

CS5630 Physically Based Realistic Rendering

Steve Marschner
Spring 2026

21 Rendering hair and fur



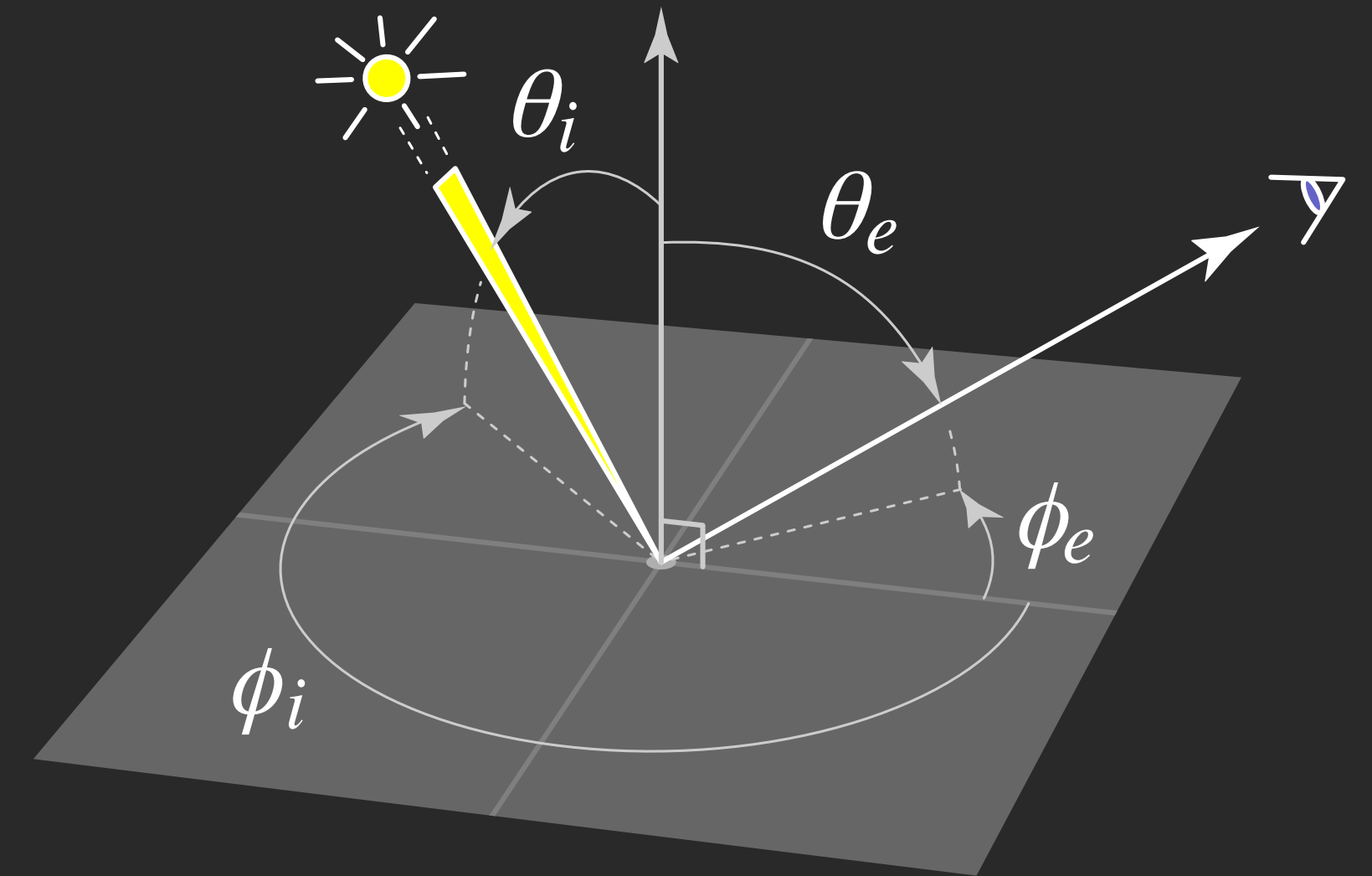




Scattering from surfaces vs. fibers

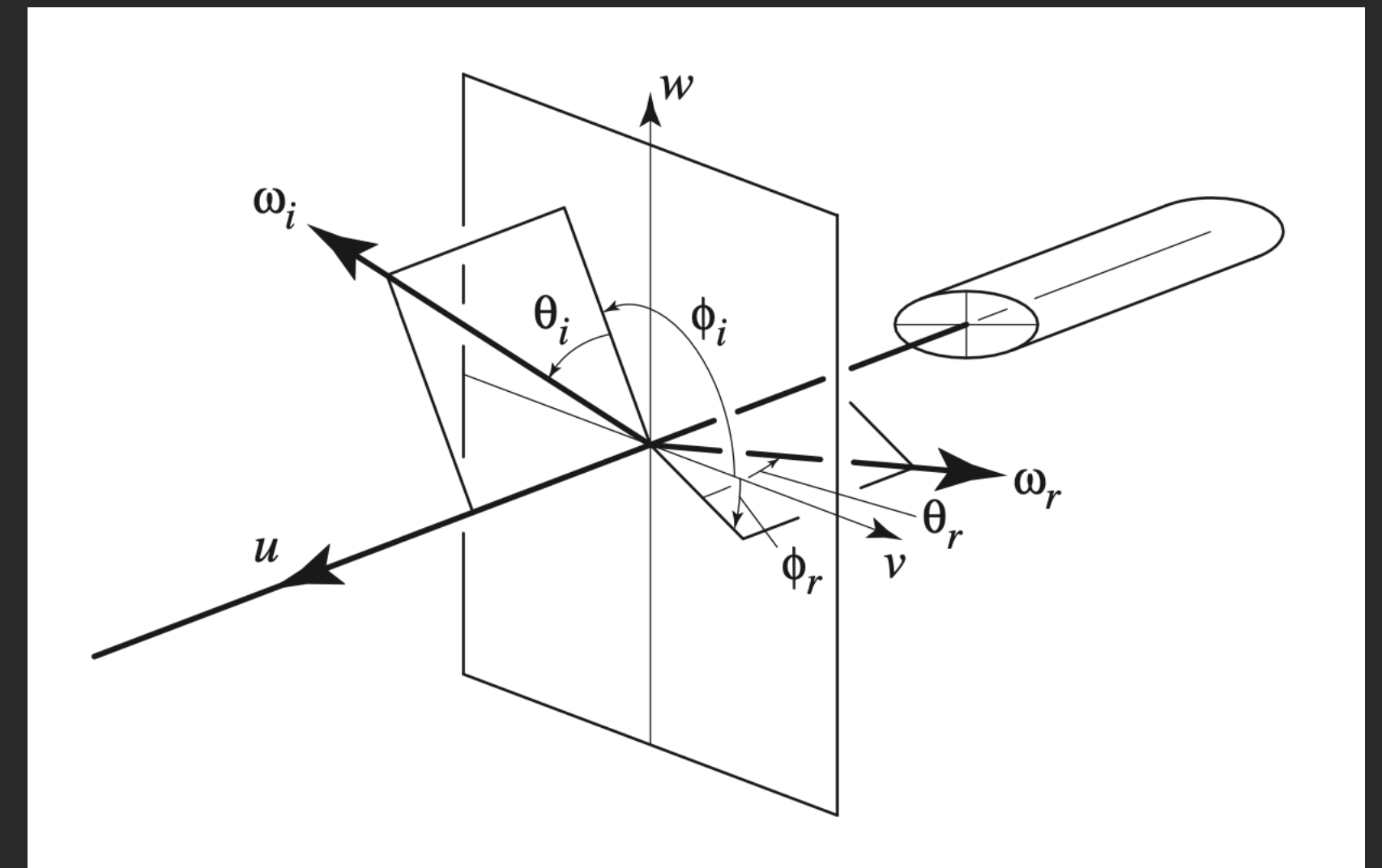
$$L_r(\omega_r) = \int_{H^2} f_r(\omega_i, \omega_r) L_i(\omega_i) \cos \theta_i d\omega_i$$

/ reflected radiance
 | BRDF
 / incident radiance
 / cosine with normal **vector**

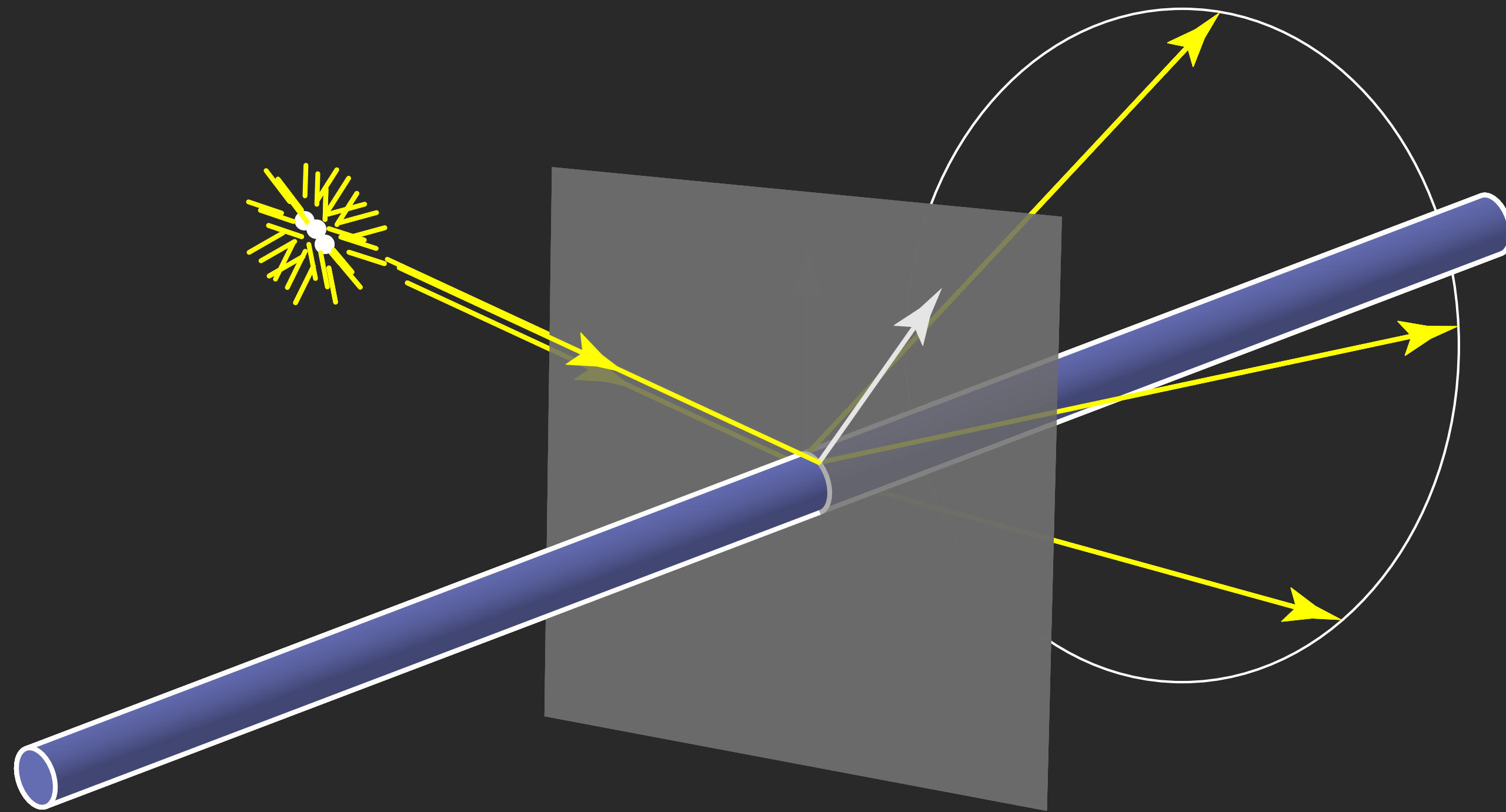


$$L_r(\omega_r) = \int_{S^2} f_s(\omega_i, \omega_r) L_i(\omega_i) \cos \theta_i d\omega_i$$

/ reflected radiance
 (constant across fiber)
 | fiber BSDF
 (constant across fiber)
 / incident radiance
 / cosine with normal **plane**



Classic fiber reflection model



predicts a single, linear highlight



Kajiya & Kay, "Rendering fur with 3D textures," 1989

visual phenomena of hair scattering and the basic hair scattering model

Stephen R. Marschner, Henrik Wann Jensen, Mike Cammarano, Steve Worley,
and Pat Hanrahan. "Light Scattering from Human Hair Fibers." SIGGRAPH
2003.

highlight



primary



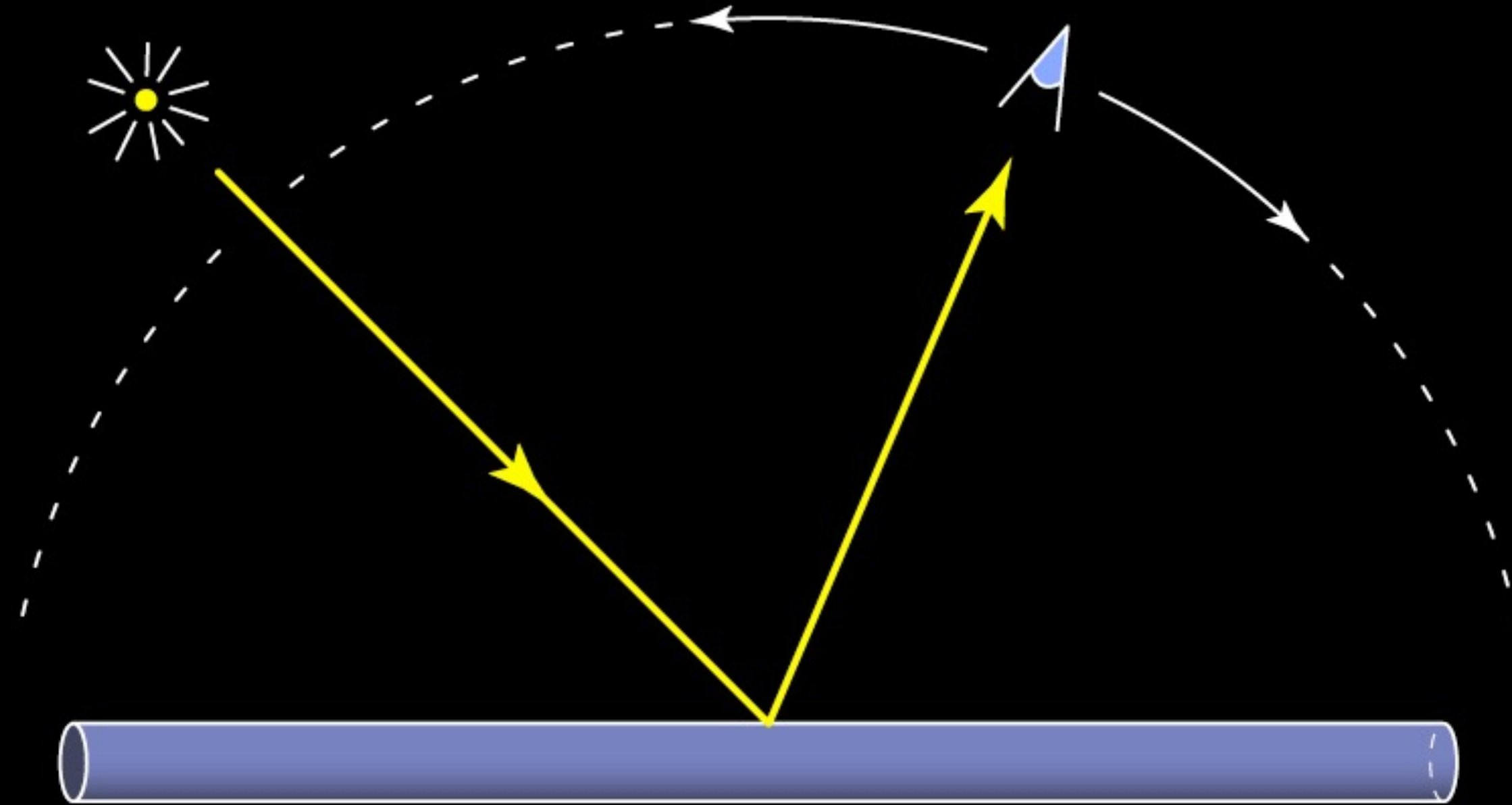
secondary

secondary

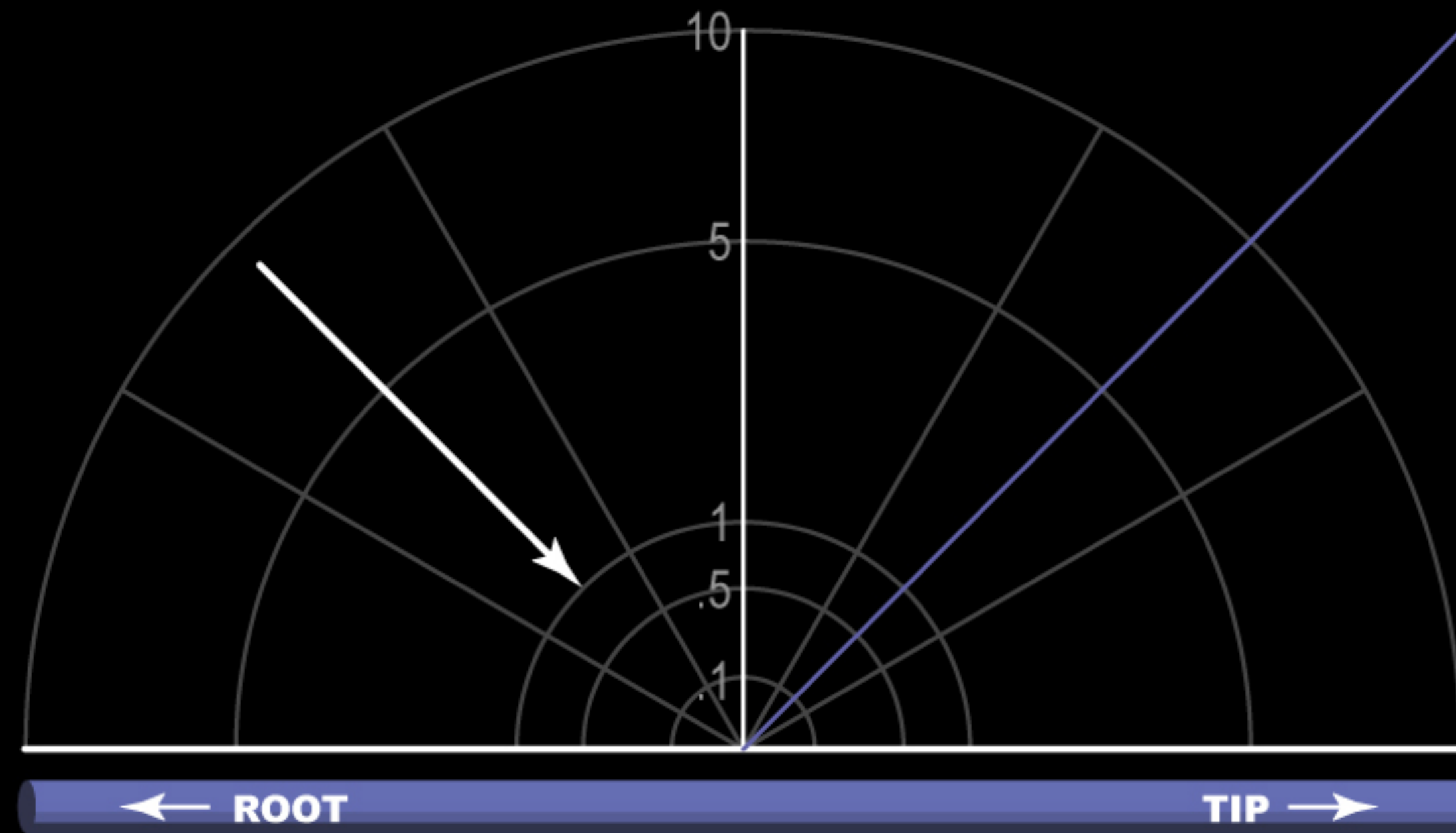
primary



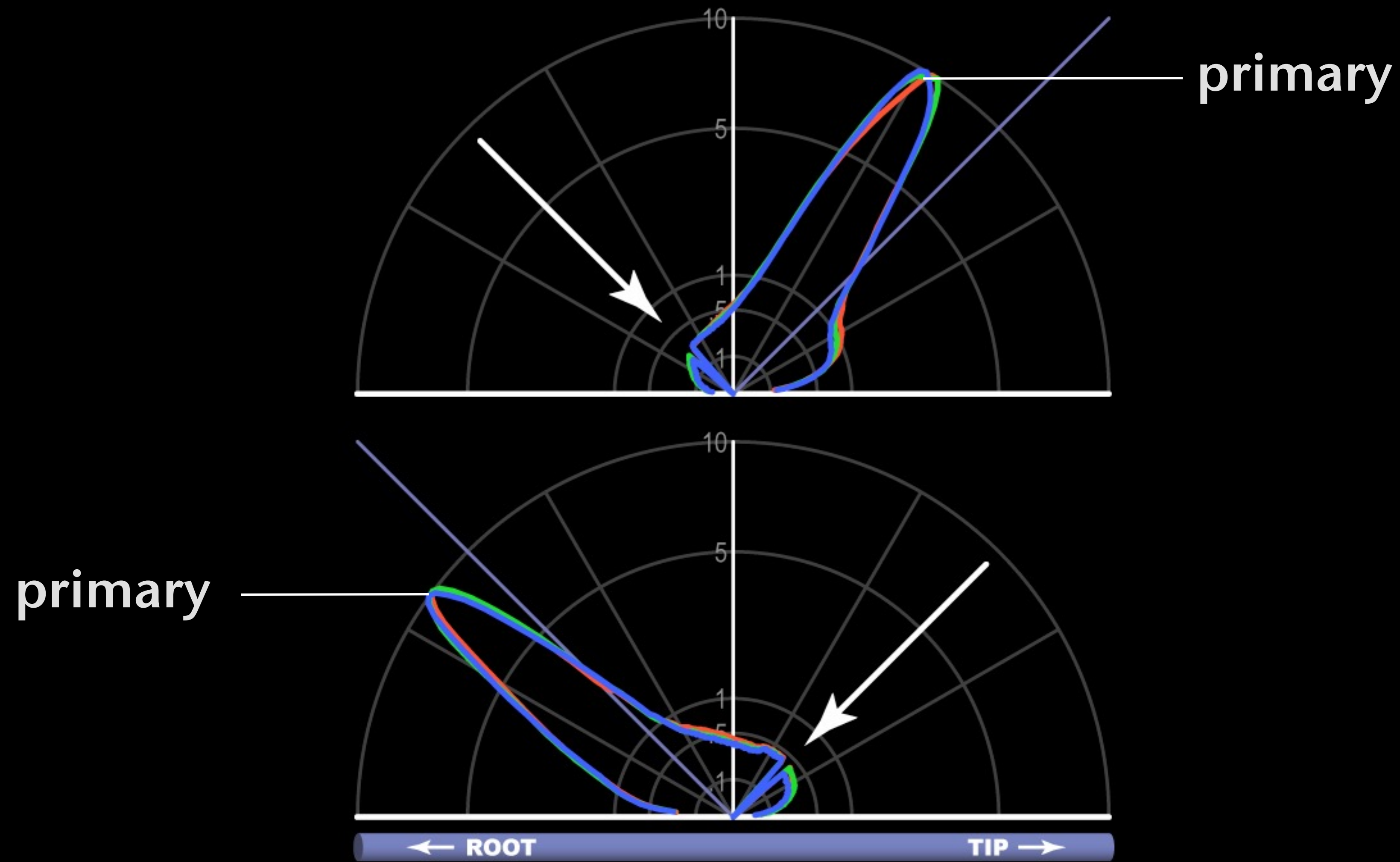
Measurement—longitudinal (θ)



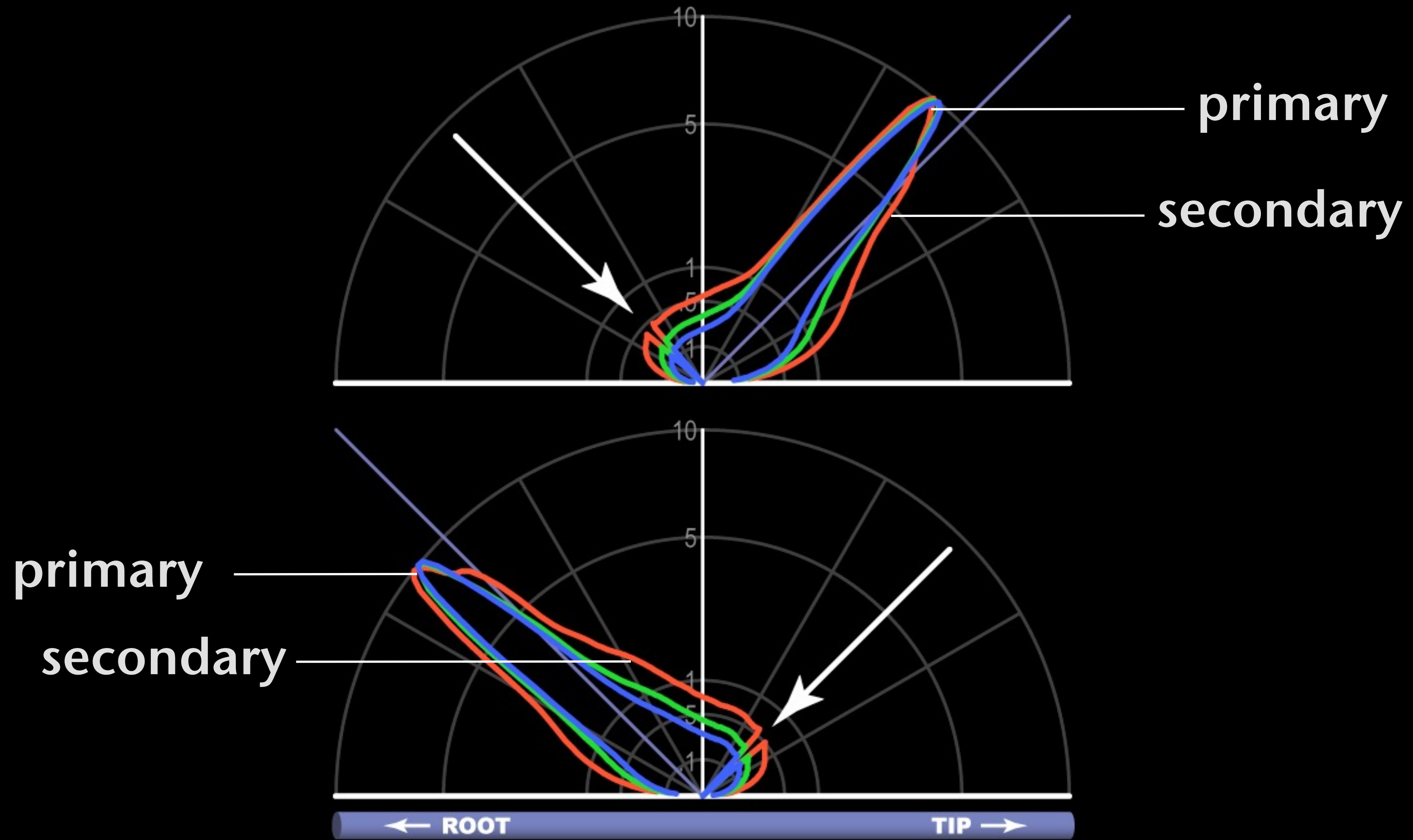
Measurement—longitudinal



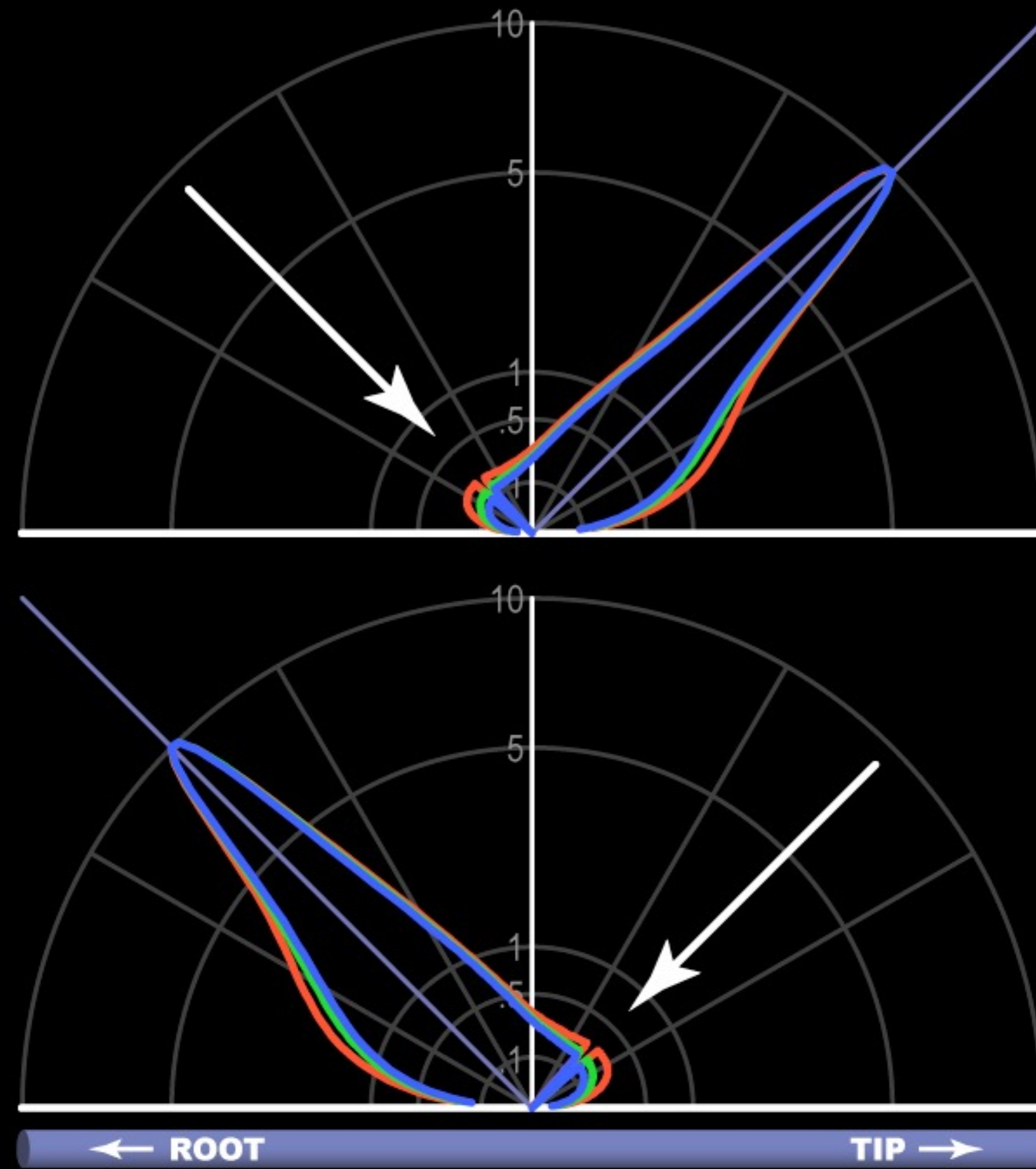
Black hair

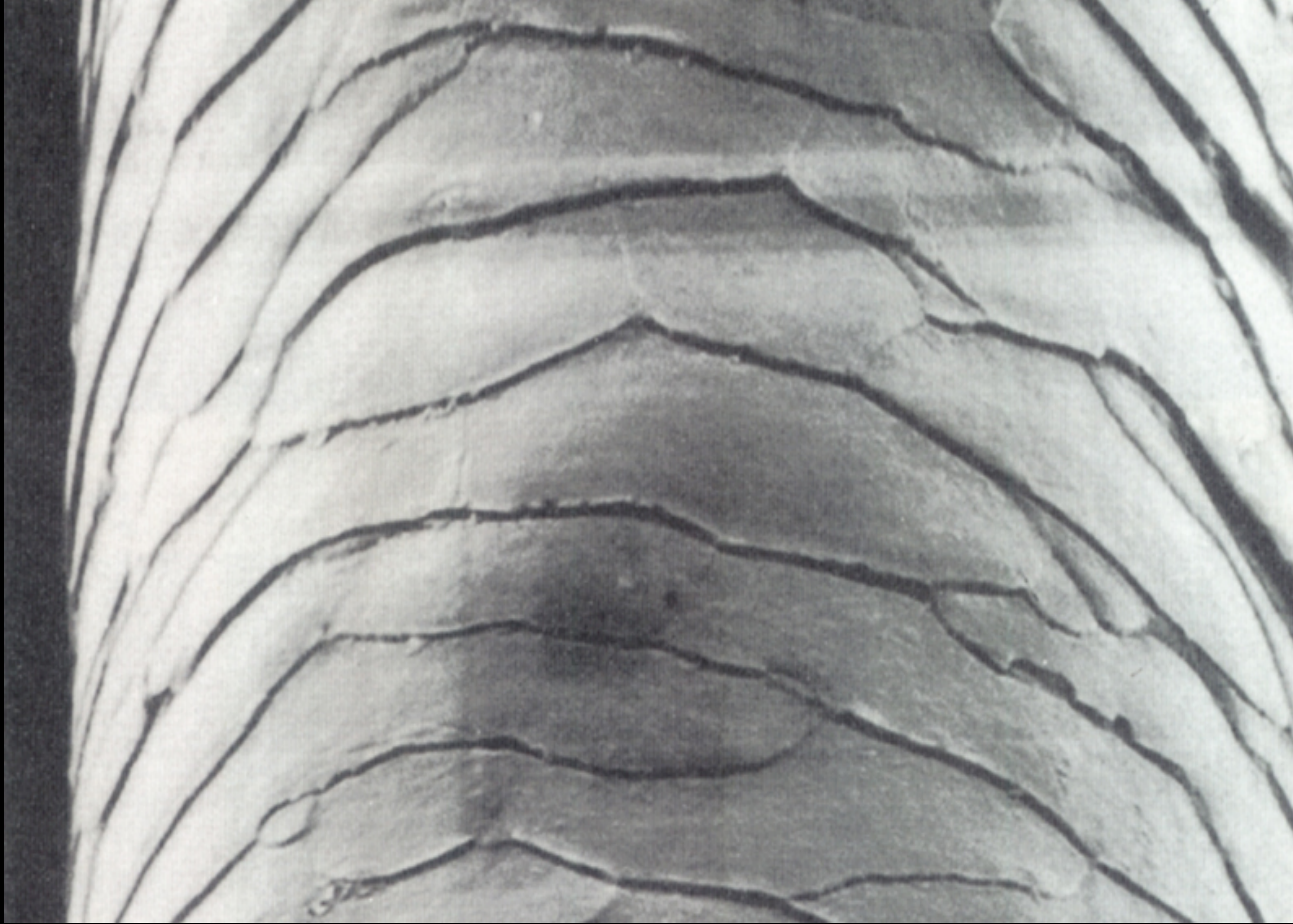


Blond hair

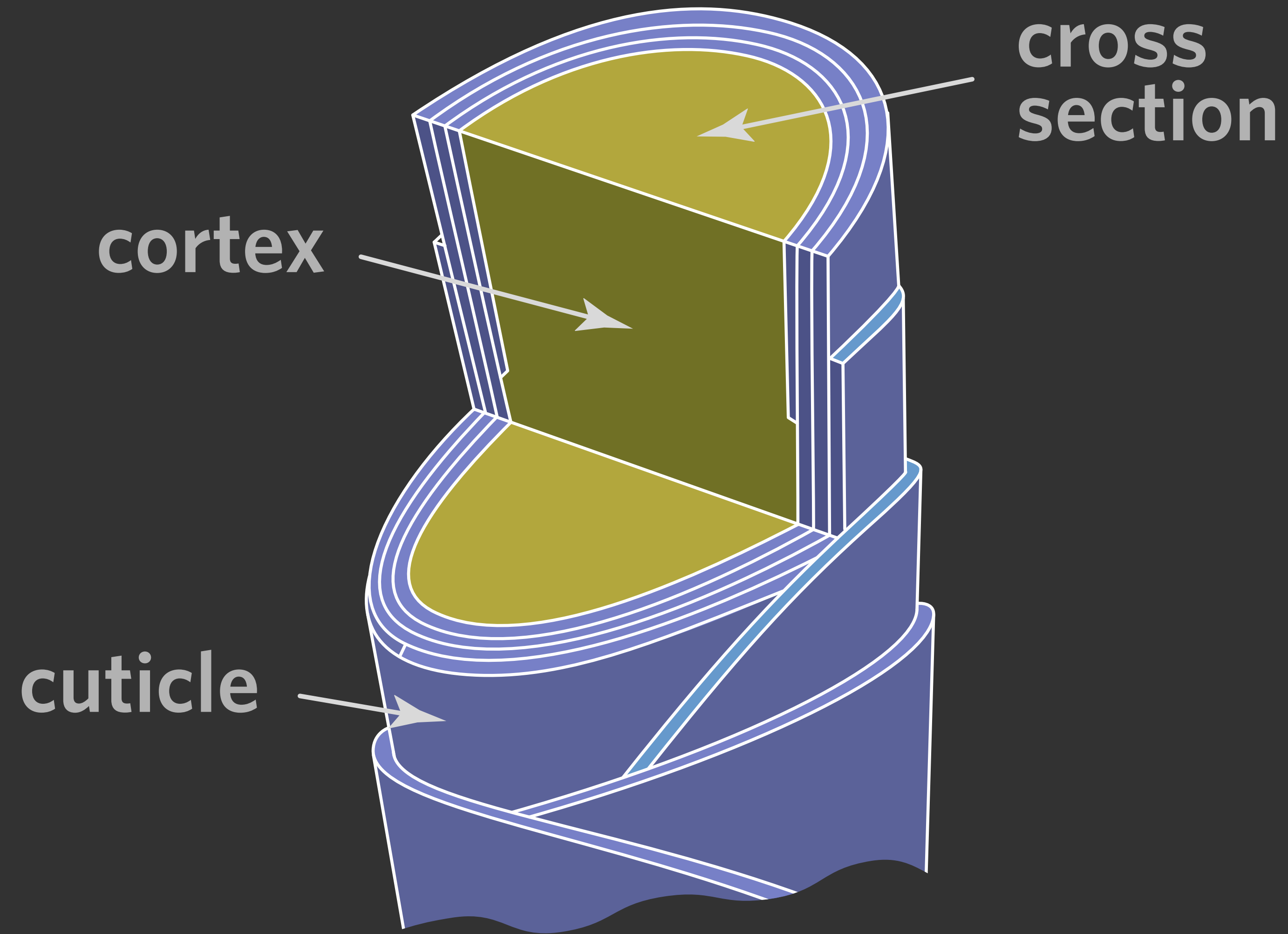


Synthetic hair

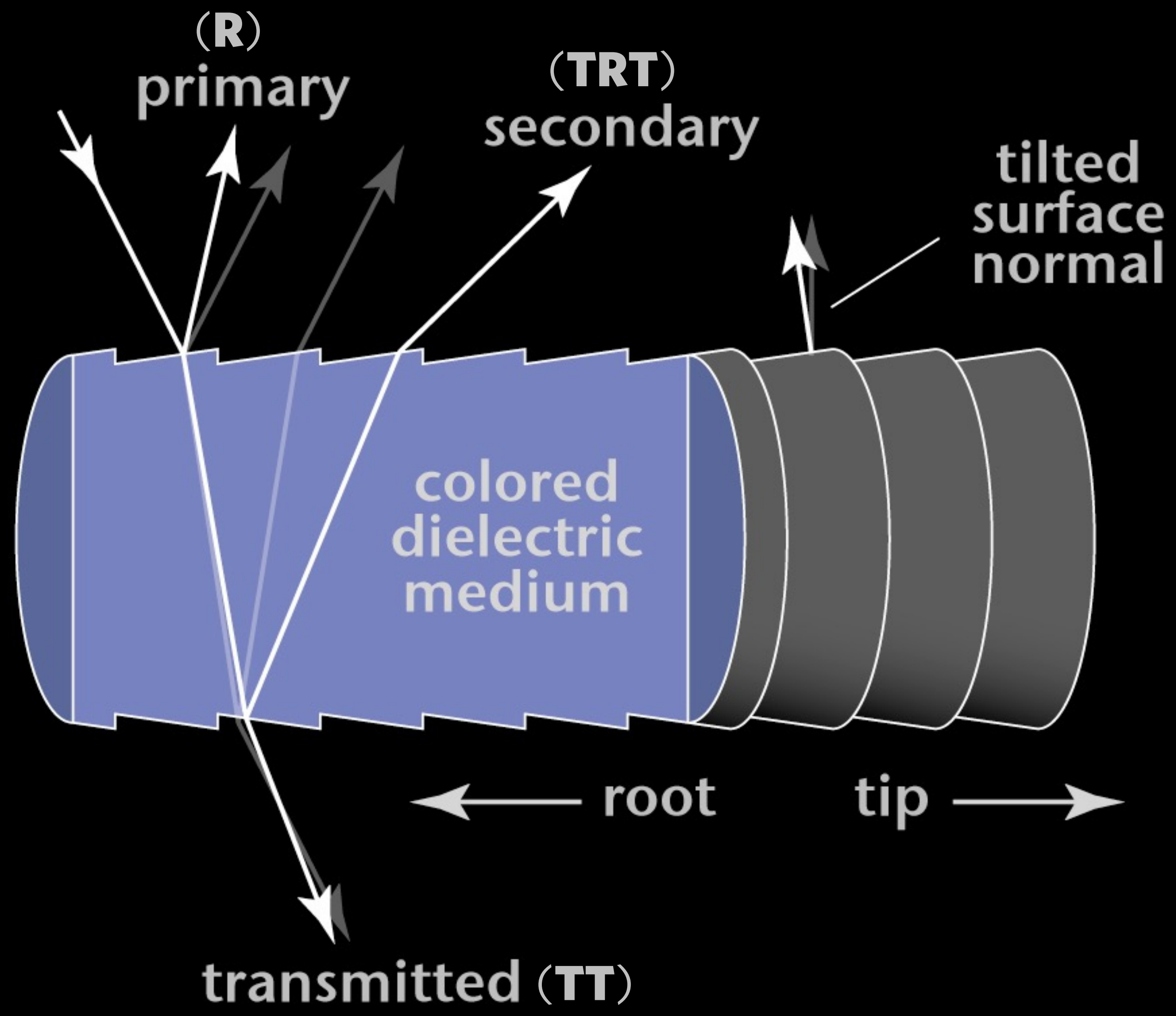




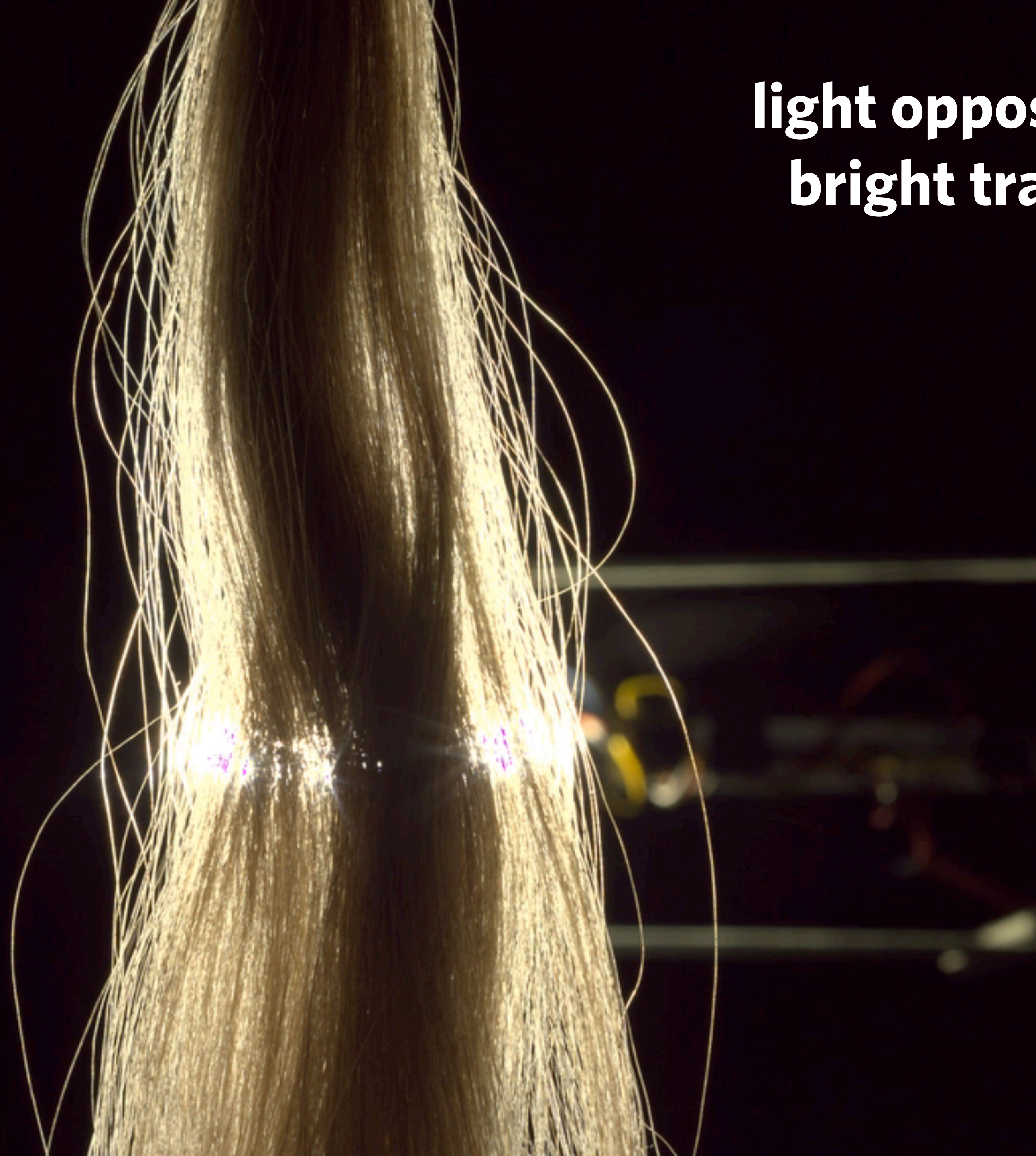
[Robbins 1994]



(after [Robbins 1994])



**light opposite camera:
bright transmission**



**light $\sim 90^\circ$ from camera:
glinty appearance**



**light behind camera:
smooth appearance**



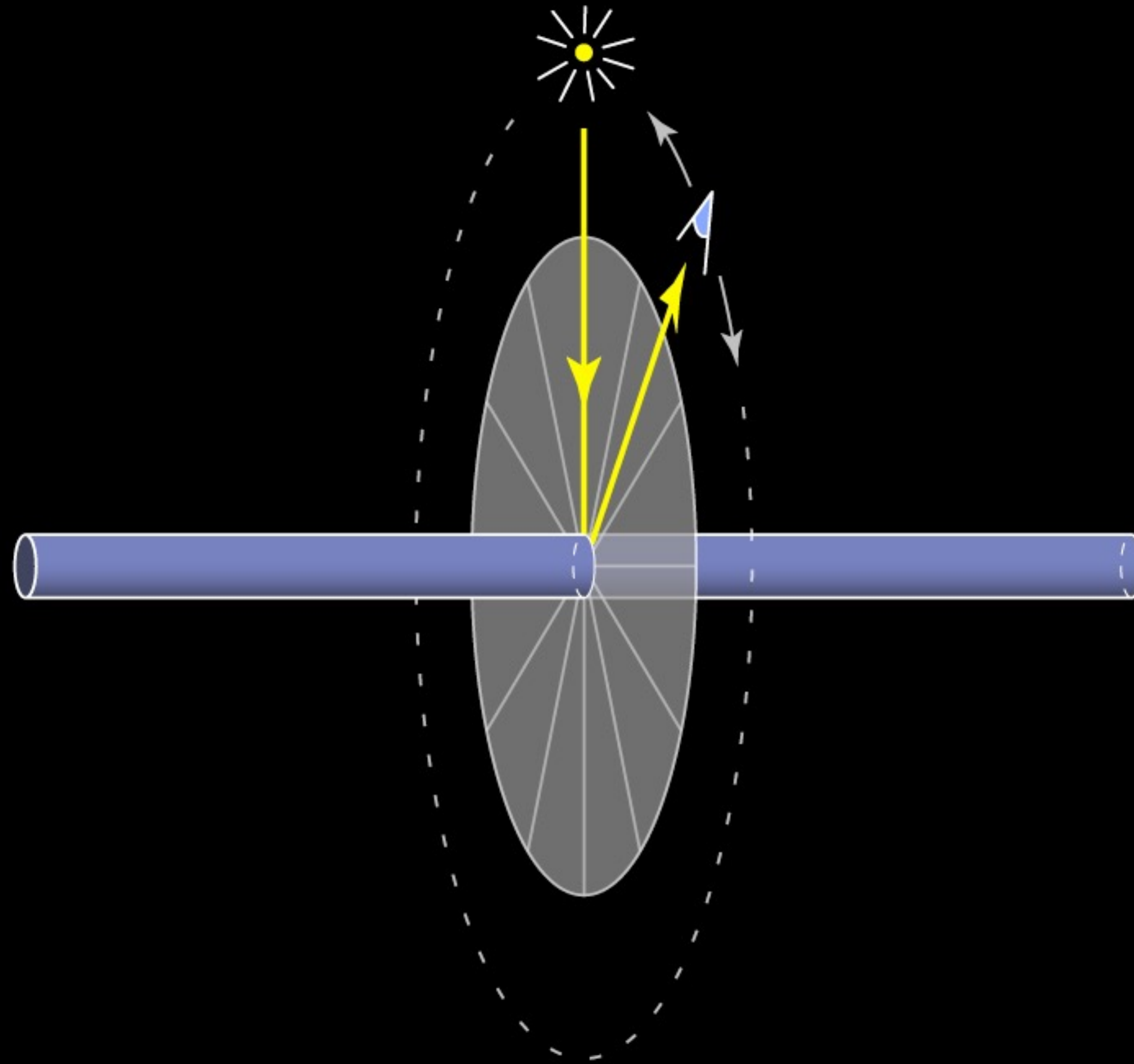
**light ~90° from camera:
glinty appearance**



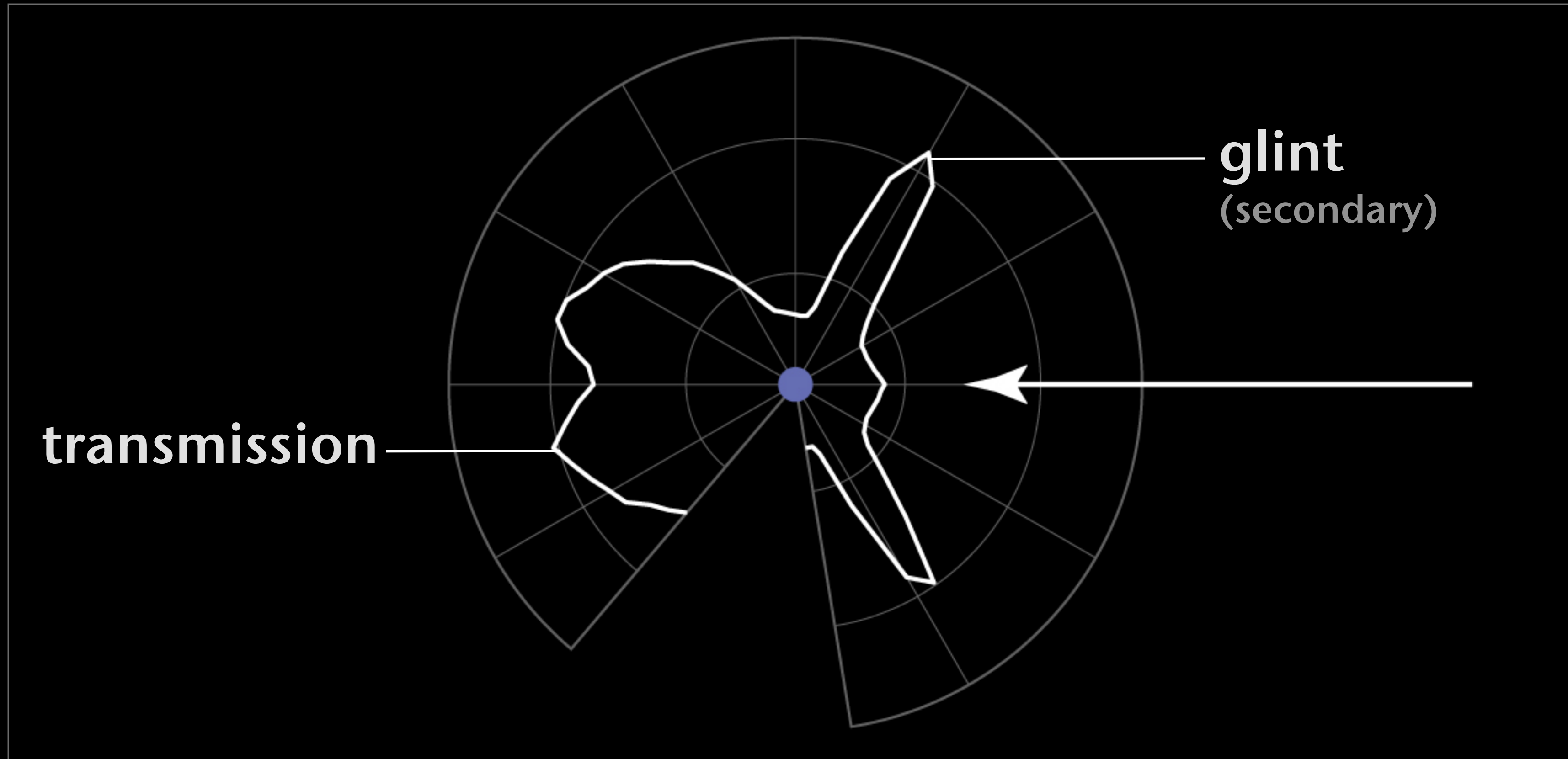
**light behind camera:
smooth appearance**



Measurement—azimuthal (ϕ)

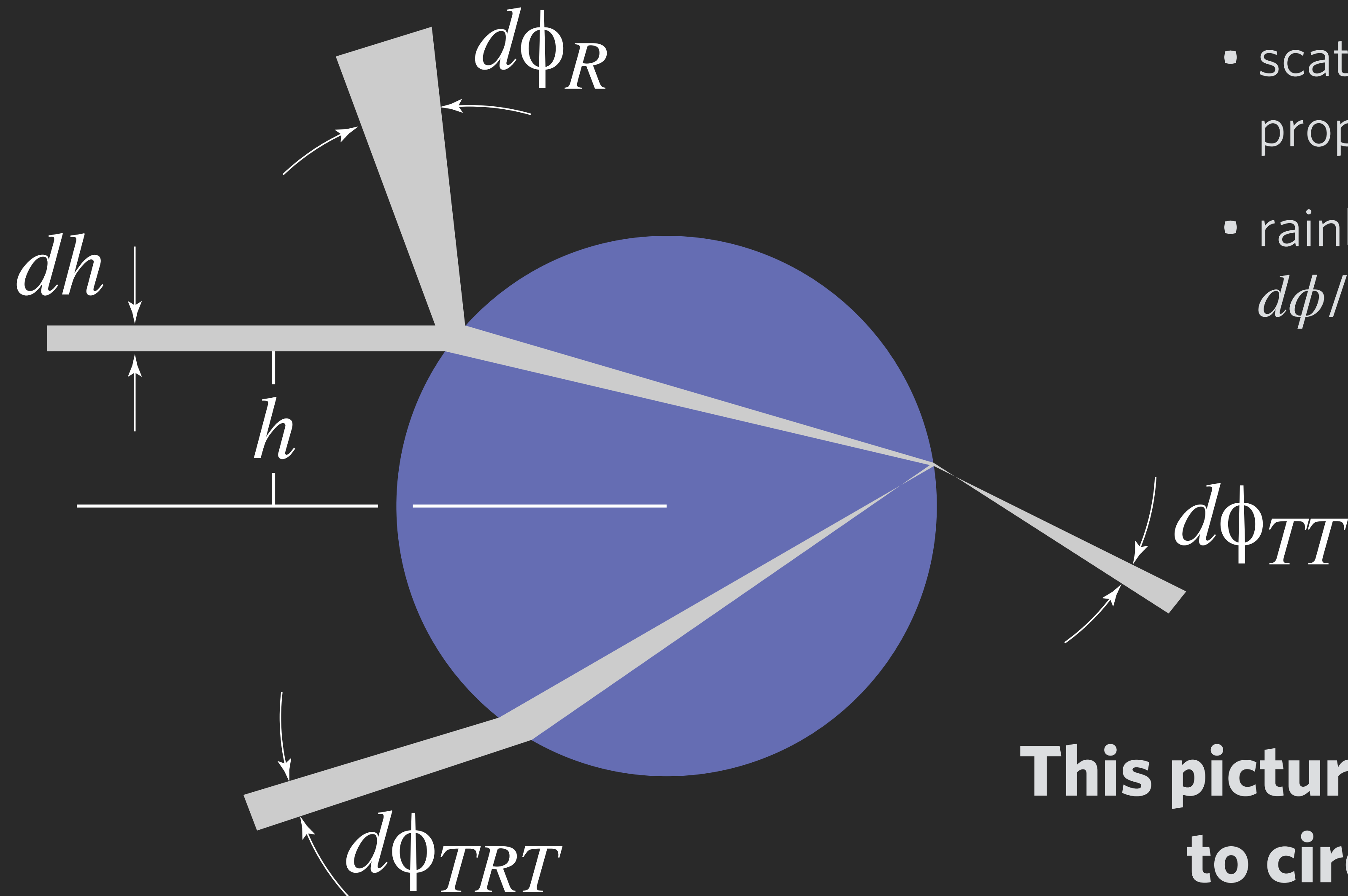


Blond hair





Descartes's rainbow model



- ϕ is a function of h
- scattered intensity is inversely proportional to $d\phi/dh$
- rainbow occurs when $d\phi/dh$ is zero

This picture applies just as well to circular cylinders!

light scattered from fiber
(for some fixed incident direction)

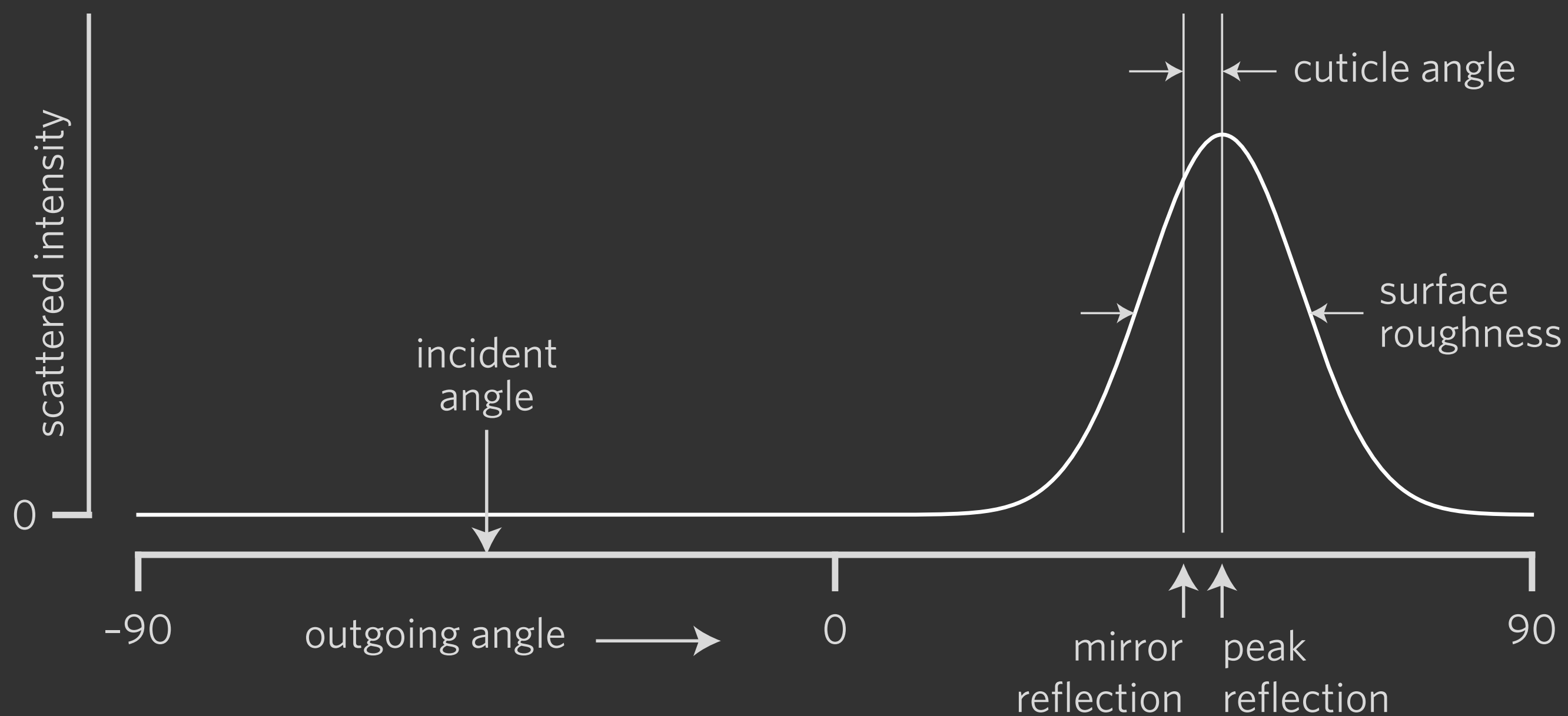
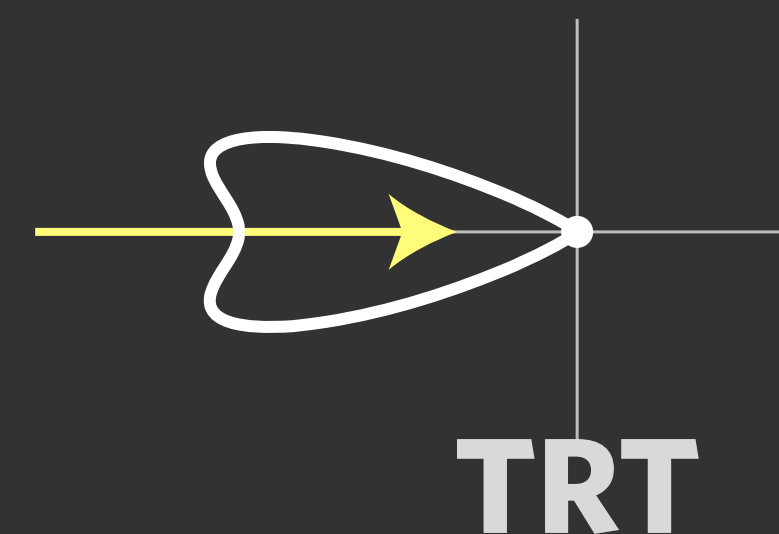
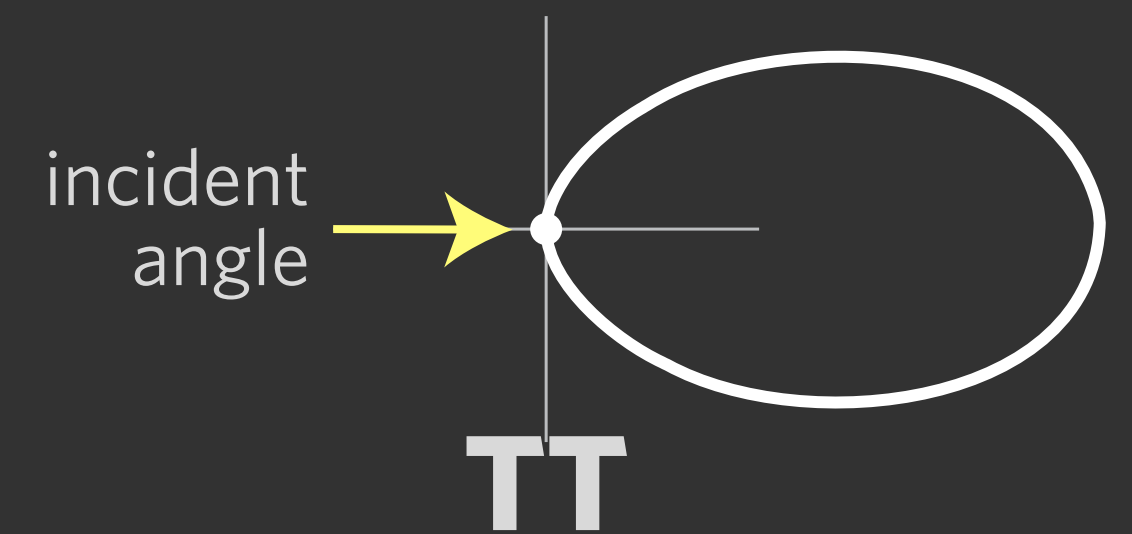
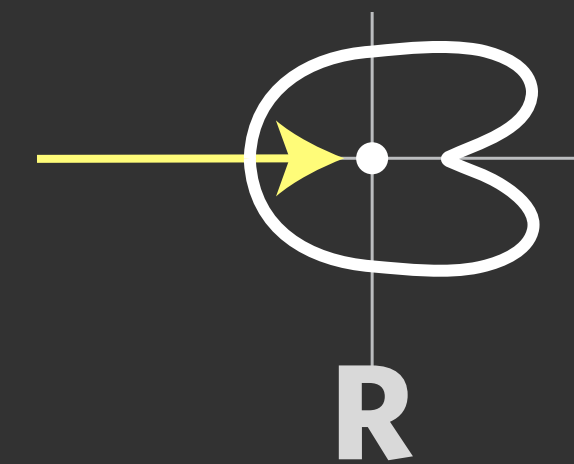
$$f_s(\omega_i, \omega_r) = S_R(\omega_i, \omega_r) + S_{TT}(\omega_i, \omega_r) + S_{TRT}(\omega_i, \omega_r)$$

$$S_X(\theta_i, \phi_i, \theta_r, \phi_r) = M_X(\theta_i, \theta_r) N_X(\phi_i, \phi_r; \theta_i, \theta_r)$$

X is R, TT, or TRT

longitudinal factor

azimuthal factor



extensions and fixes to the basic model

Multiple scattering

Hair scattering is directed strongly forward

Large fraction of light scatters more than once in lighter colored hair

Multiple scattering plays key role in overall hair color



single scattering only



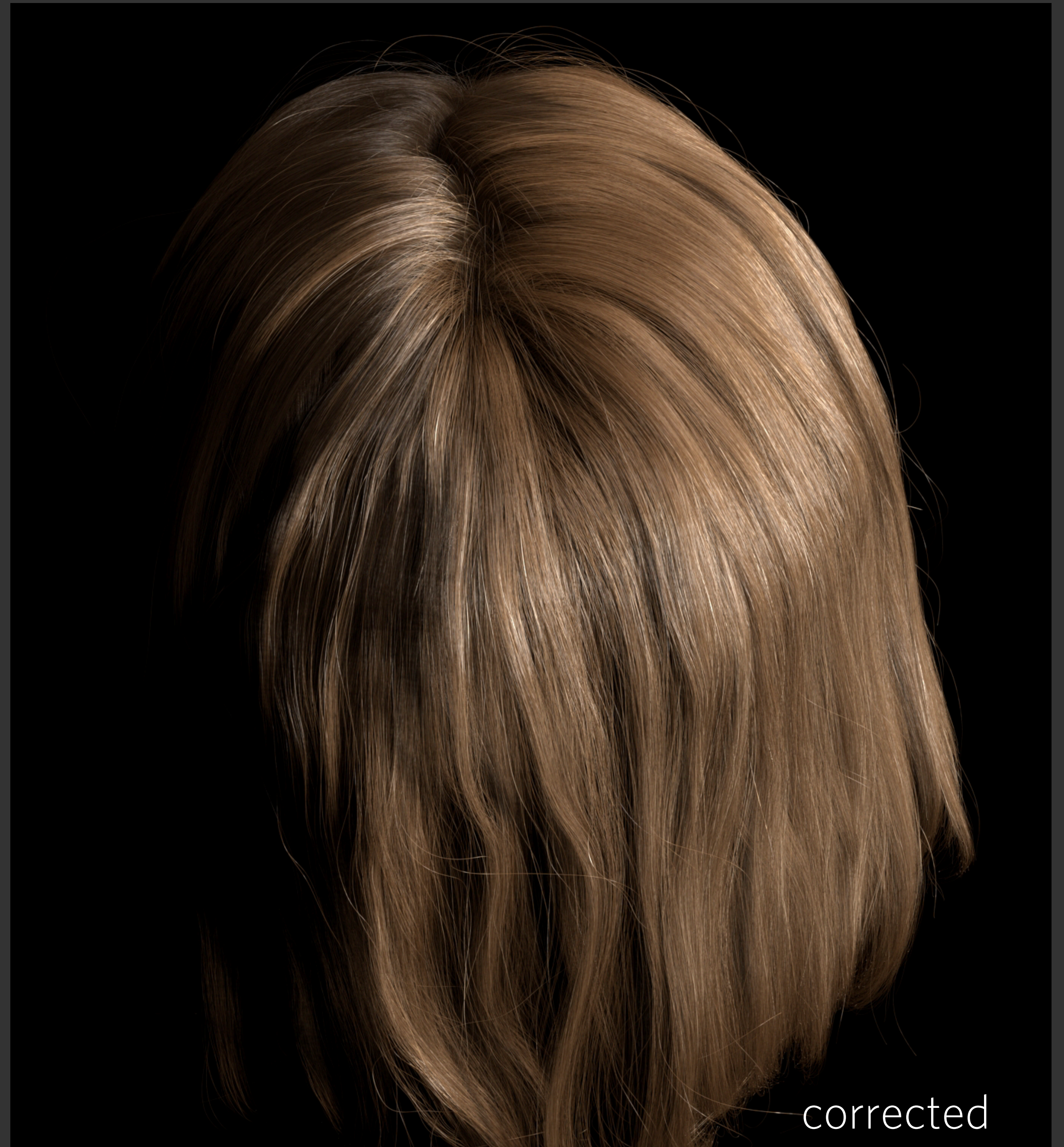
all paths

Fix: energy conservation

Original model tended to lose energy

D'Eon et al. proposed an elegant replacement for $M(\theta)$

d'Eon et al. | "An Energy-Conserving Hair Reflectance Model" (2011)

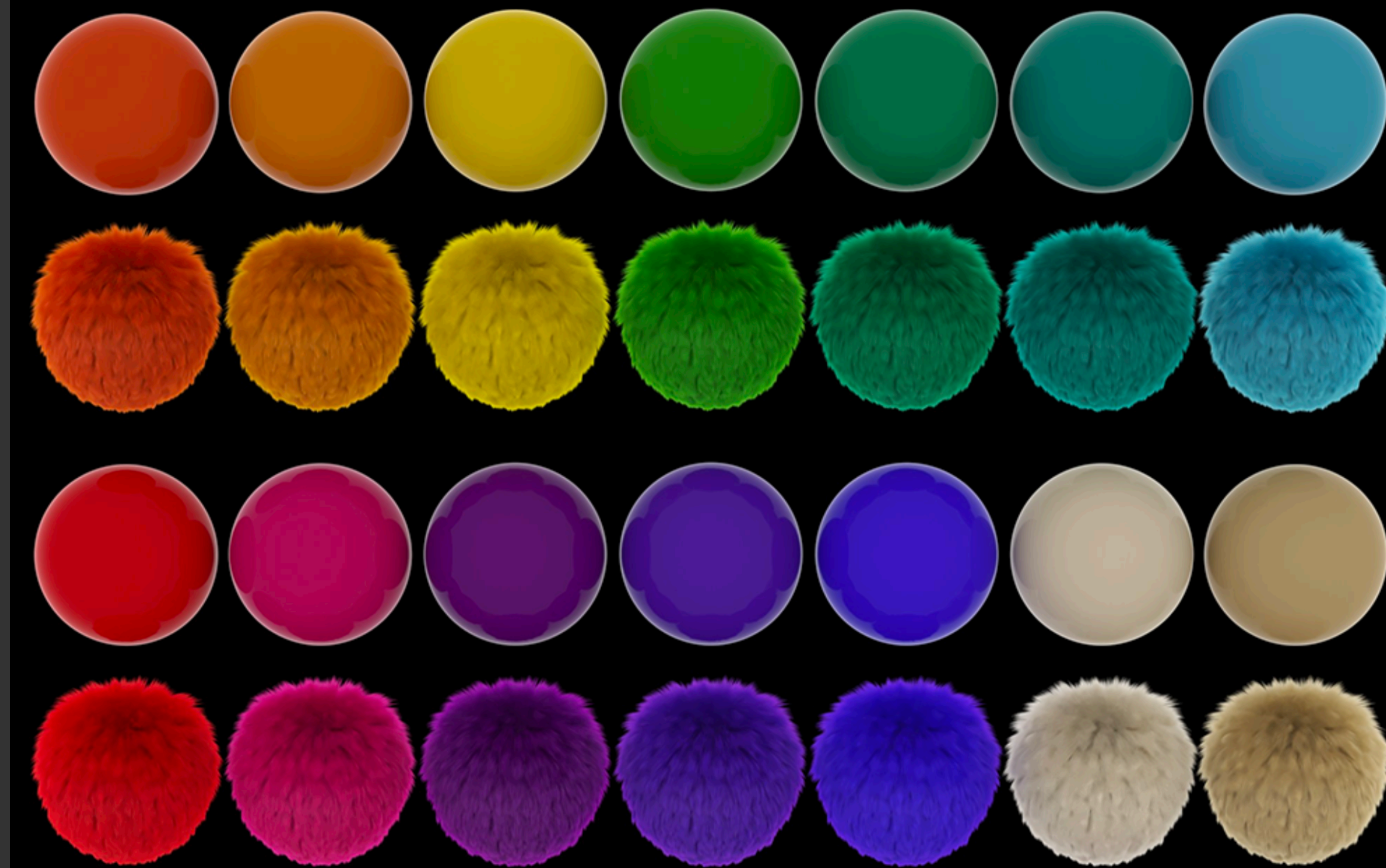


Refinements: usability

Physical parameters like absorptivity, refractive index are not easy to adjust

For production, more indirect controls are better

overall color
glint angle, ...



Hery et al. | "Pixar's Foundation for Materials" (2017)

Sadeghi et al. | "An Artist Friendly Hair Shading System" (2011)



Refinement: near-field

original model is for fibers smaller than a pixel

no variation across the fiber

models that keep that variation can actually be simpler

Zinke & Weber | "Light Scattering from Filaments" (2007)



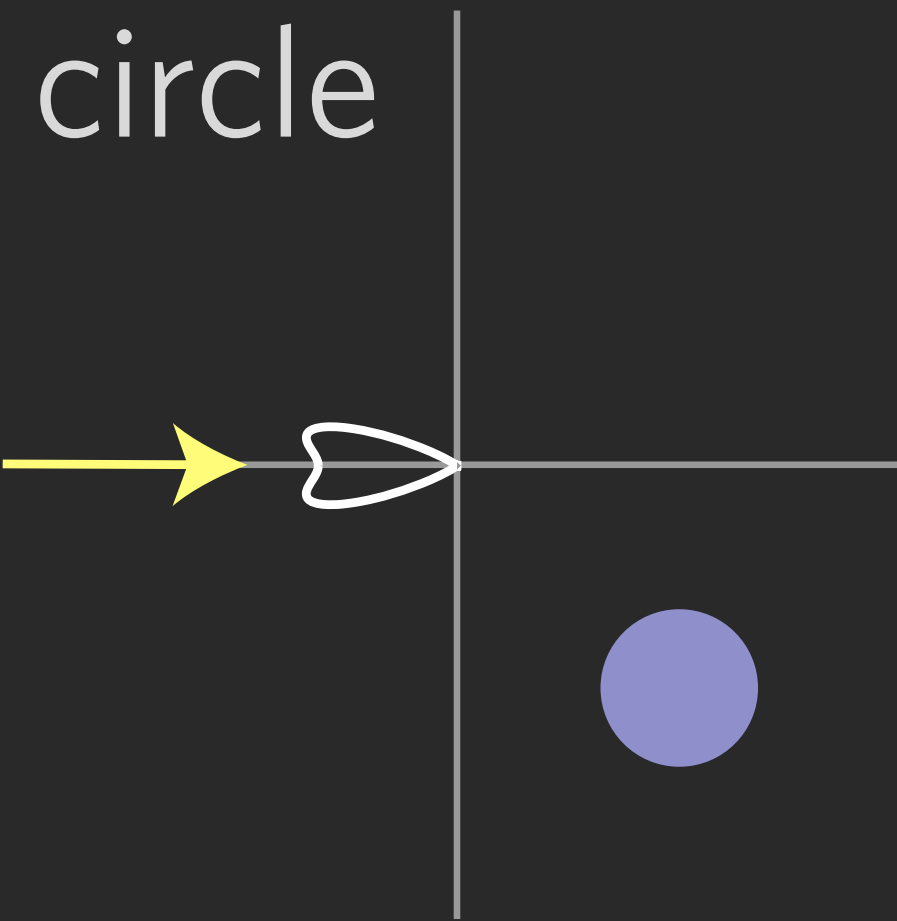
Chiang et al. | "A Practical and Controllable Hair and Fur Model for Production Path Tracing" (2016)



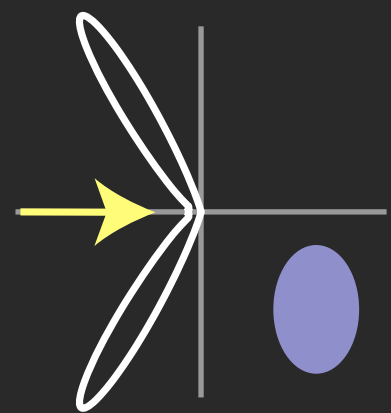
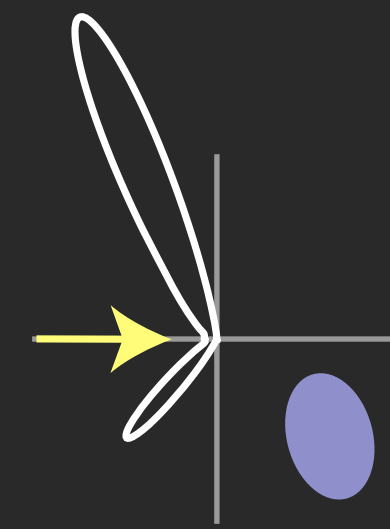
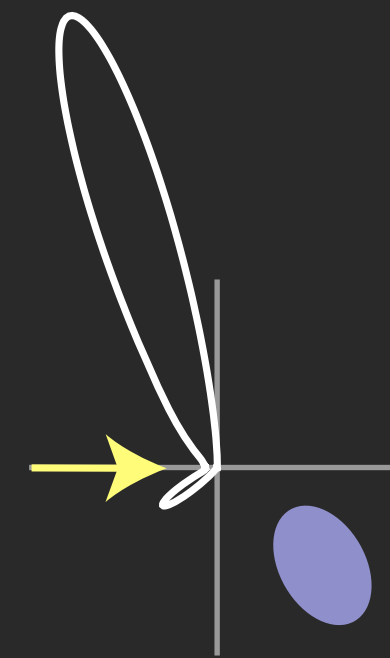
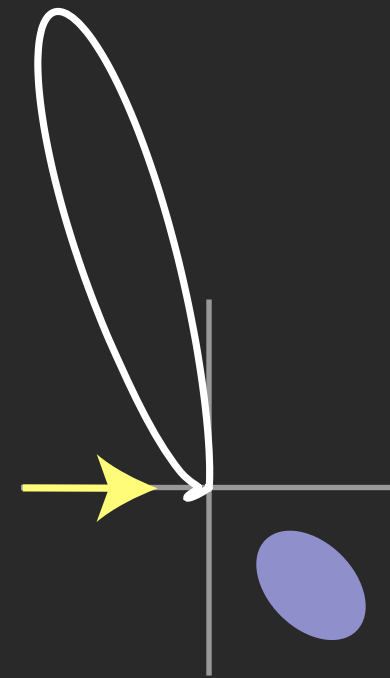
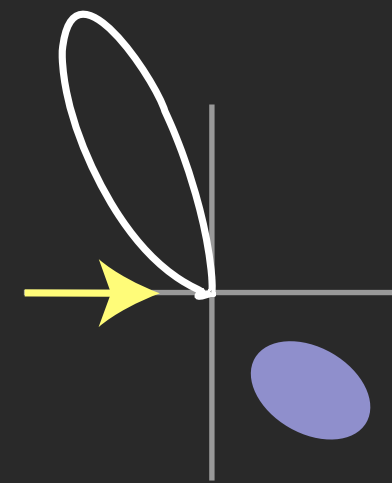
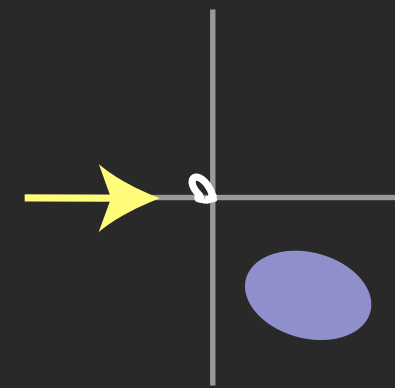
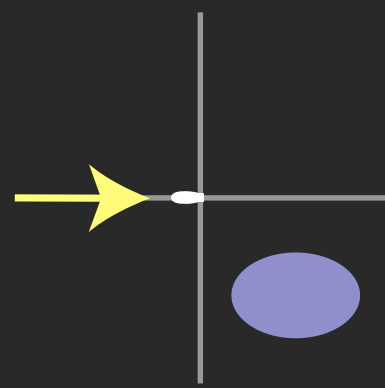
ray optics of elliptical fibers

Pramook Khungurn and Steve Marschner. "Azimuthal Scattering from Elliptical Hair Fibers." *ACM Transactions on Graphics* 36:2 (April 2017).

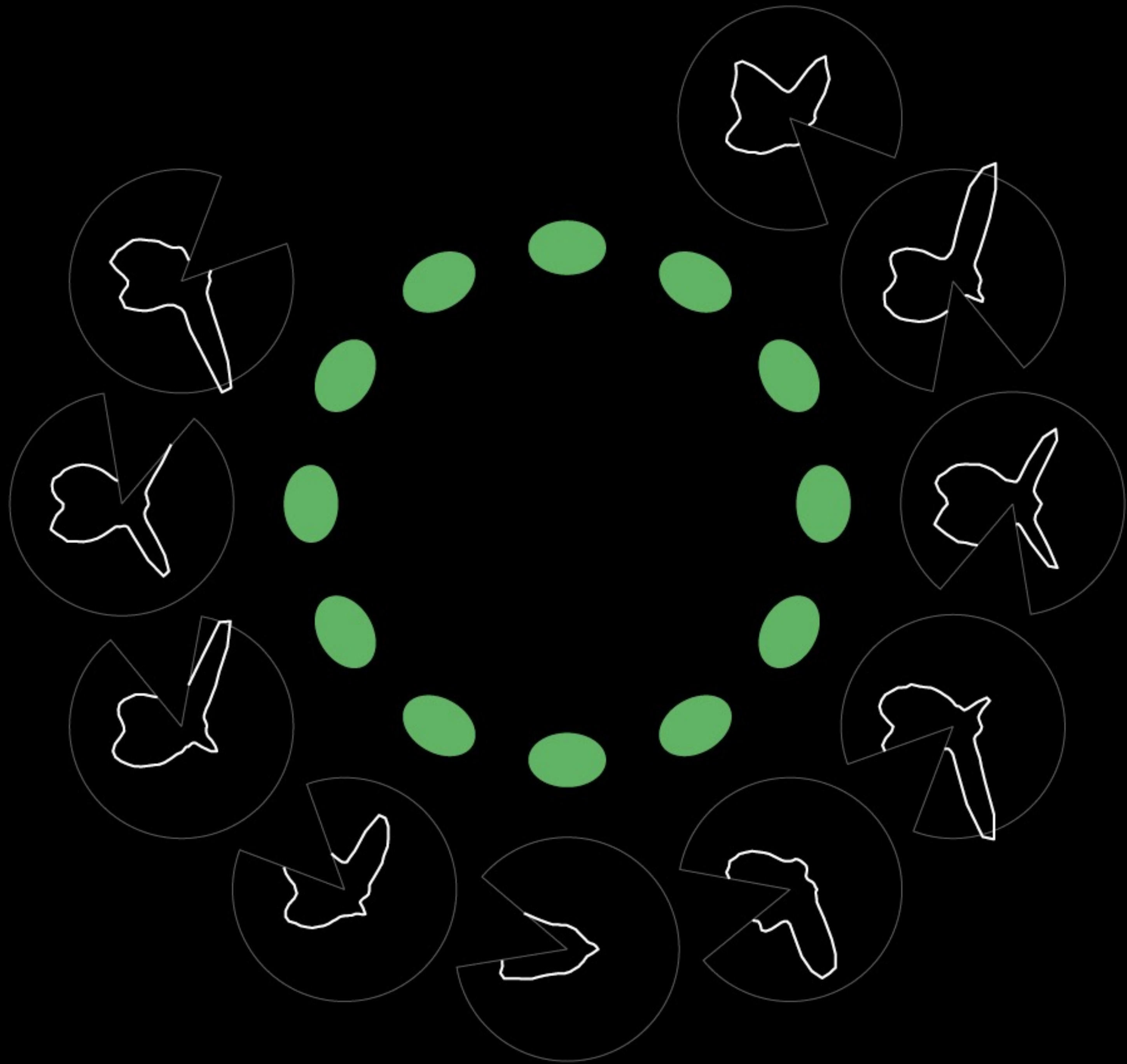
Azimuthal distributions



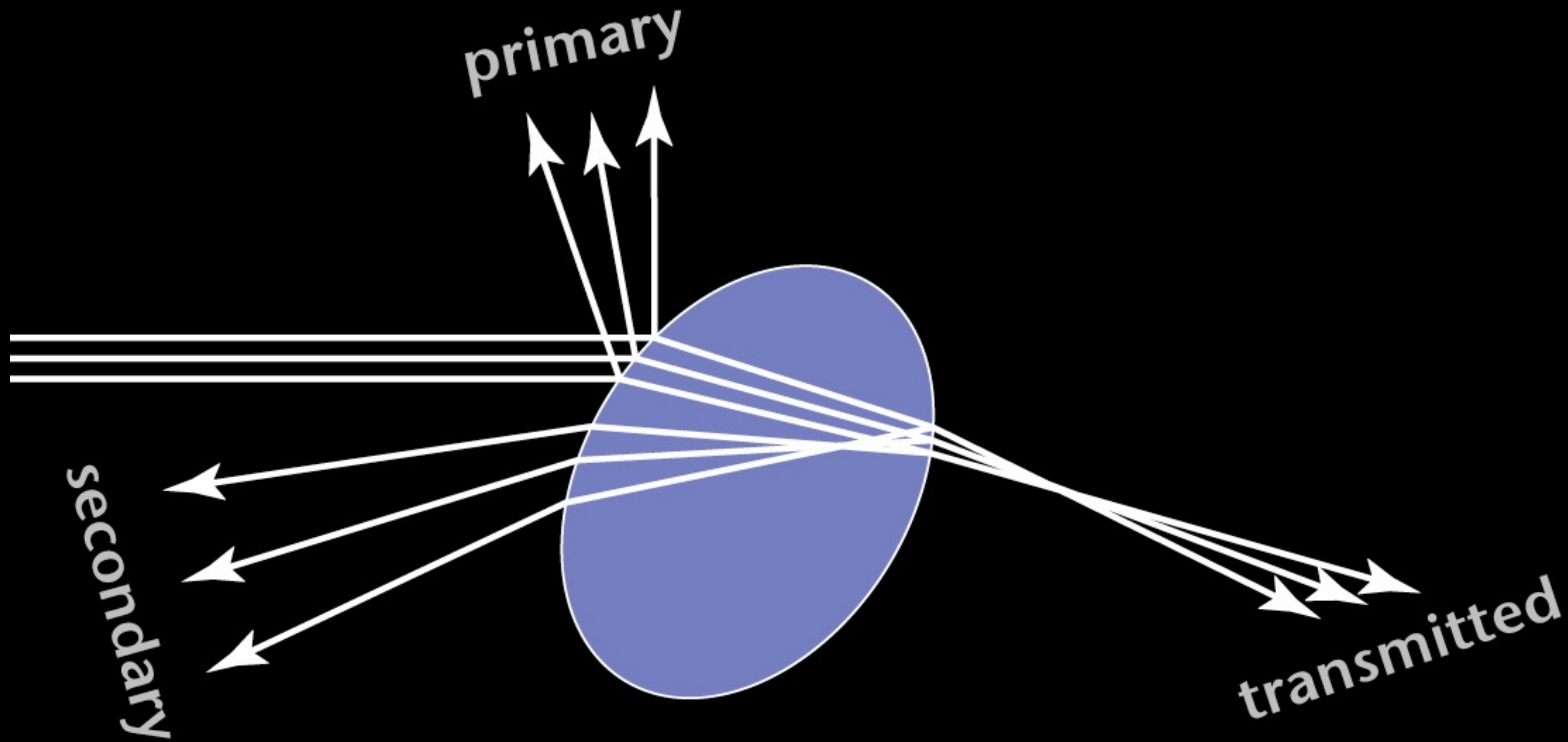
ellipse (1.5:1)



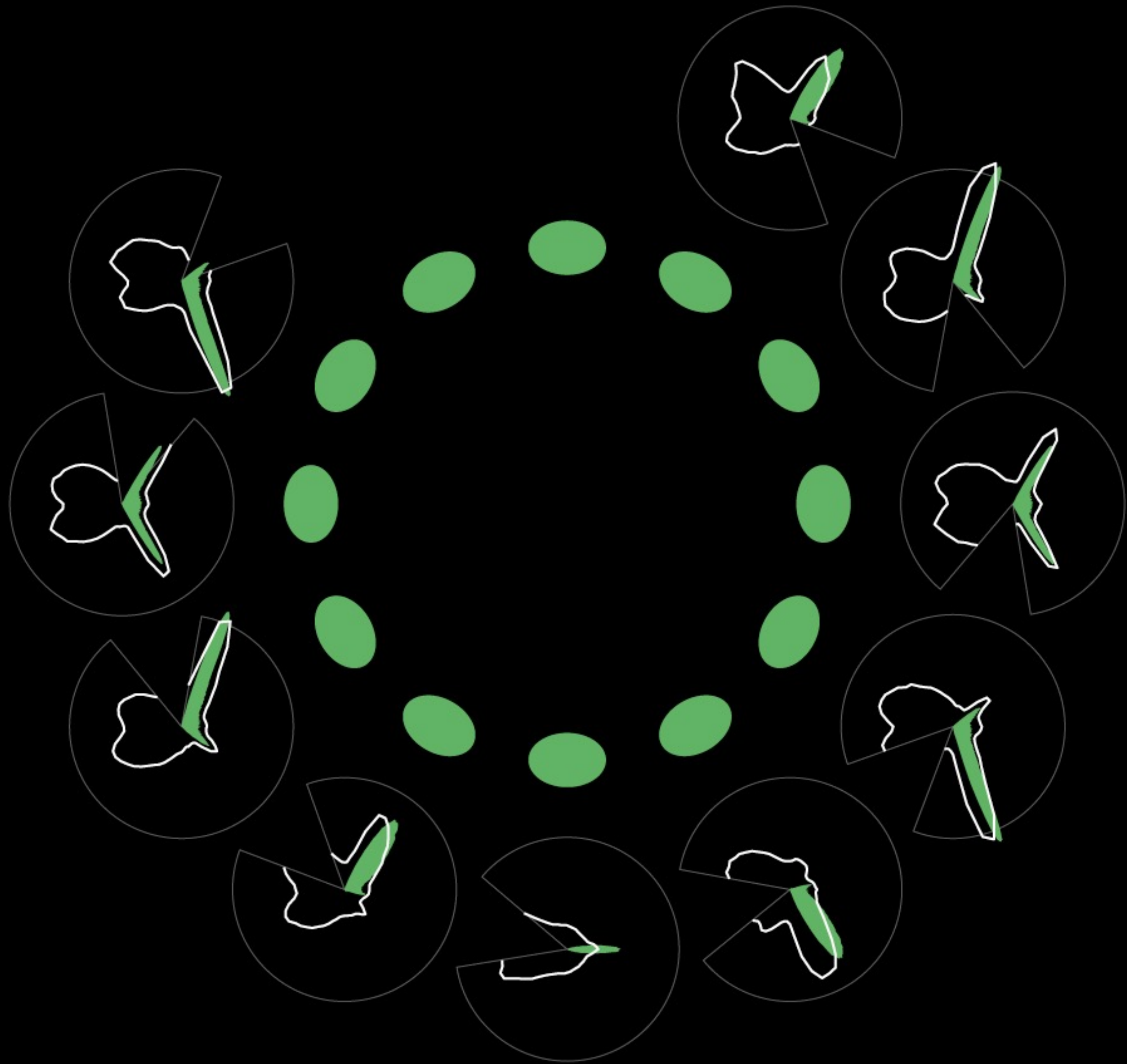
hair
rotation



illumination

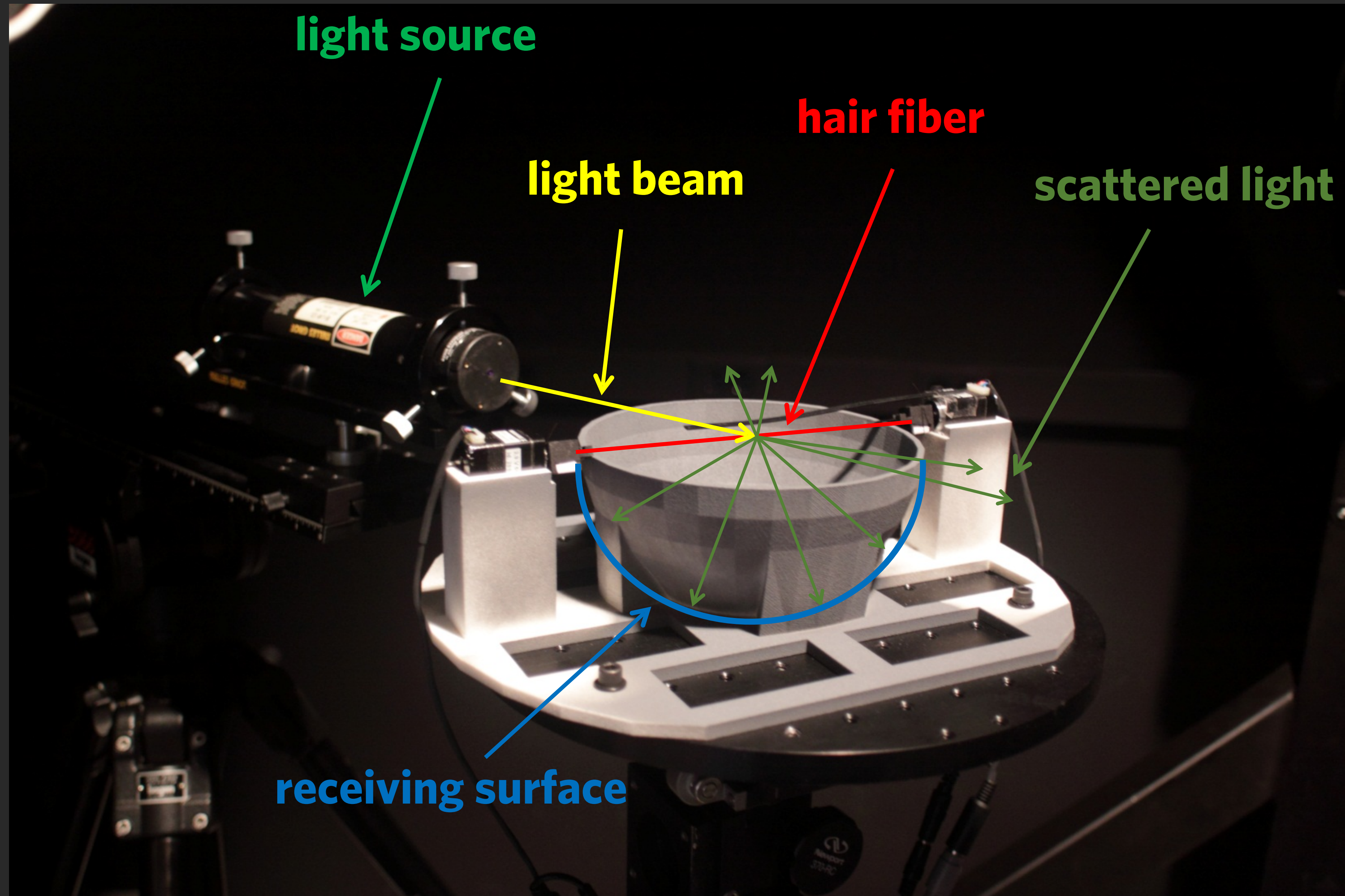


hair
rotation



illumination

Image-based Measurement



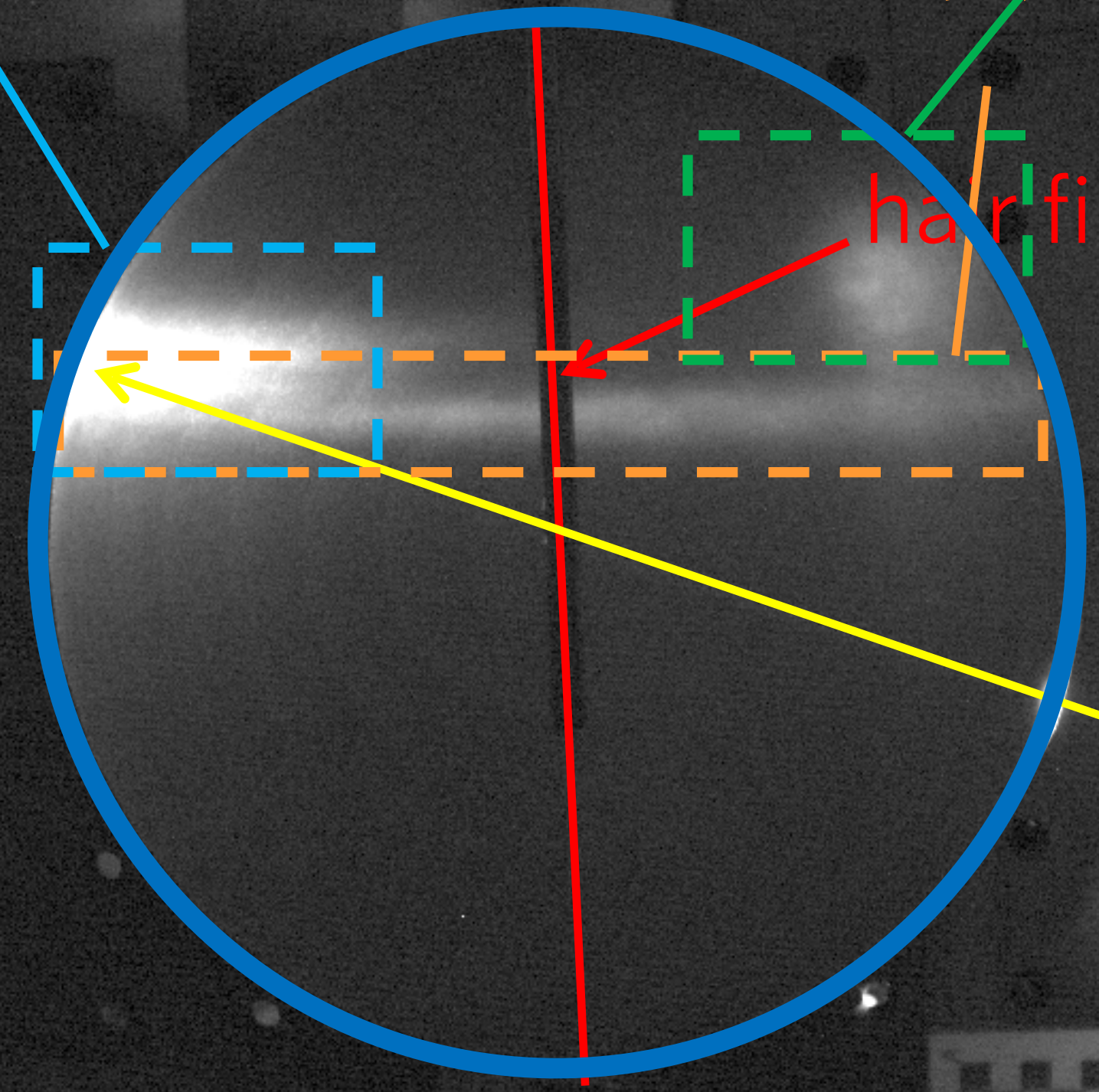
forward
transmission
(**TT**)

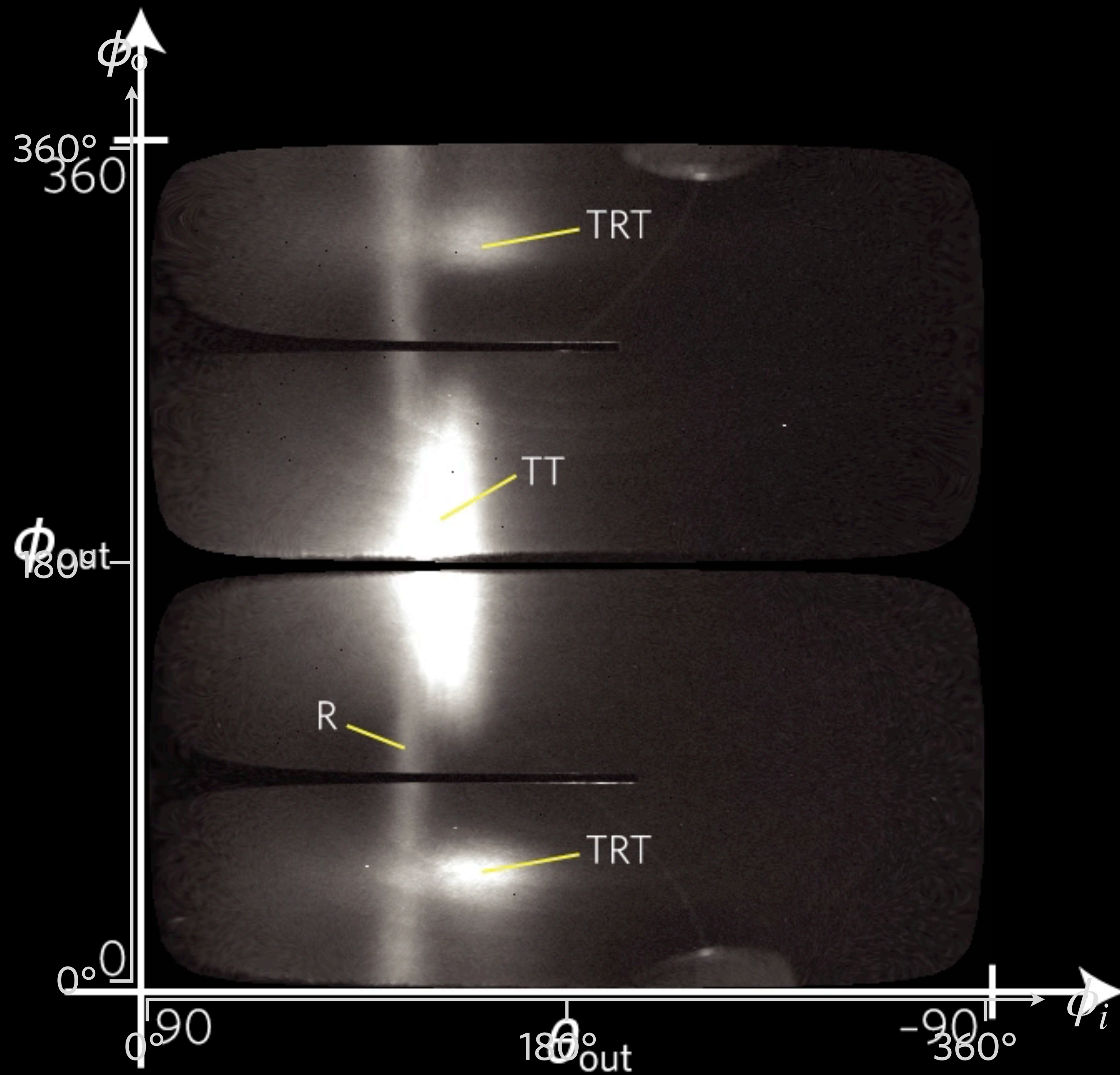
specular
reflection
(**R**)




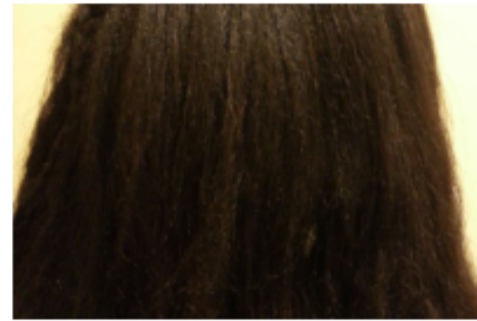


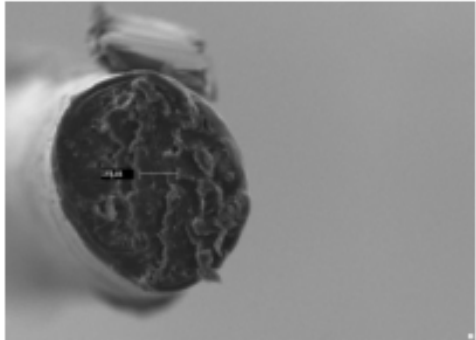
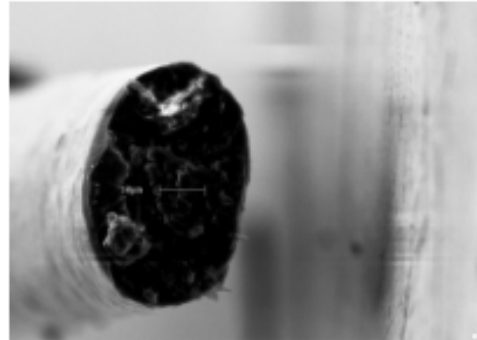
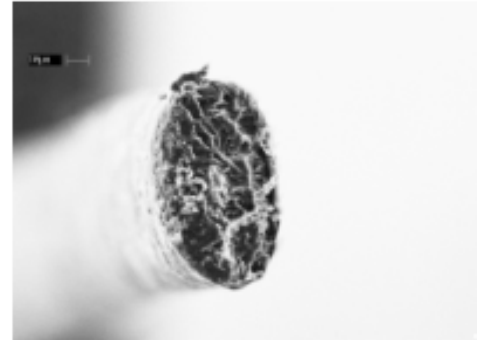
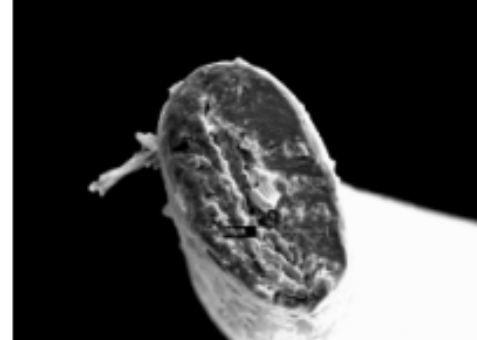
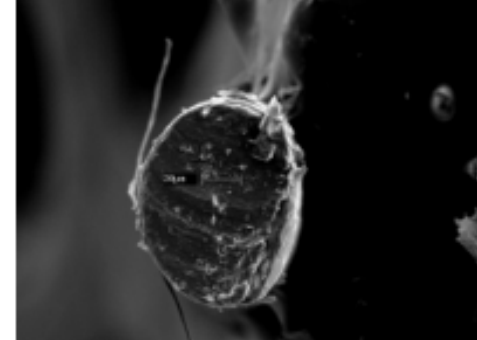
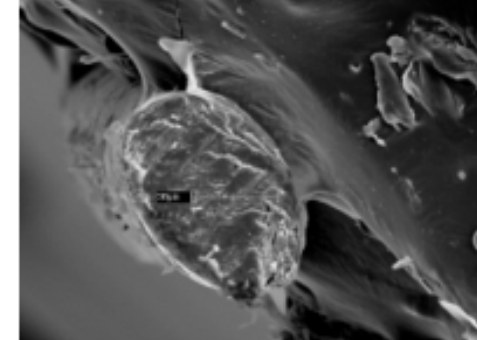
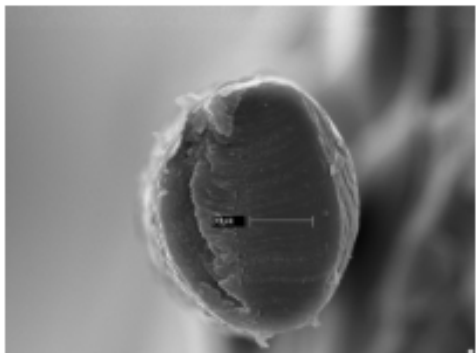
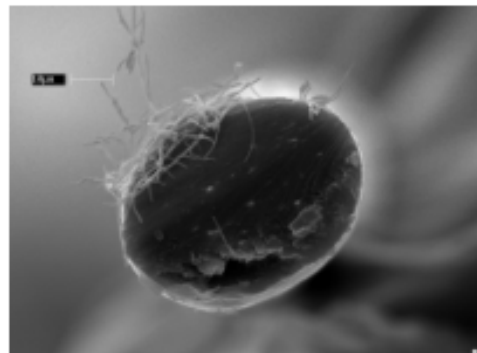
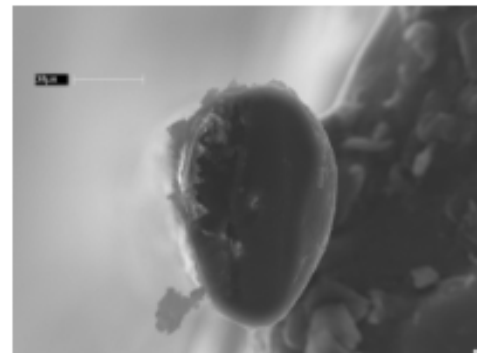
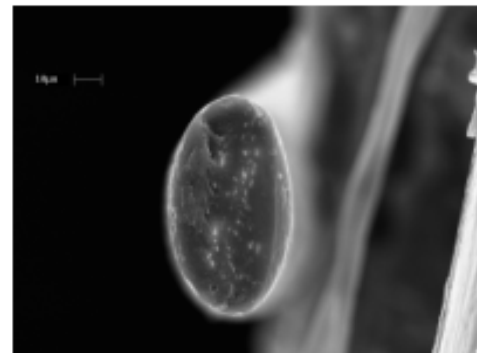
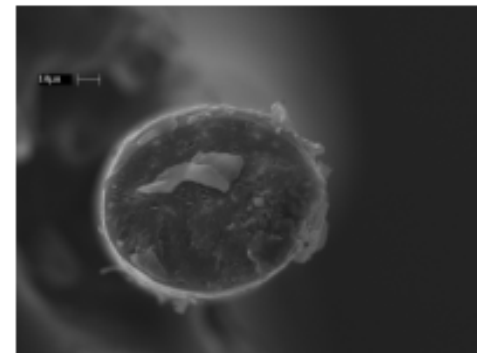
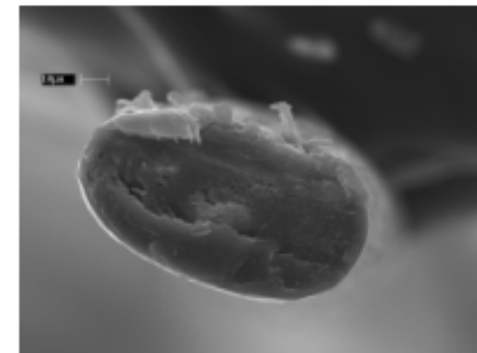
backward
transmission
(**TRT**)

hair fiber

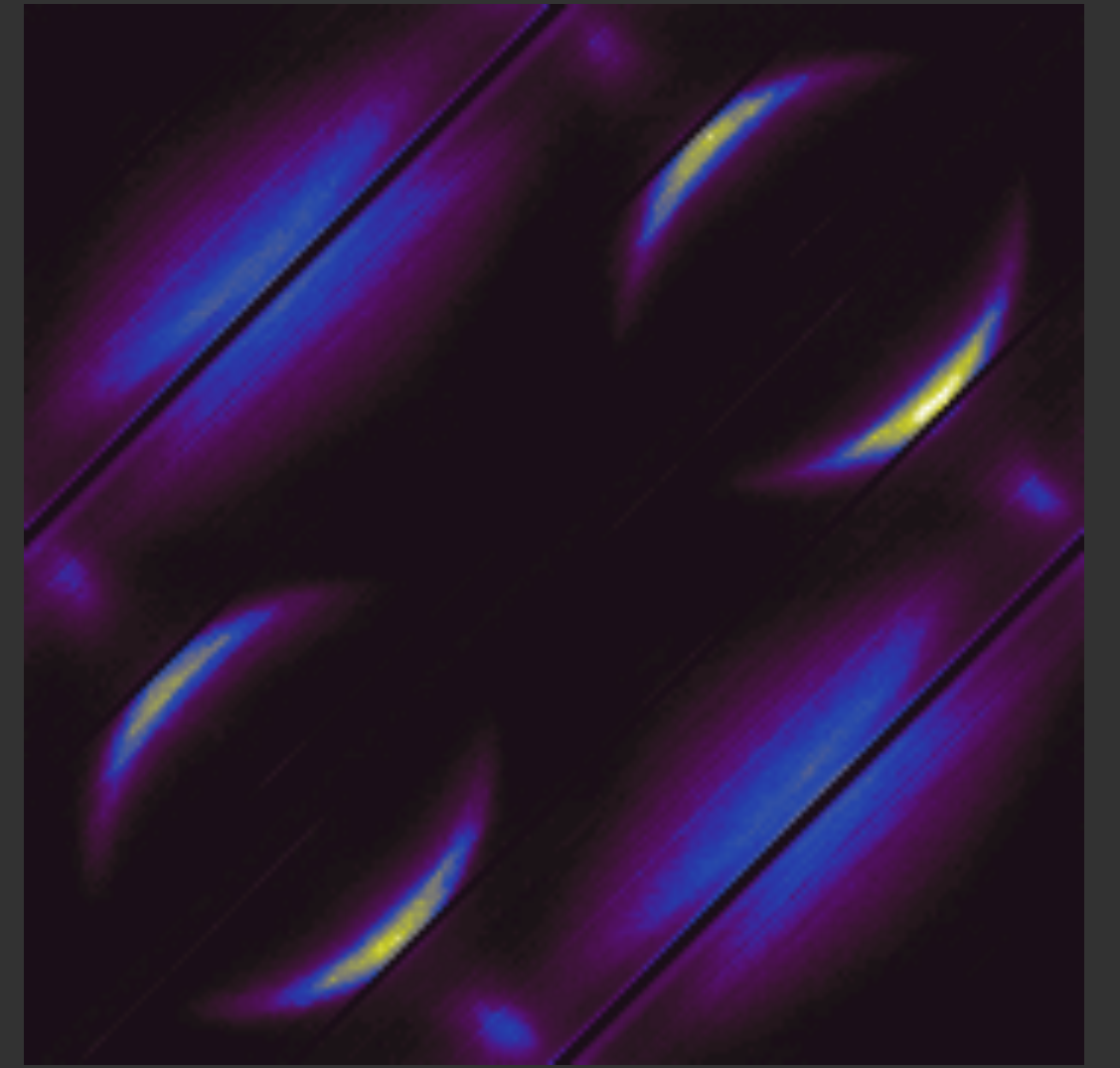
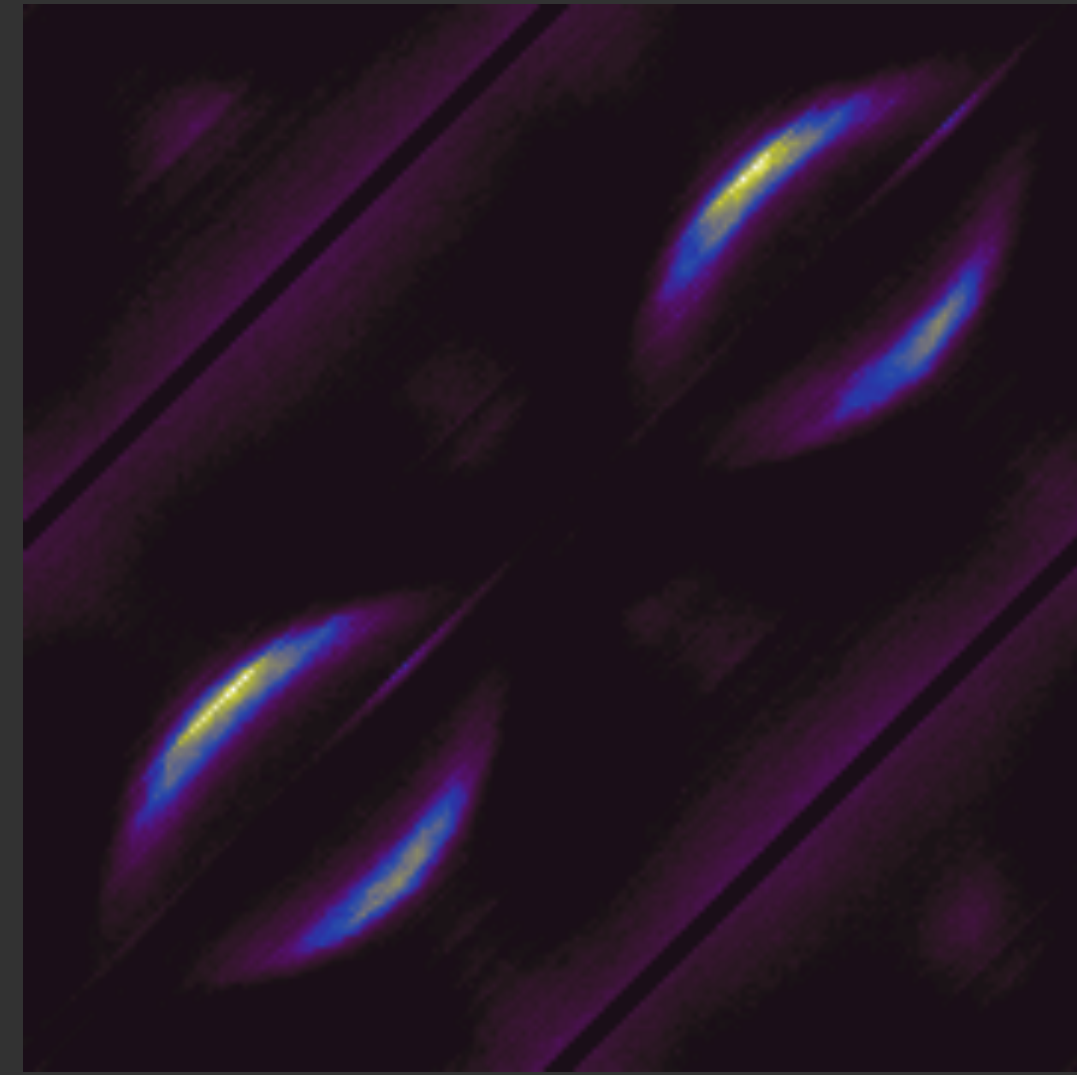
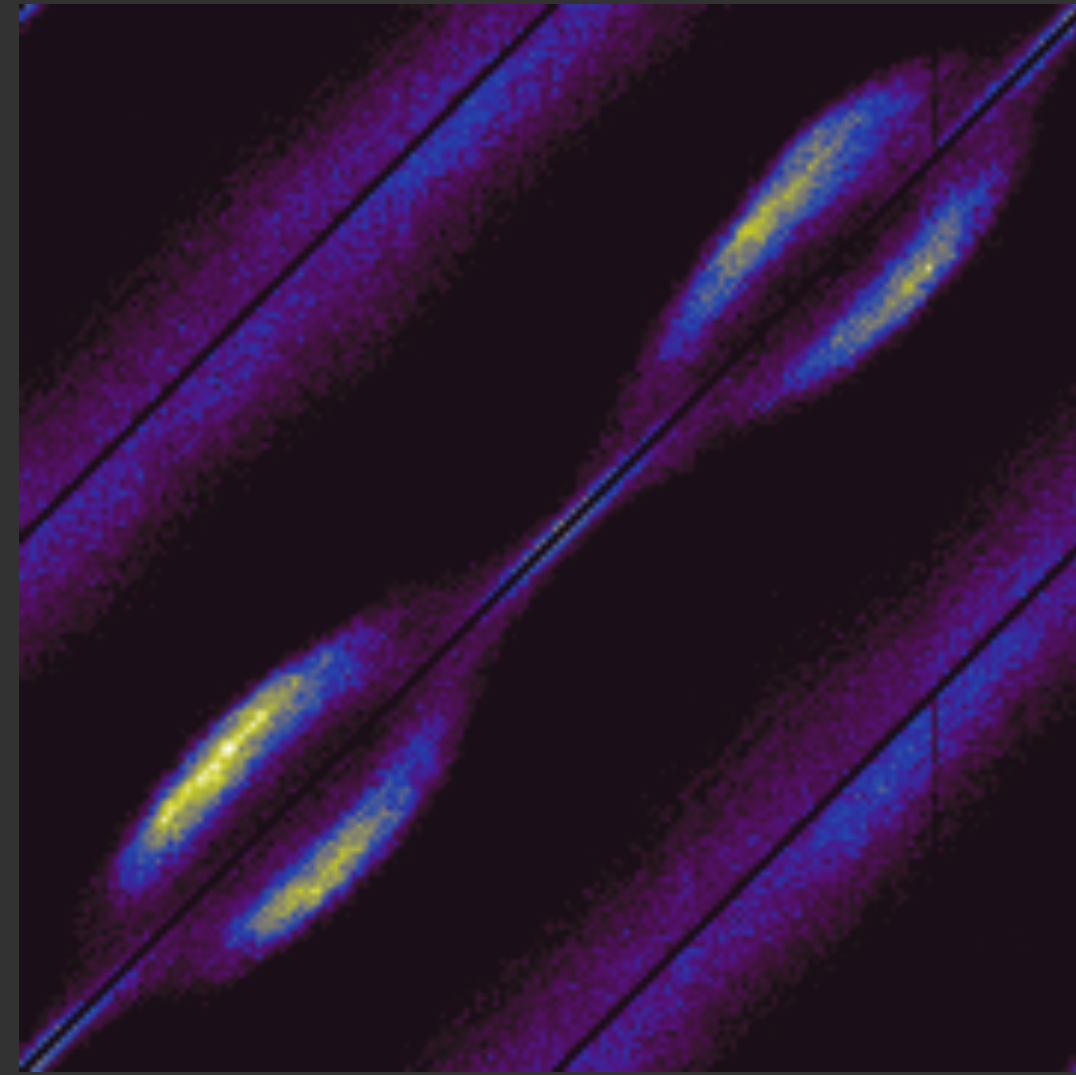
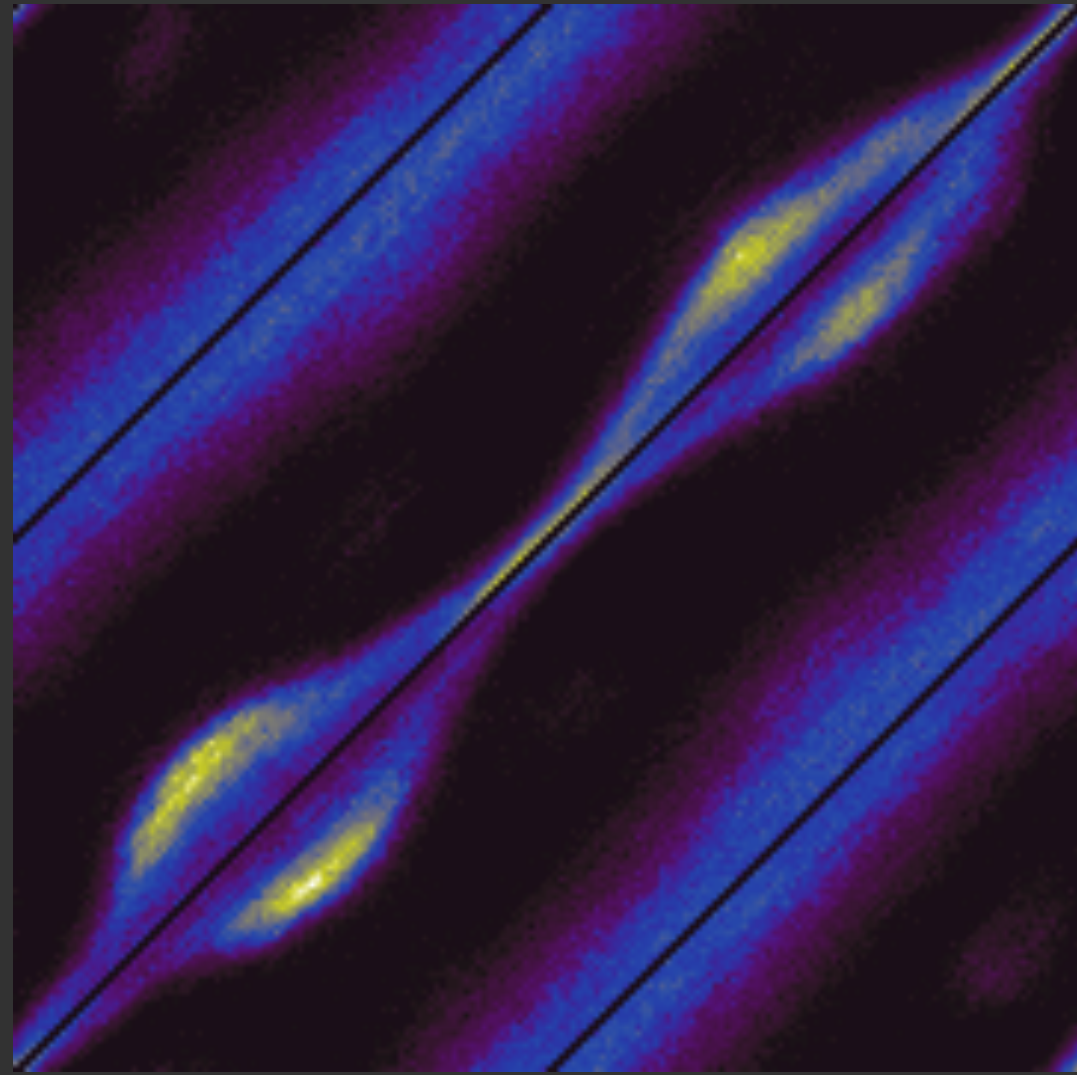
light beam





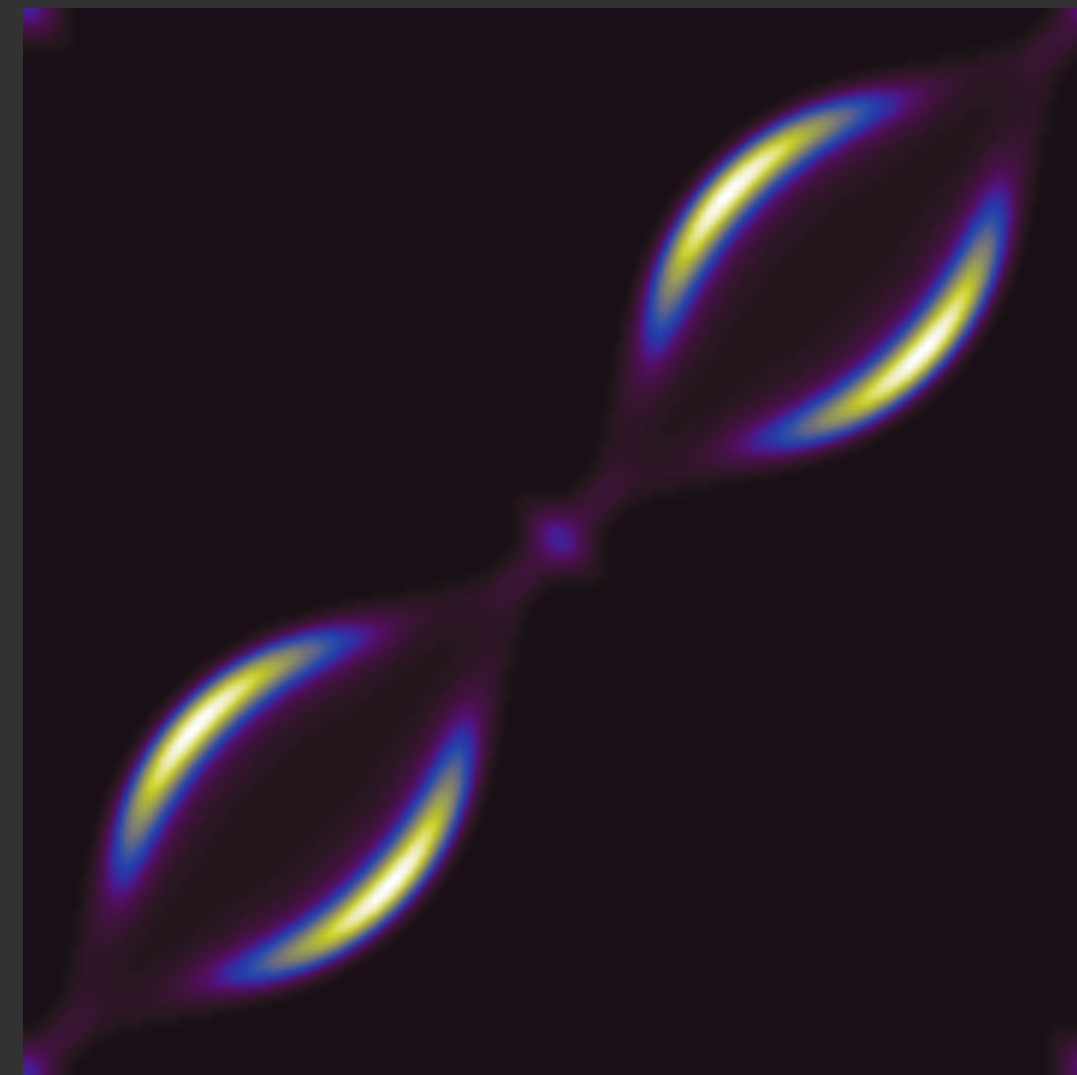
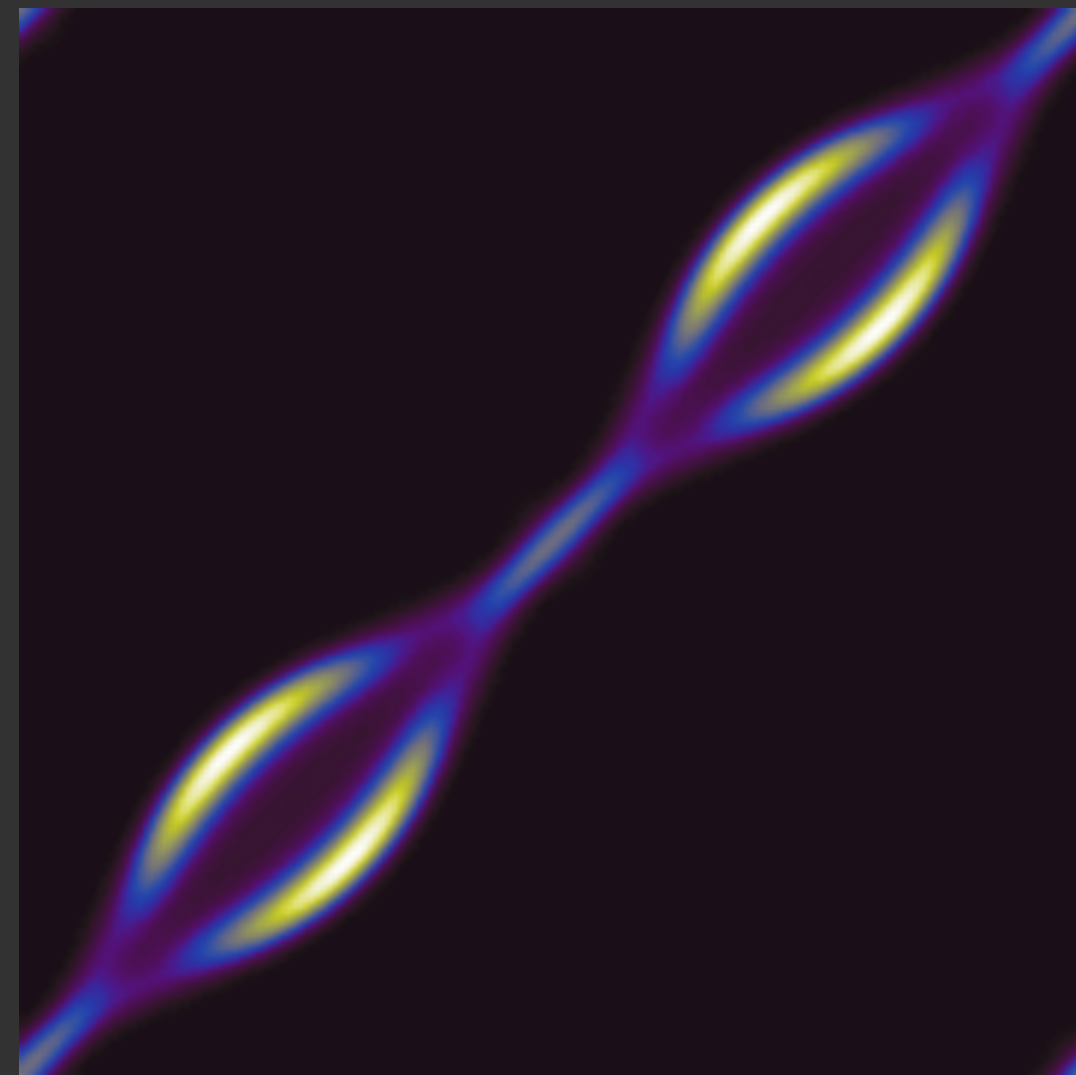
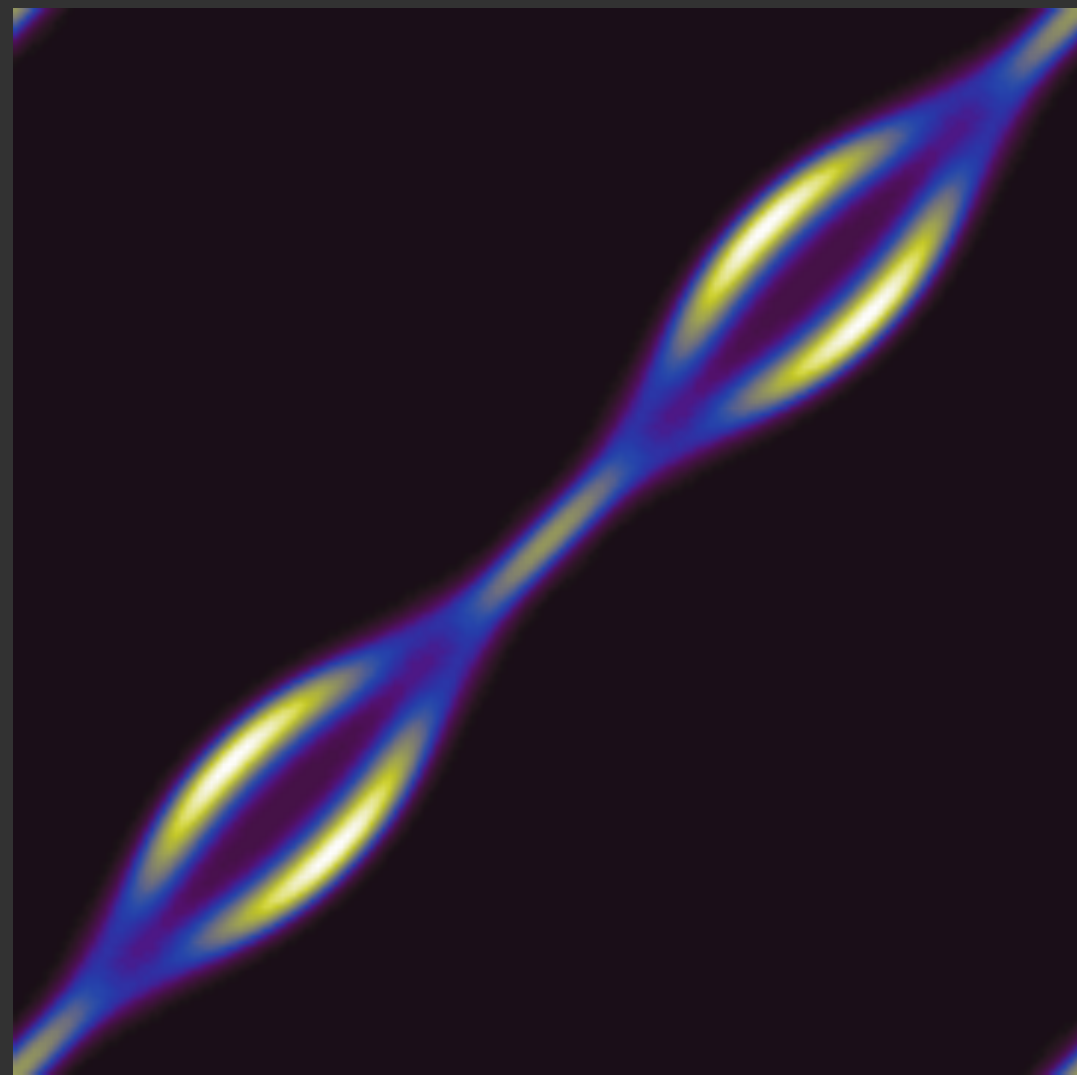
Source	Caucasian	Caucasian	Caucasian	African	Chinese	Indian
Photographs						
SEM images (Fiber 1)						
Measured aspect ratios (Fiber 1)	1.136 1.213 1.469 ($\bar{x} = 1.273$)	1.446 1.501 ($\bar{x} = 1.473$)	1.603 1.717 ($\bar{x} = 1.660$)	1.834 1.923 1.933 ($\bar{x} = 1.897$)	1.308 1.361 ($\bar{x} = 1.334$)	1.310 1.482 ($\bar{x} = 1.396$)
SEM images (Fiber 2)						
Measured aspect ratios (Fiber 2)	1.256 1.268 1.411 1.429 ($\bar{x} = 1.341$)	1.077 1.195 1.246 1.353 1.436 1.552 ($\bar{x} = 1.310$)	1.548 1.553 1.746 1.762 ($\bar{x} = 1.605$)	1.561 1.647 1.789 ($\bar{x} = 1.666$)	1.035 1.123 1.142 1.183 ($\bar{x} = 1.121$)	1.503 1.524 1.609 1.616 1.780 1.839 ($\bar{x} = 1.645$)

measured azimuthal scattering



fibers of increasing eccentricity →

modeled azimuthal scattering



$a = 1.0$

$a = 1.2$

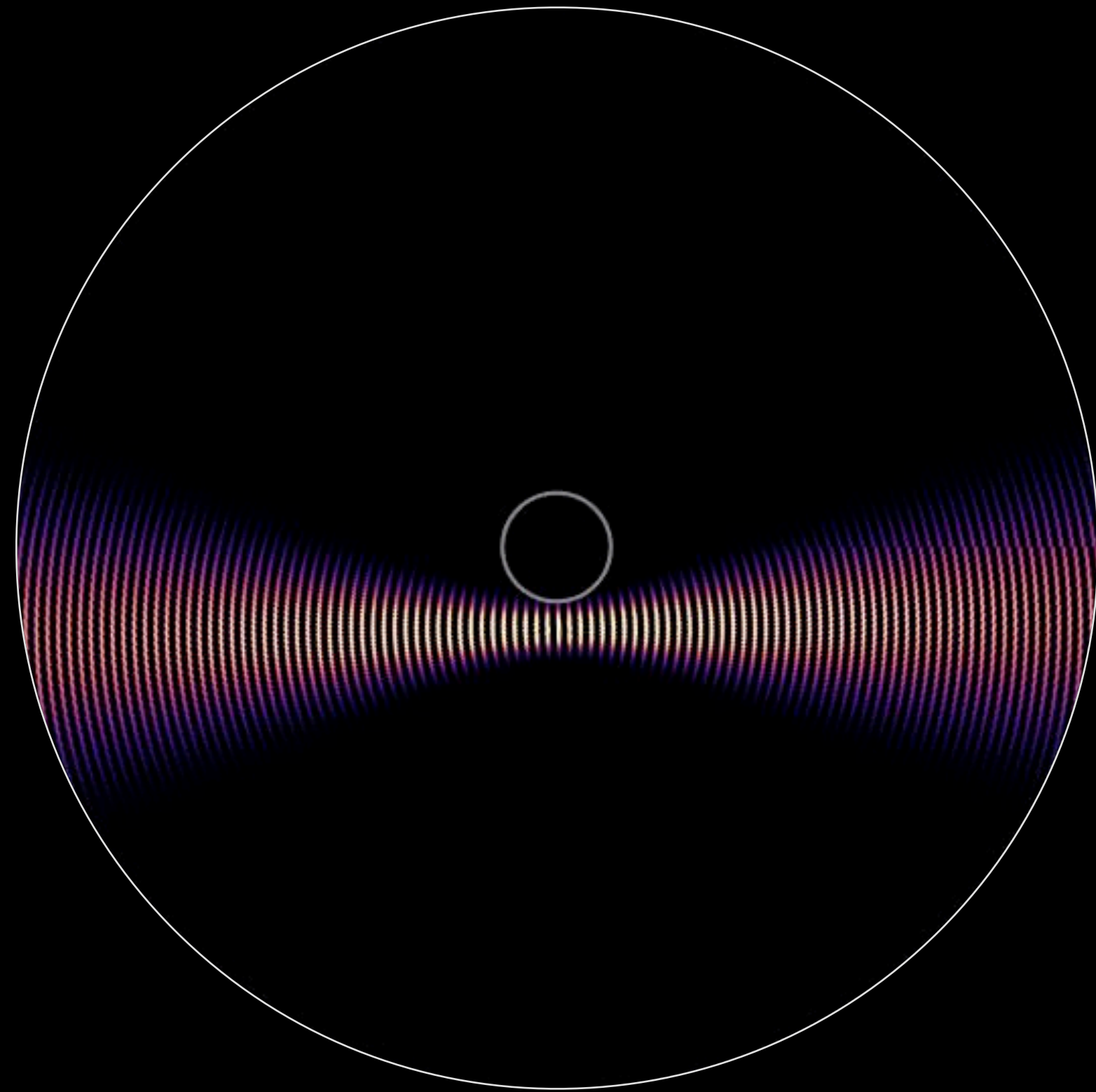
$a = 1.5$



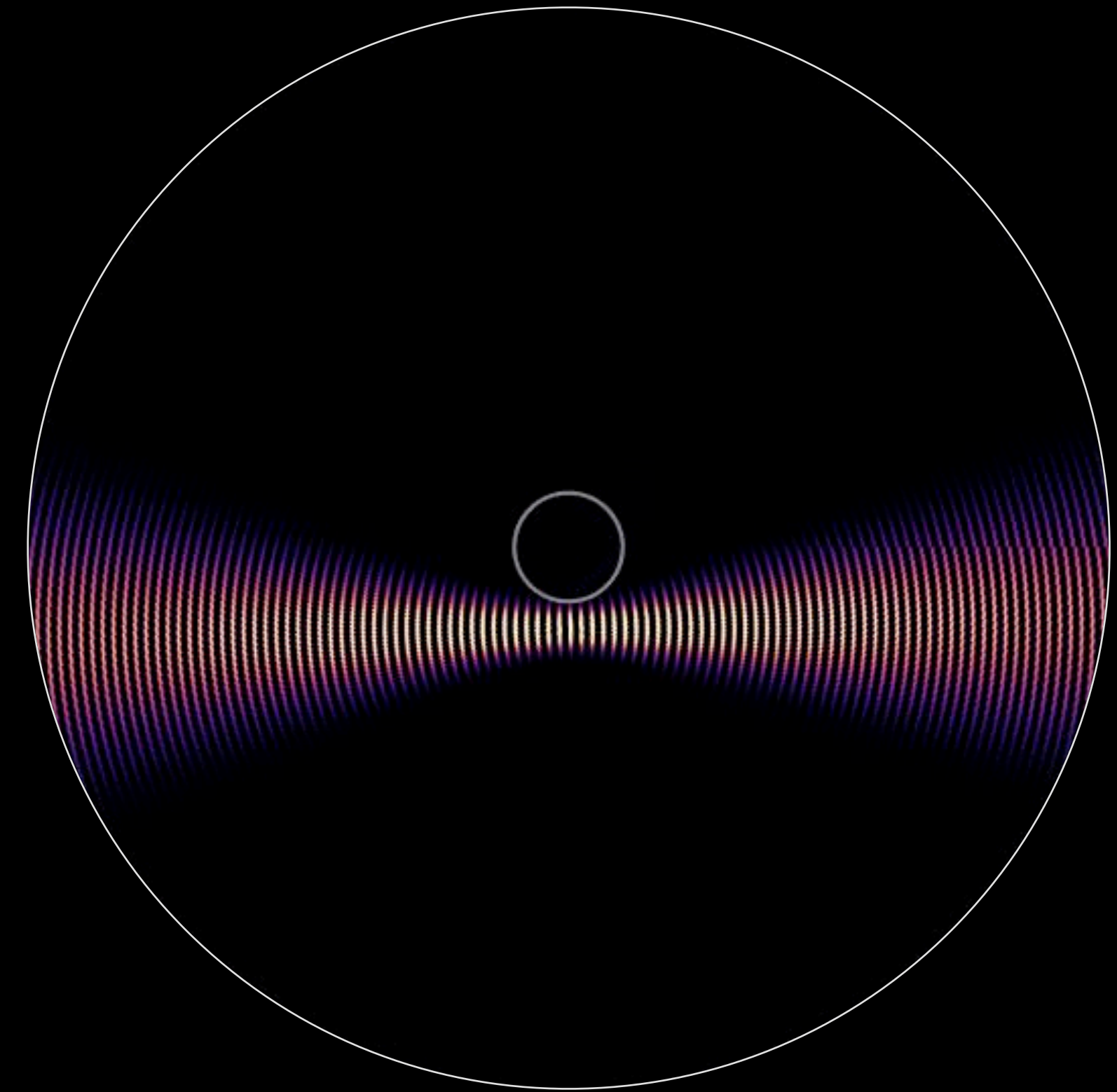
wave optics of fibers

Mengqi (Mandy) Xia, Bruce Walter, Eric Michielssen, David Bindel, and Steve Marschner. "A wave optics based fiber scattering model." *SIGGRAPH Asia* 2020.

Wave simulation

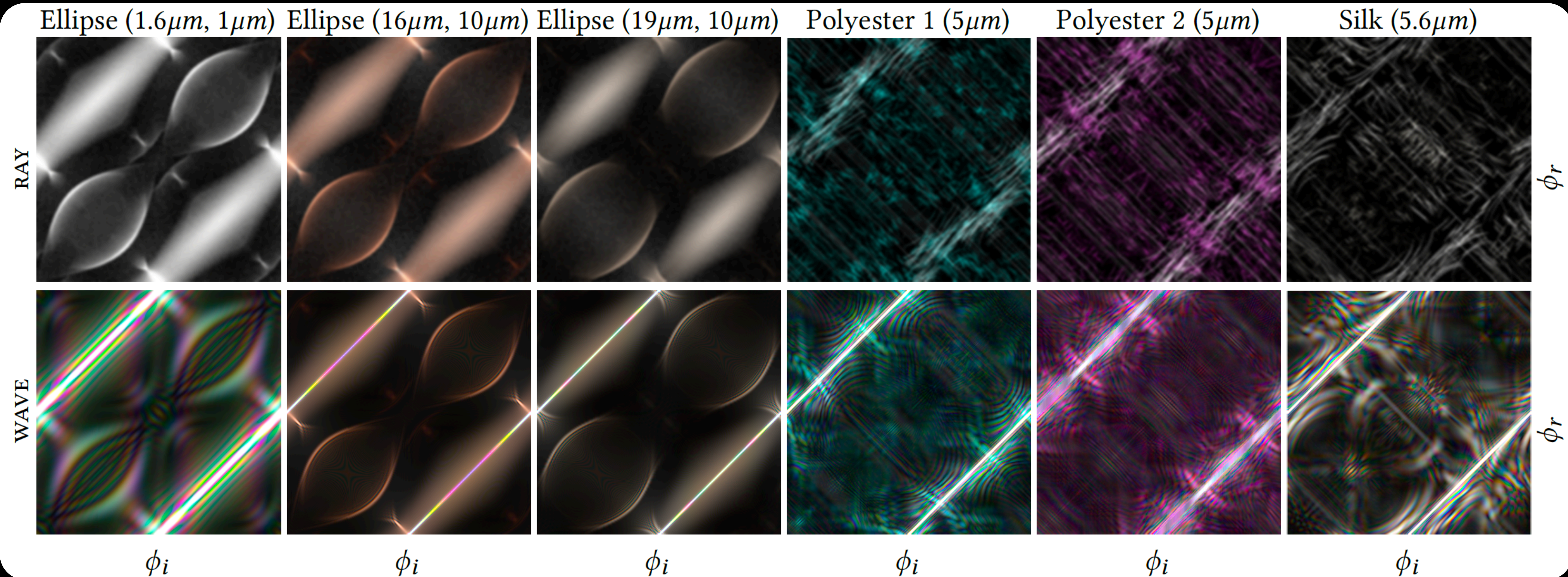


Incident fields

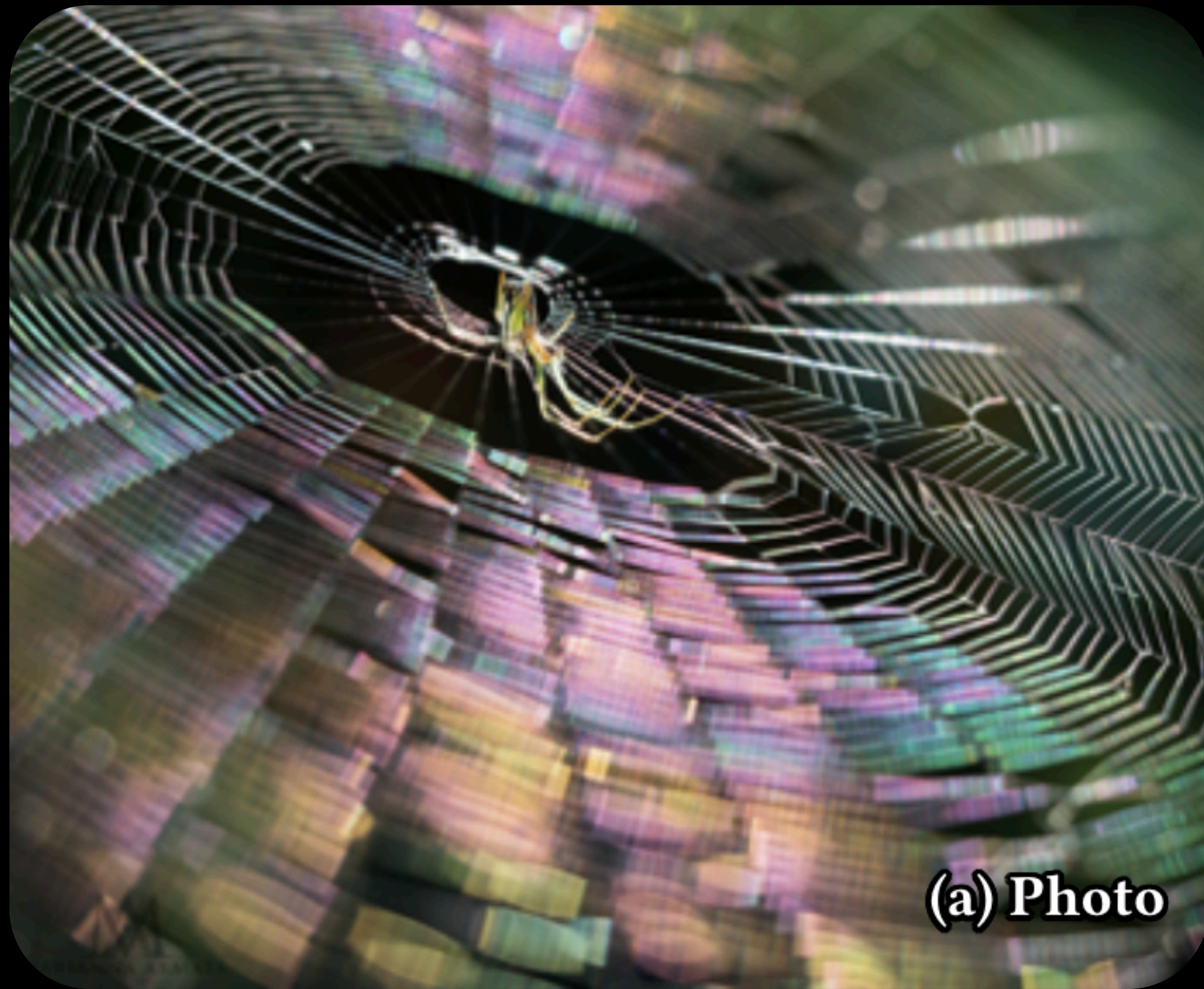


Total fields

Azimuthal scattering function



Strong color effects



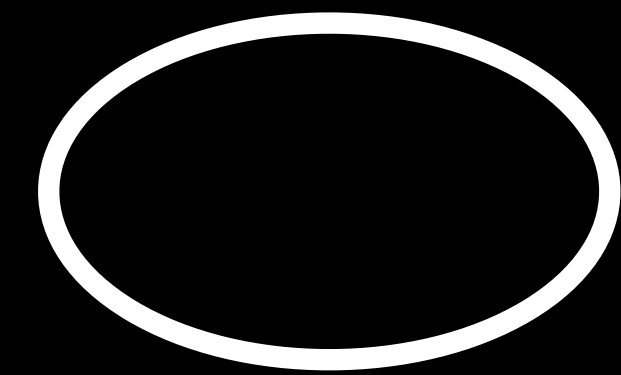
Strong forward peak



Ray



Wave



Ellipse
16 μm , 10 μm

Acknowledgments

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Wenzel Jakob

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Henrik Wann Jensen

Pat Hanrahan

Bruce Walter

Eric Michielssen

David Bindel

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Sloan Foundation

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