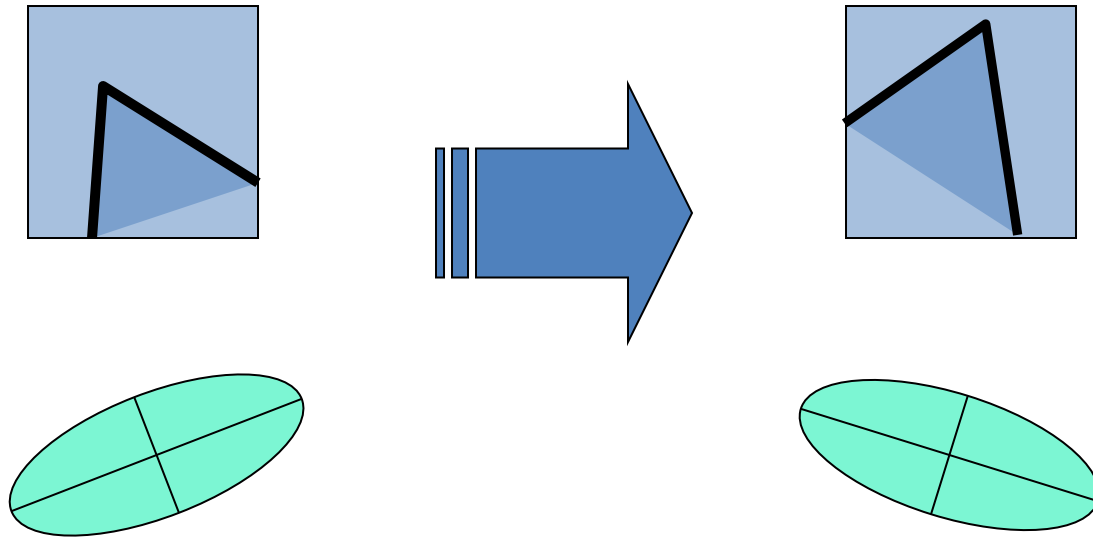
- Derivatives and window function are equivariant

Corner location is equivariant w.r.t. translation



# Harris detector: Invariance properties

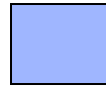
## -- Image rotation



Second moment ellipse rotates but its shape (i.e. eigenvalues) remains the same

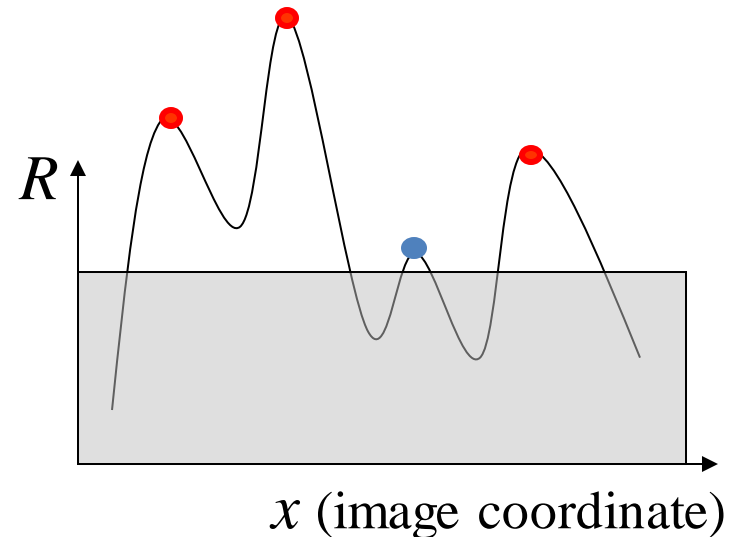
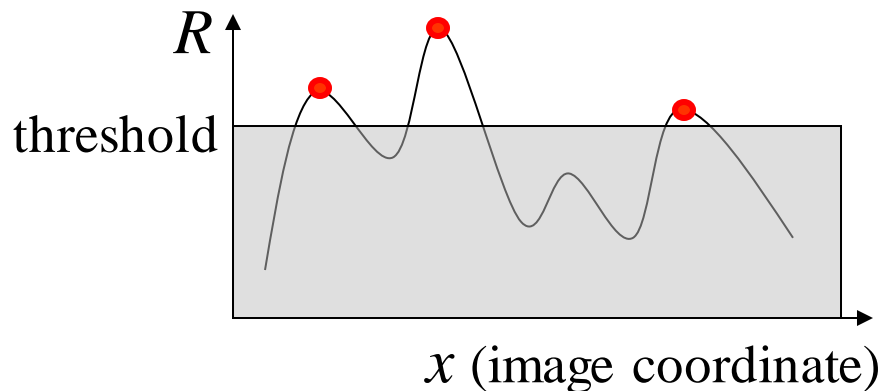
Corner location is equivariant w.r.t. image rotation

# Harris detector: Invariance properties – Affine intensity change



$$I \rightarrow aI + b$$

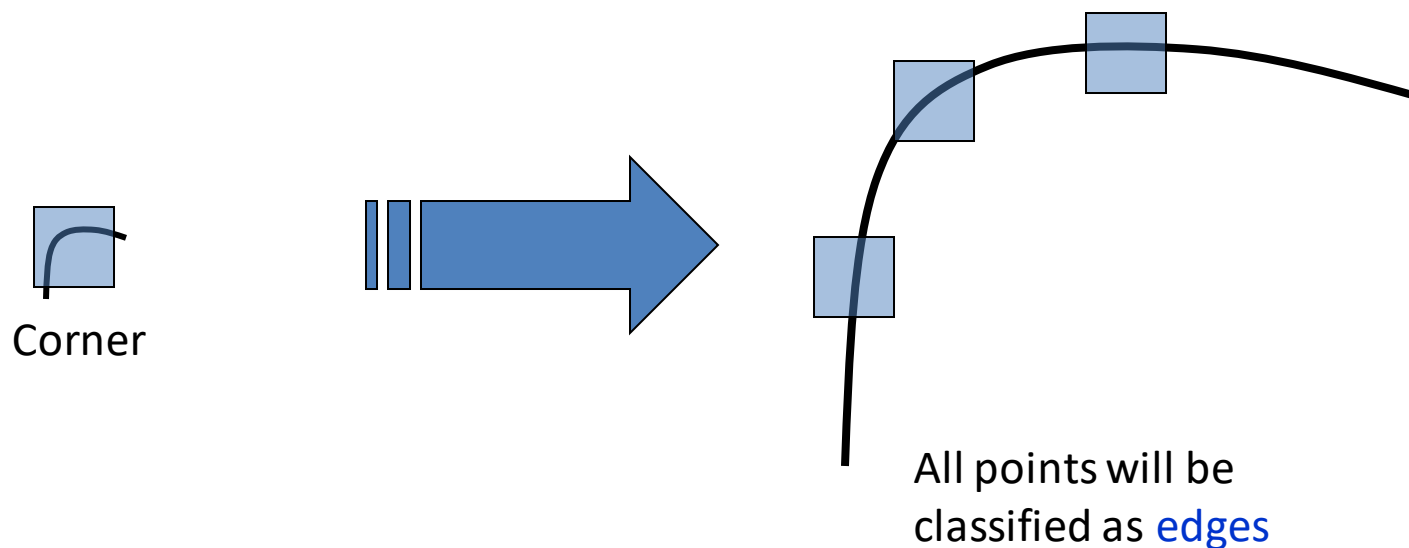
- Only derivatives are used => invariance to intensity shift  $I \rightarrow I + b$
- Intensity scaling:  $I \rightarrow aI$



*Partially invariant to affine intensity change*

# Harris Detector: Invariance Properties

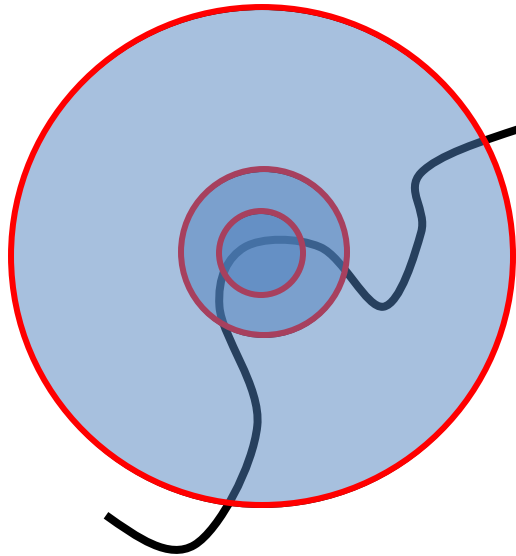
- Scaling



*Neither invariant nor equivariant to scaling*

# Scale invariant detection

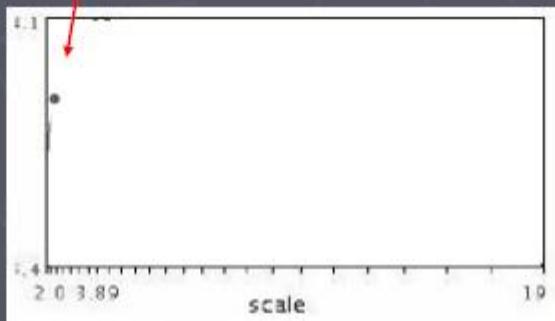
Suppose you're looking for corners



- Key idea: find scale that gives local maximum of  $f$
- in both position and scale
  - One definition of  $f$ : the Harris operator

# Automatic scale selection

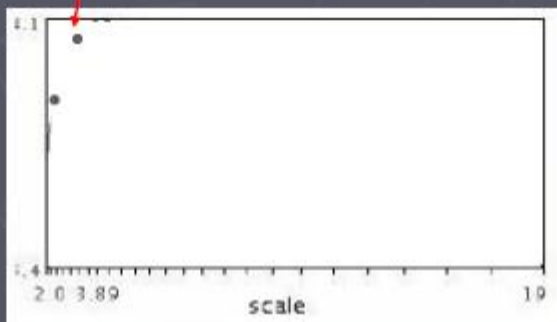
Lindeberg et al., 1996



$$f(I_{l...l_m}(x, \sigma))$$

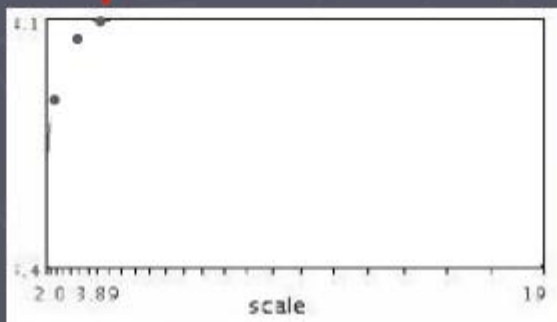
Slide from Tinne Tuytelaars

# Automatic scale selection



$$f(I_{l_1...l_m}(x, \sigma))$$

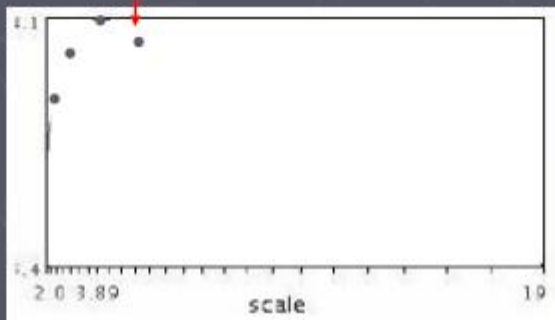
# Automatic scale selection



$$f(I_{l_1...l_m}(x, \sigma))$$

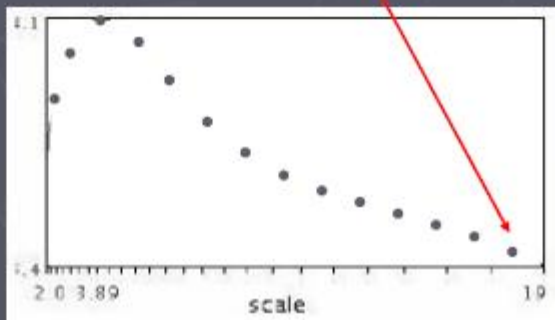


# Automatic scale selection



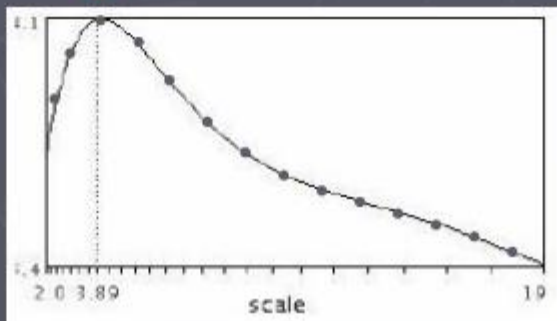
$$f(I_{l...l_m}(x, \sigma))$$

# Automatic scale selection



$$f(I_{l \dots l_m}(x, \sigma))$$

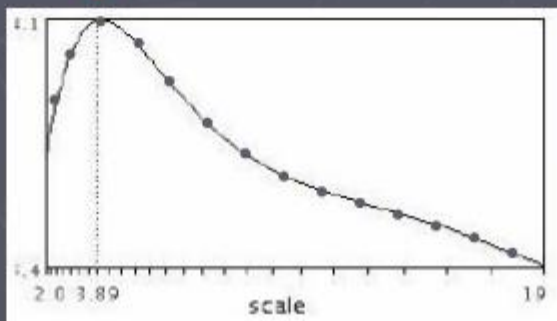
# Automatic scale selection



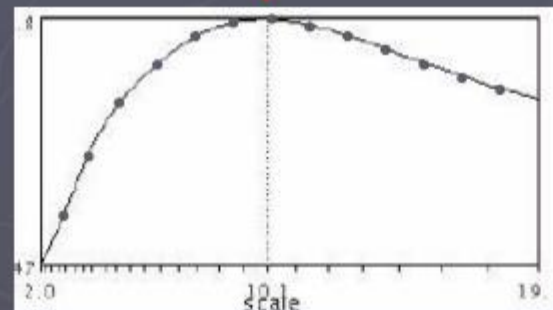
$$f(I_{l \dots l_m}(x, \sigma))$$



# Automatic scale selection



$$f(I_{i_1 \dots i_m}(x, \sigma))$$



$$f(I_{i_1 \dots i_m}(x', \sigma'))$$

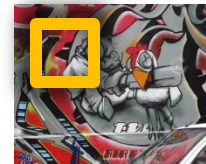
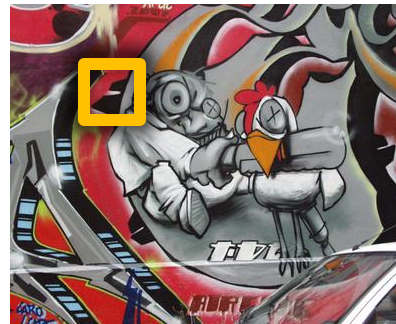
# Automatic scale selection

Normalize: rescale to fixed size



# Implementation

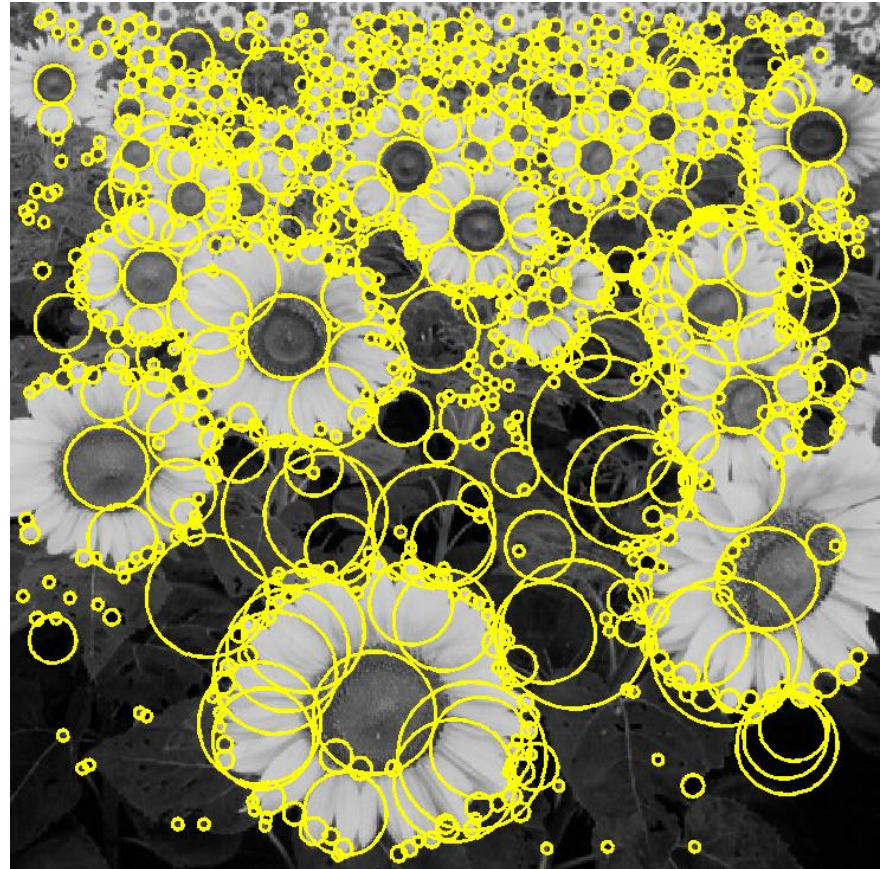
- Instead of computing  $f$  for larger and larger windows, we can implement using a fixed window size with a Gaussian pyramid



(sometimes need to create in-between levels, e.g. a  $\frac{3}{4}$ -size image)



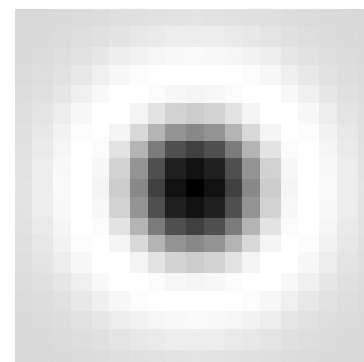
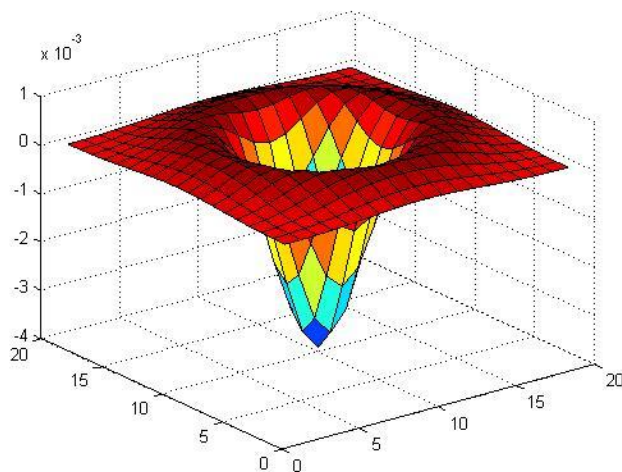
# Feature extraction: Corners and blobs





# Another common definition of $f$

- The *Laplacian of Gaussian (LoG)*



$$\nabla^2 g = \frac{\partial^2 g}{\partial x^2} + \frac{\partial^2 g}{\partial y^2}$$

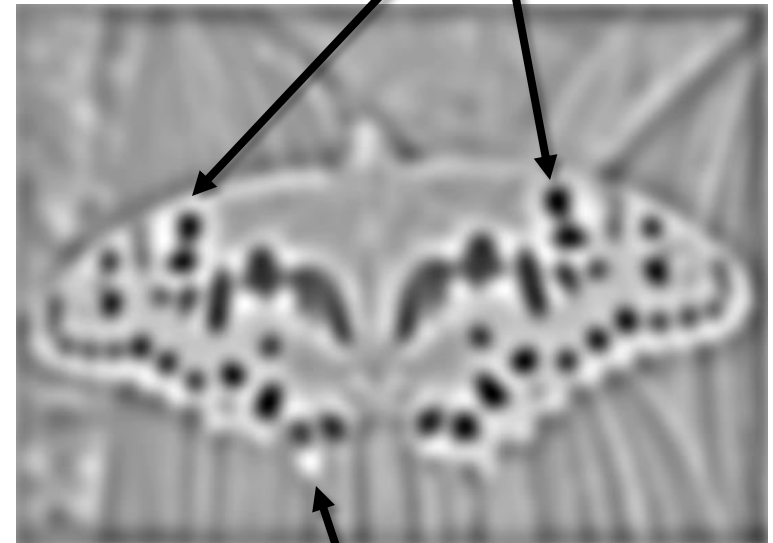
(very similar to a Difference of Gaussians (DoG) –  
i.e. a Gaussian minus a slightly smaller Gaussian)

# Laplacian of Gaussian

- “Blob” detector



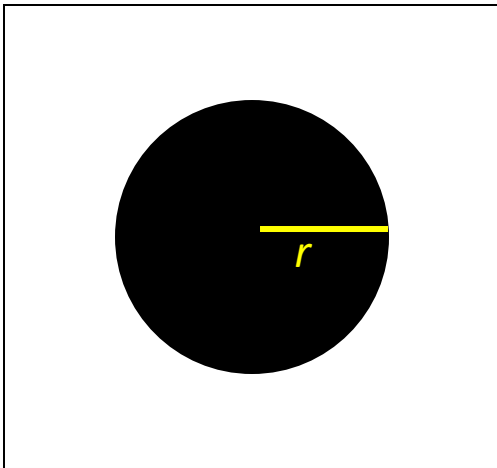
$$* \text{ [Gaussian Kernel] } =$$



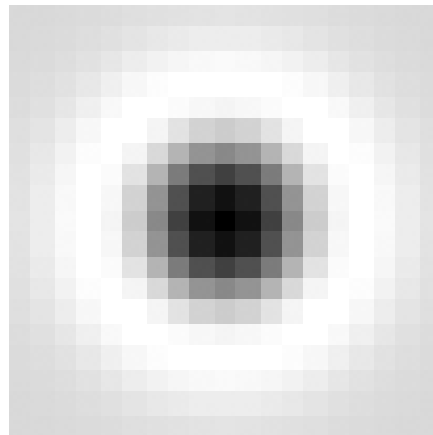
- Find maxima *and minima* of LoG operator in space and scale

# Scale selection

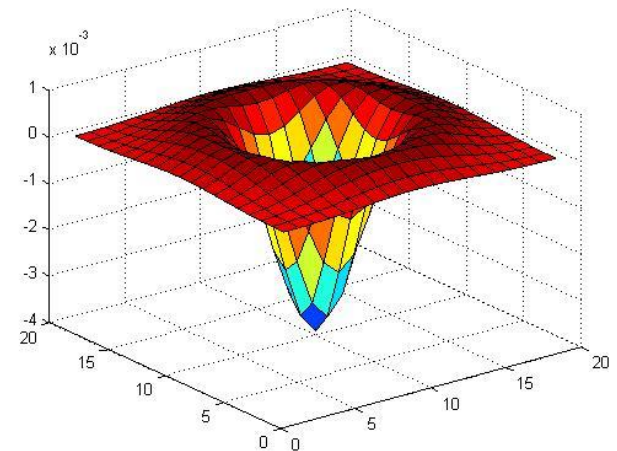
- At what scale does the Laplacian achieve a maximum response for a binary circle of radius  $r$ ?



image

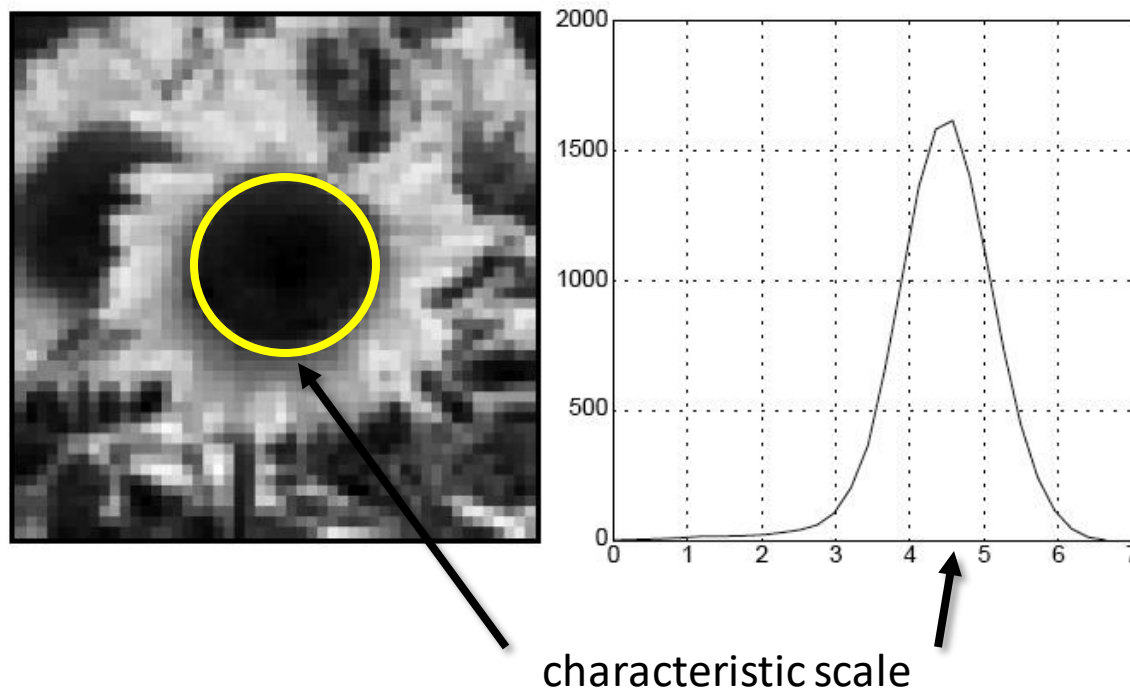


Laplacian



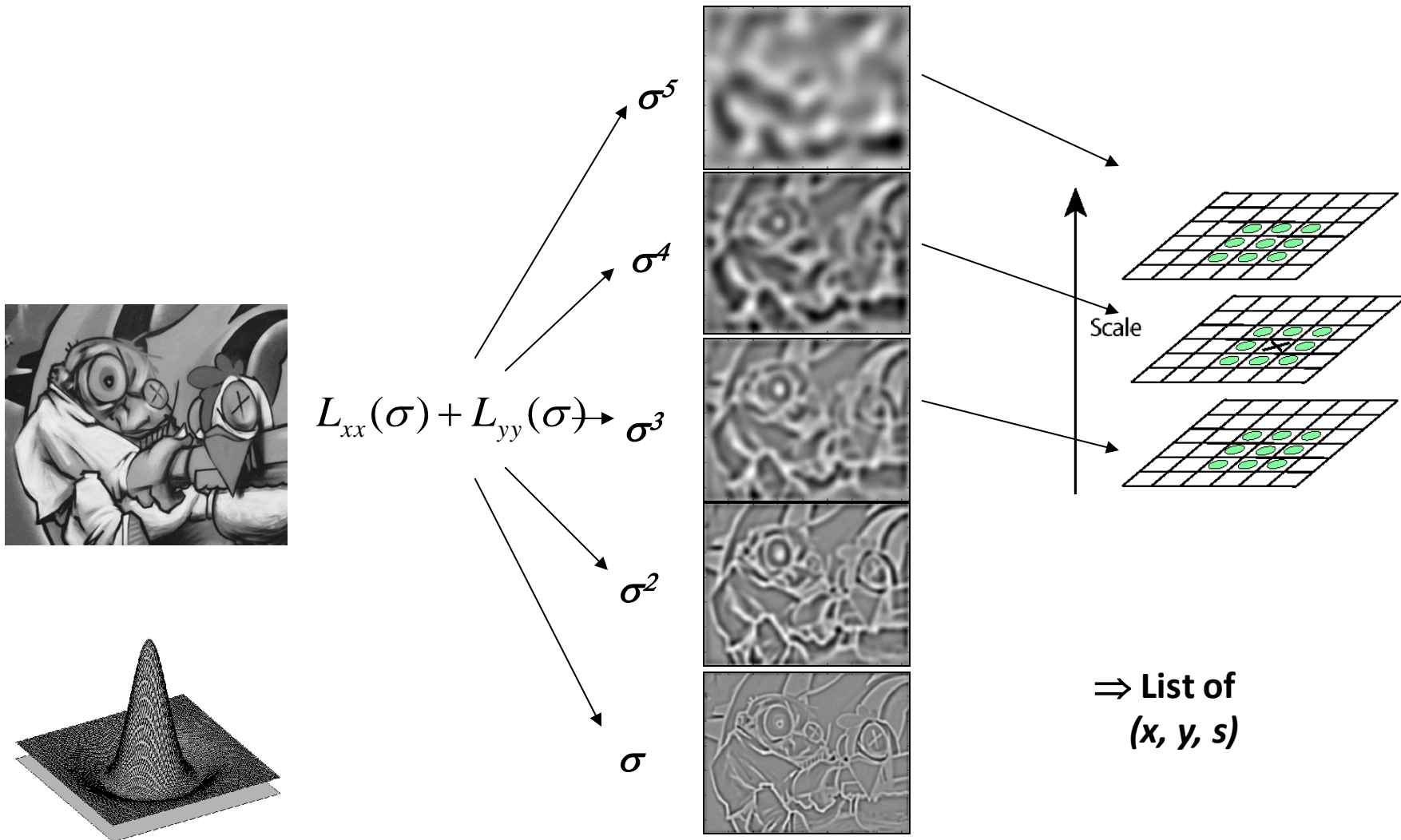
# Characteristic scale

- We define the characteristic scale as the scale that produces peak of Laplacian response



T. Lindeberg (1998). ["Feature detection with automatic scale selection."](#)  
*International Journal of Computer Vision* **30** (2): pp 77--116.

# Find local maxima in 3D position-scale space



# Scale-space blob detector: Example



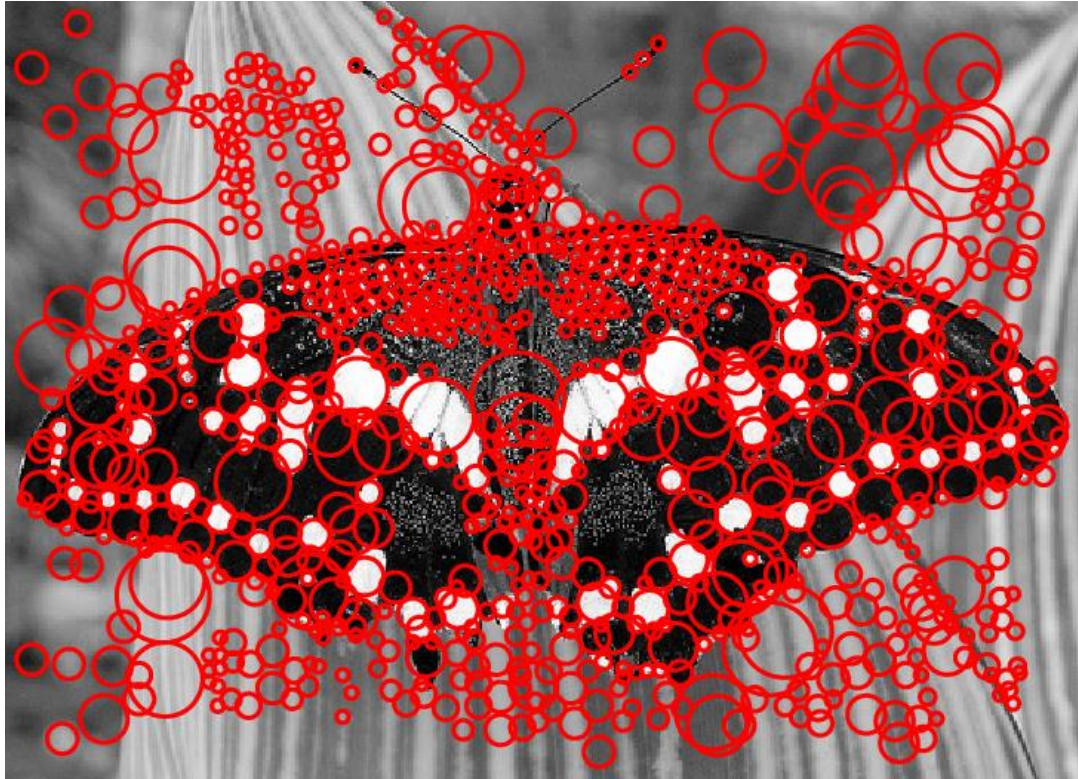
# Scale-space blob detector: Example



sigma = 11.9912



# Scale-space blob detector: Example



# Scale Invariant Detection

- Functions for determining scale

$$f = \text{Kernel} * \text{Image}$$

Kernels:

$$\nabla^2 g = \frac{\partial^2 g}{\partial x^2} + \frac{\partial^2 g}{\partial y^2}$$

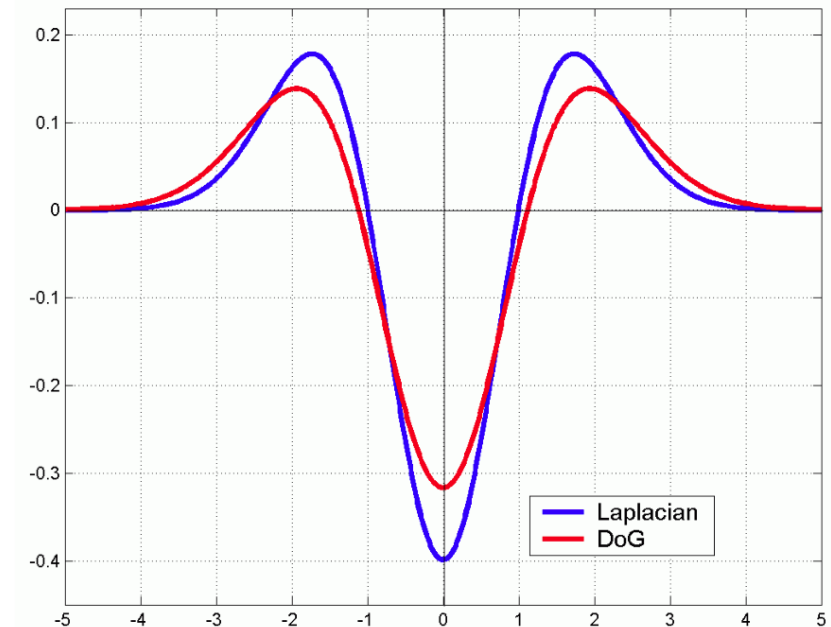
(Laplacian)

$$\text{DoG} = G(x, y, k\sigma) - G(x, y, \sigma)$$

(Difference of Gaussians)

where Gaussian

$$G(x, y, \sigma) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2 + y^2}{2\sigma^2}}$$



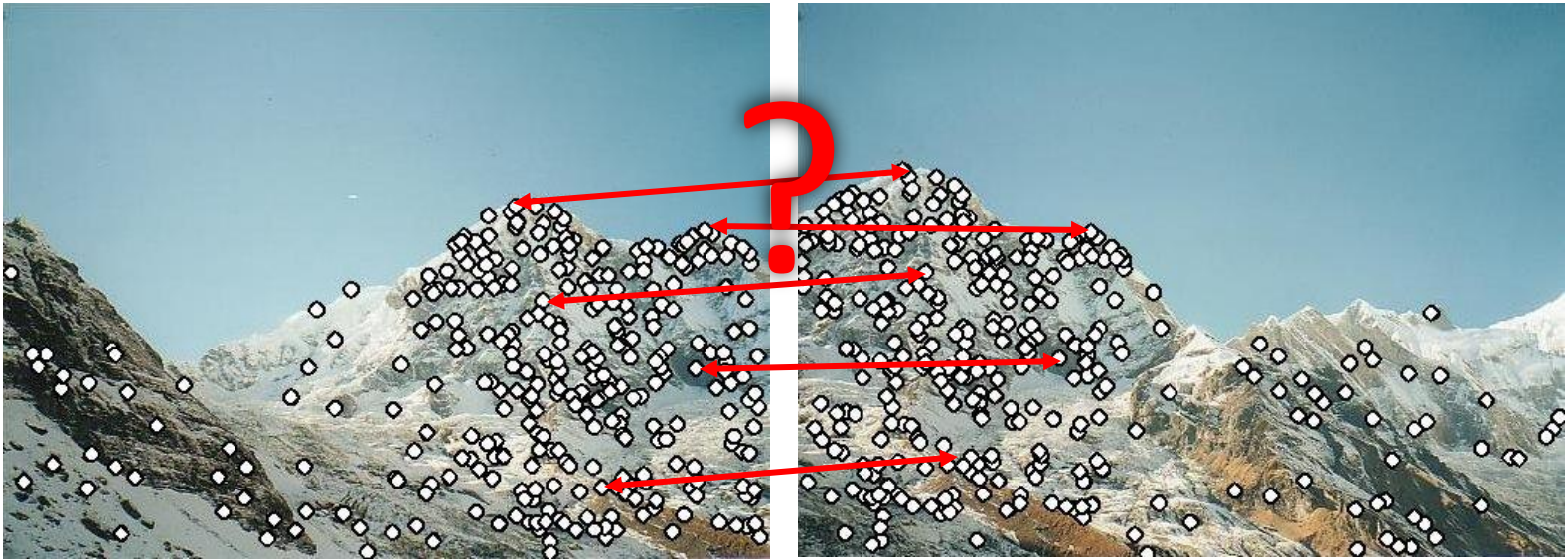
Note: The LoG and DoG operators are both rotation equivariant

# Questions?

# Feature descriptors

We know how to detect good points

Next question: **How to match them?**



**Answer:** Come up with a *descriptor* for each point,  
find similar descriptors between the two images