Microfacet models for reflection and refraction

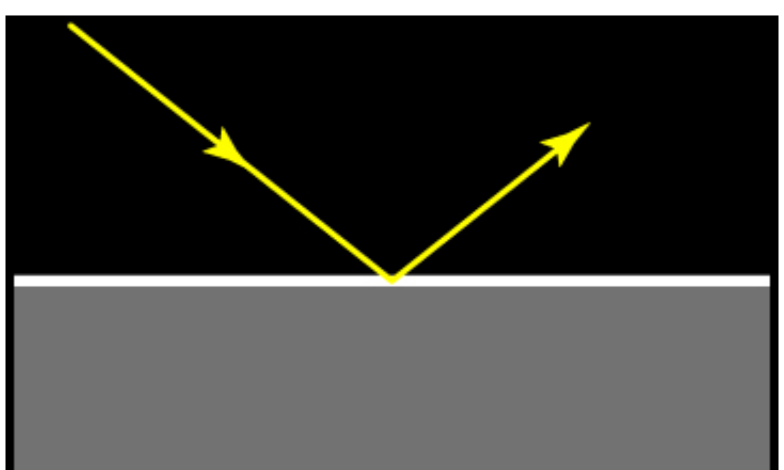
Steve Marschner

Cornell University CS 5630 Spring 2026
Lecture 10

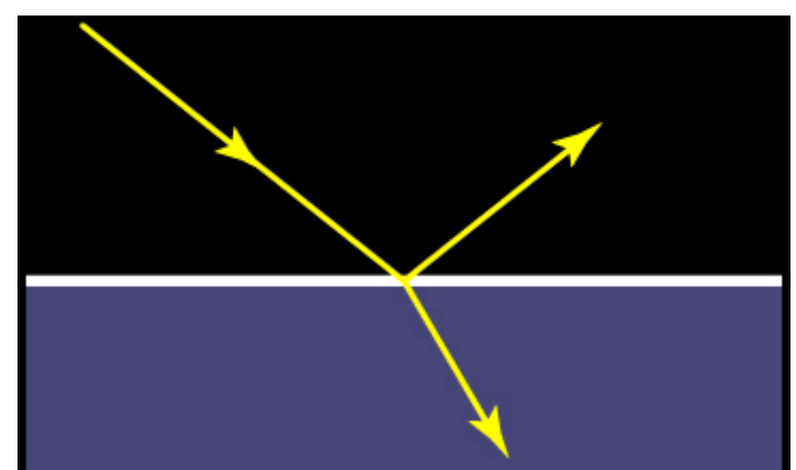
(based on presentation for
Walter, Marschner, Li, and Torrance EGSR '07)

Smooth surfaces

metal



dielectric



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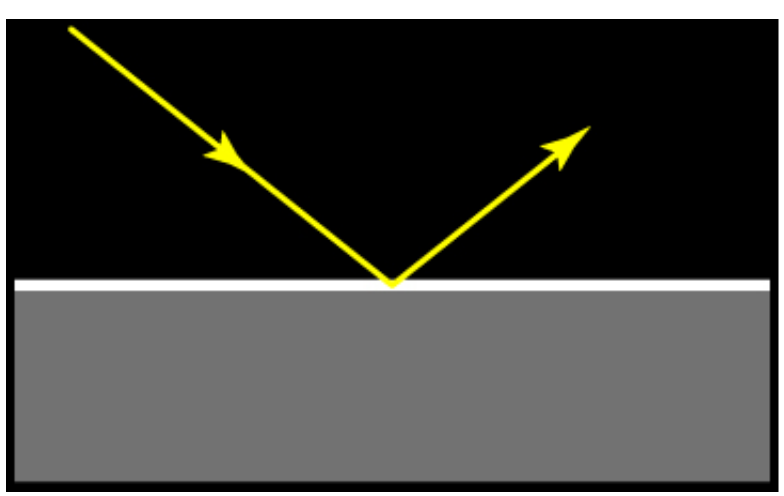
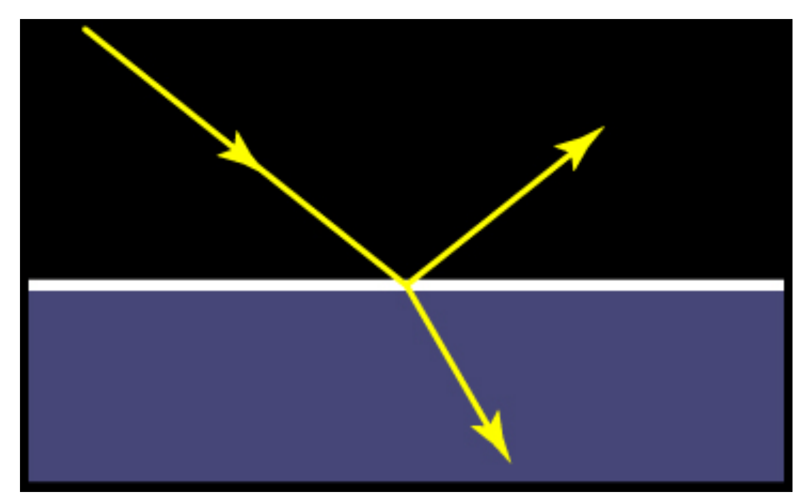
Ideal specular reflection

- **Smooth surfaces of pure materials have ideal specular reflection**
  - Metals (conductors) and dielectrics (insulators) behave differently
- **Reflectance (fraction of light reflected) depends on angle**



metal



dielectric

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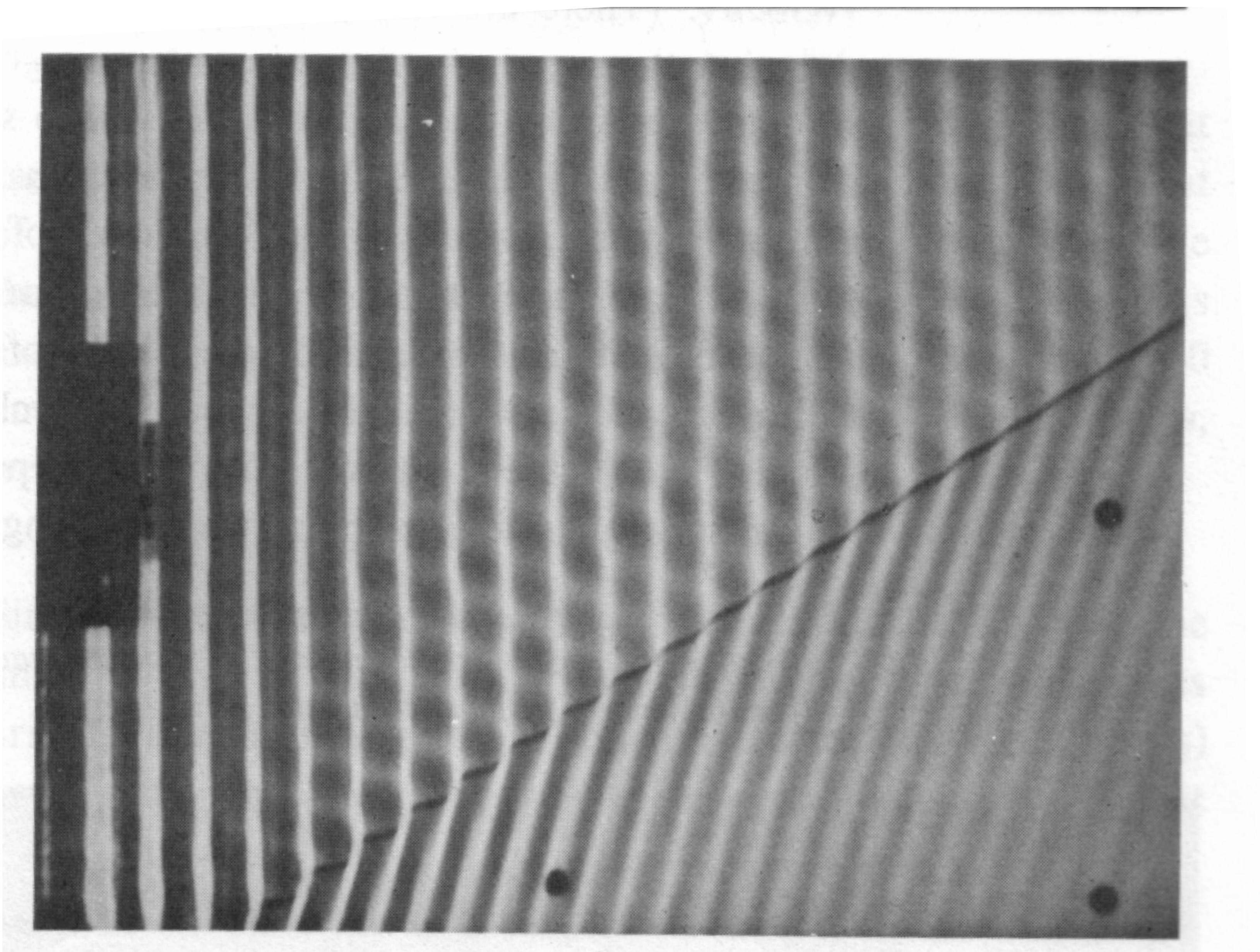
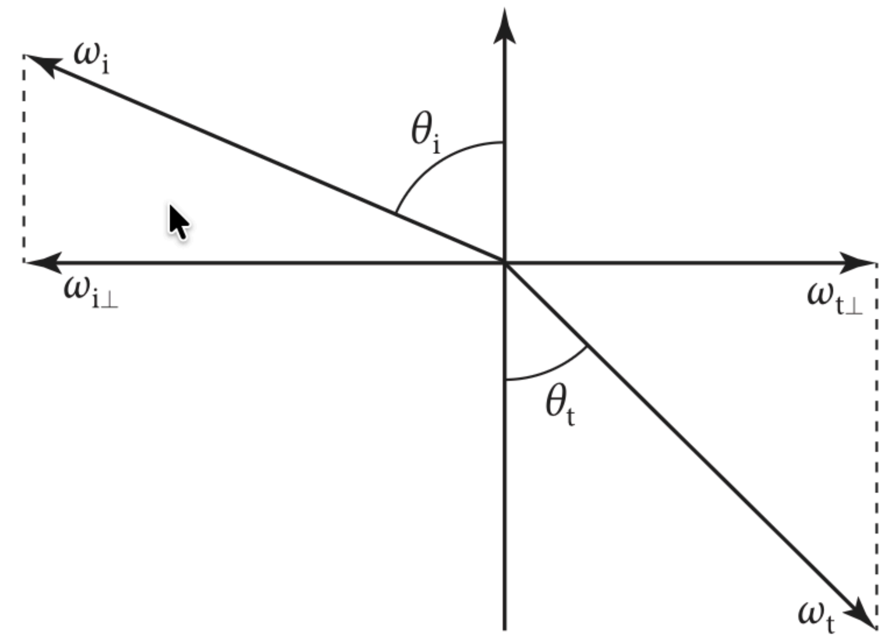
Refraction at boundary of media



Snell's law



PBR book

$$\eta_i \sin\theta_i = \eta_t \sin\theta_t$$
$$\sin\theta_i = \eta \sin\theta_t$$
$$\omega_{i,\perp} = \eta\,\omega_{t,\perp}$$

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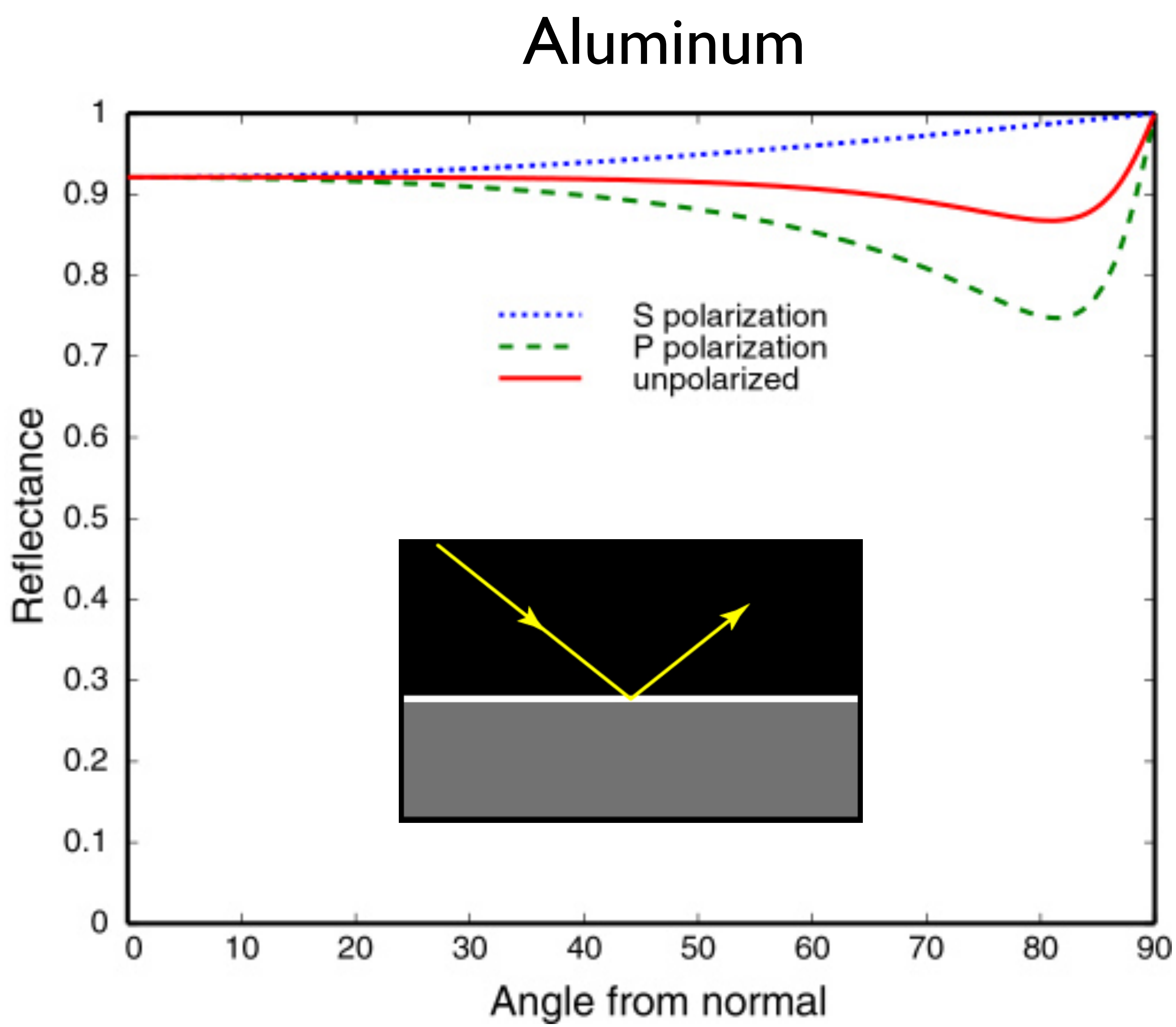
Specular reflection from metal

- **Reflectance does depend on angle**
  - but not much
  - reflectance is wavelength-dependent, strongly so in metals like copper or gold



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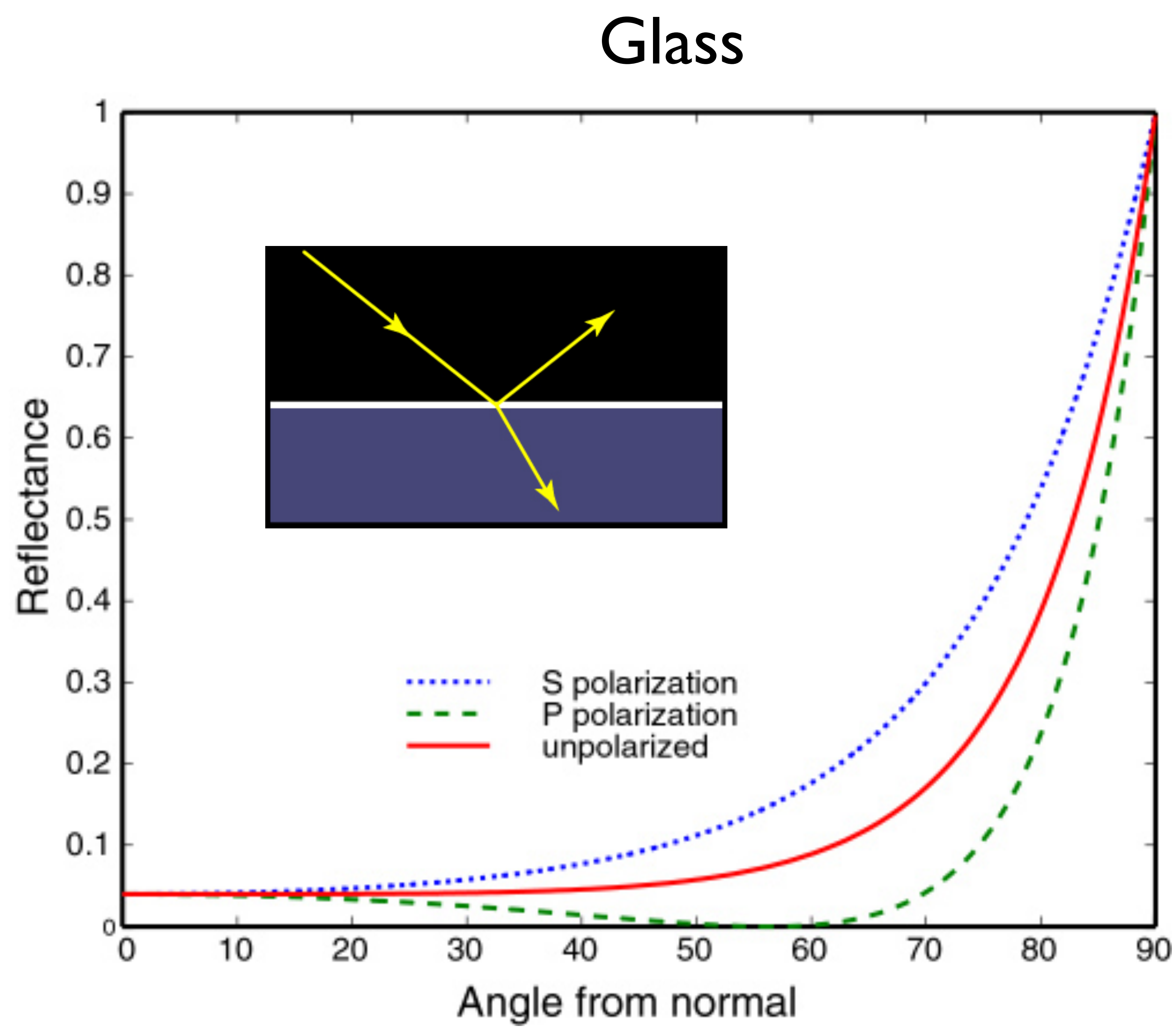
Specular reflection from glass/water

- **Dependence on angle is dramatic!**
  - about 4% at normal incidence
  - always 100% at grazing
  - remaining light is transmitted
- **This is important for proper appearance**



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Fresnel's formulas

- **They predict how much light reflects from a smooth interface between two materials**

$$F_p = \frac{\eta_2 \cos\theta_1 - \eta_1 \cos\theta_2}{\eta_2 \cos\theta_1 + \eta_1 \cos\theta_2}$$

$$F_s = \frac{\eta_1 \cos\theta_1 - \eta_2 \cos\theta_2}{\eta_1 \cos\theta_1 + \eta_2 \cos\theta_2}$$

$$R = \frac{1}{2}\left(F_p^2 + F_s^2\right)$$

where

$$\eta_1 \sin\theta_1 = \eta_2 \sin\theta_2$$

  - R is the fraction that is reflected
  - $(1 - R)$ is the fraction that is transmitted
  - for conductors, same equations, but compex-valued η

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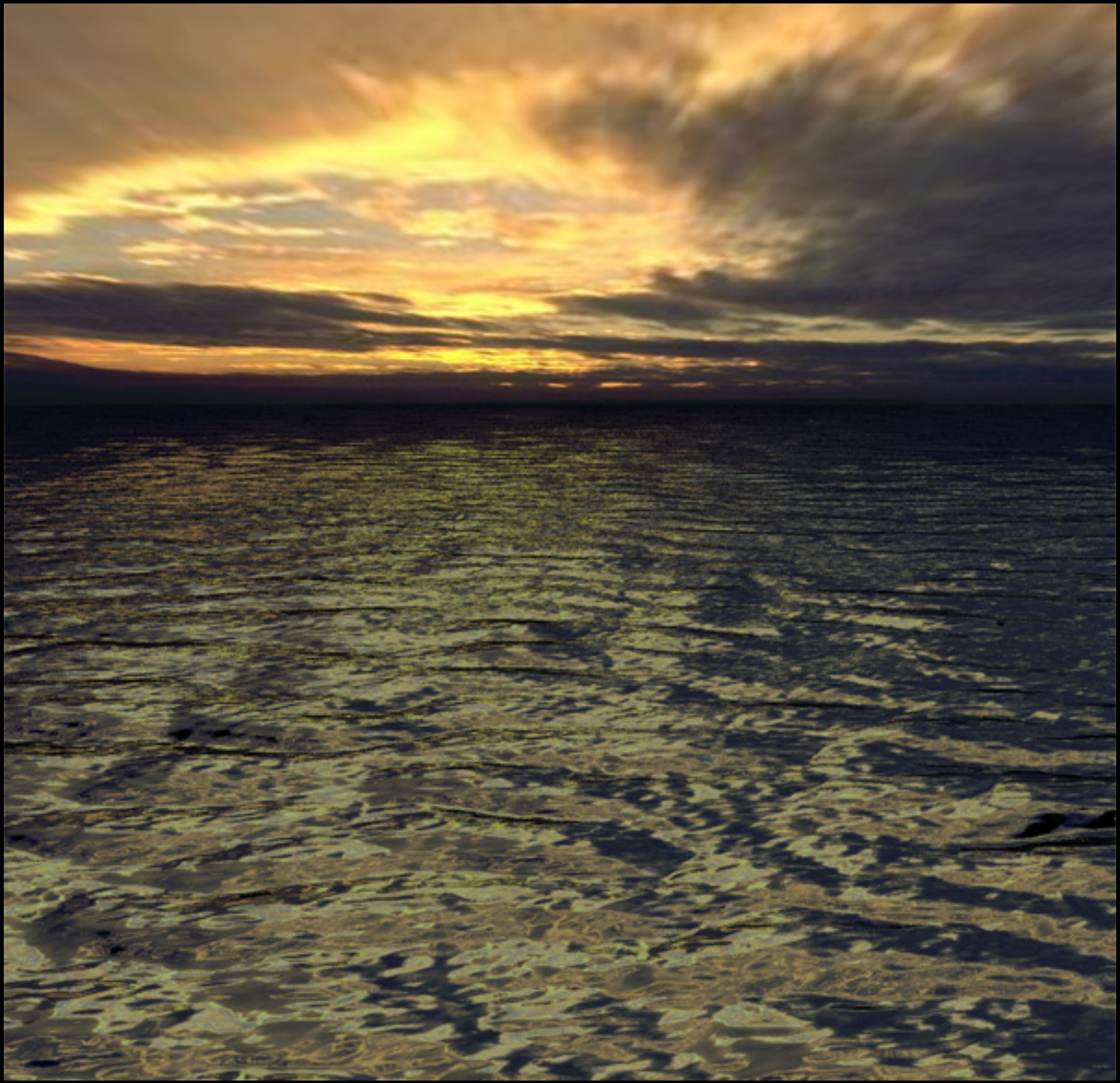
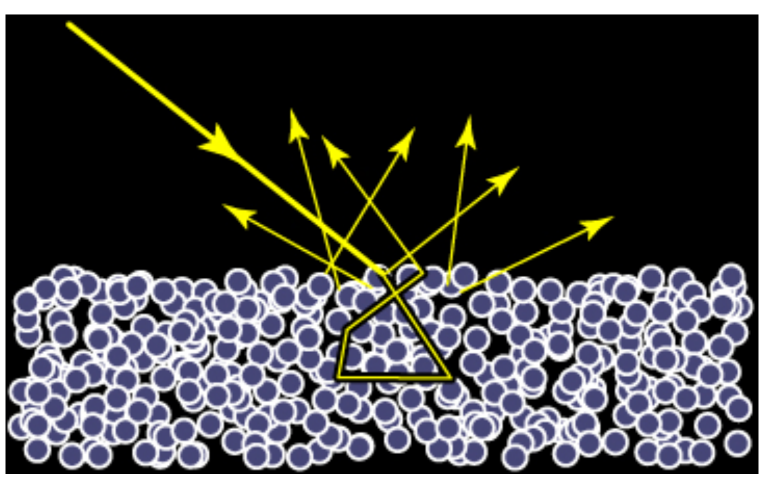
Fresnel reflection

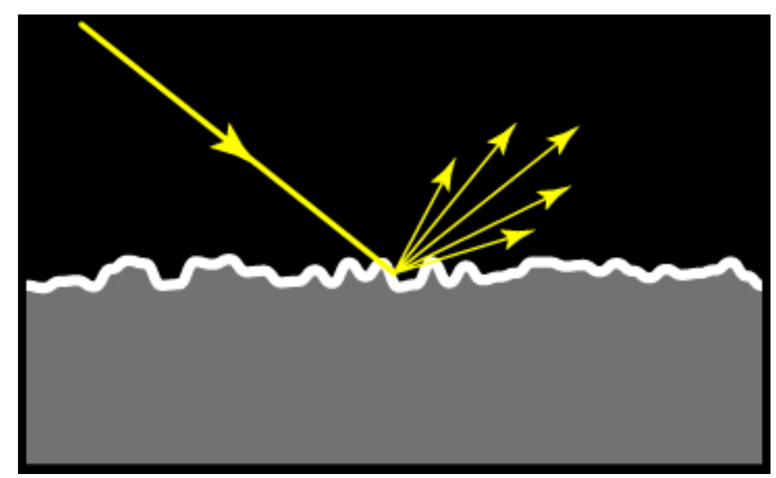


[Mike Hill & Gaain Kwan | Stanford cs348 competition 2001]

Origins of specular and diffuse



diffuse



specular

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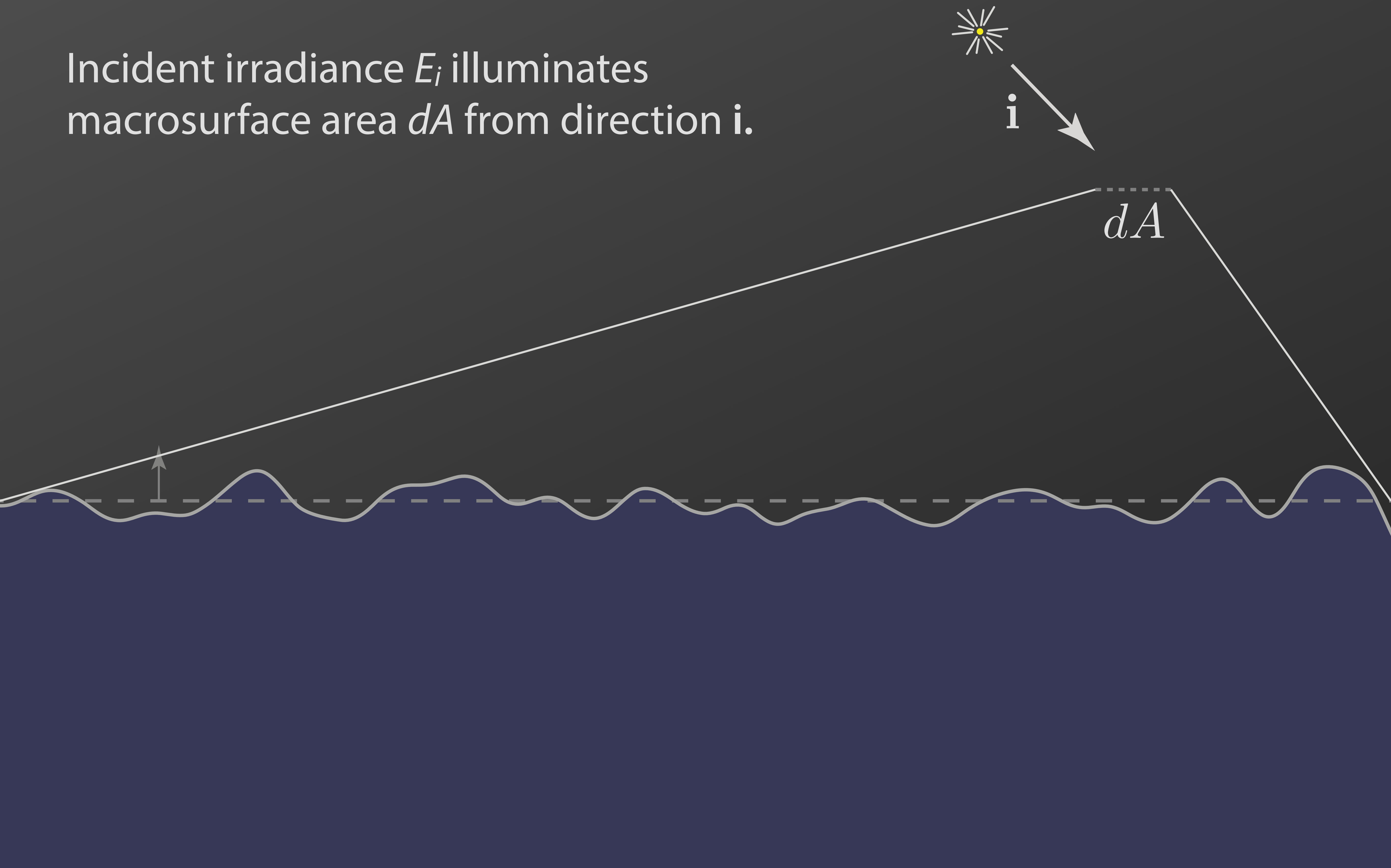
Microfacet scattering models

Rough dielectric surface

- smooth at wavelength scale
- rough at microscale
- flat at macroscale

Microfacet scattering models

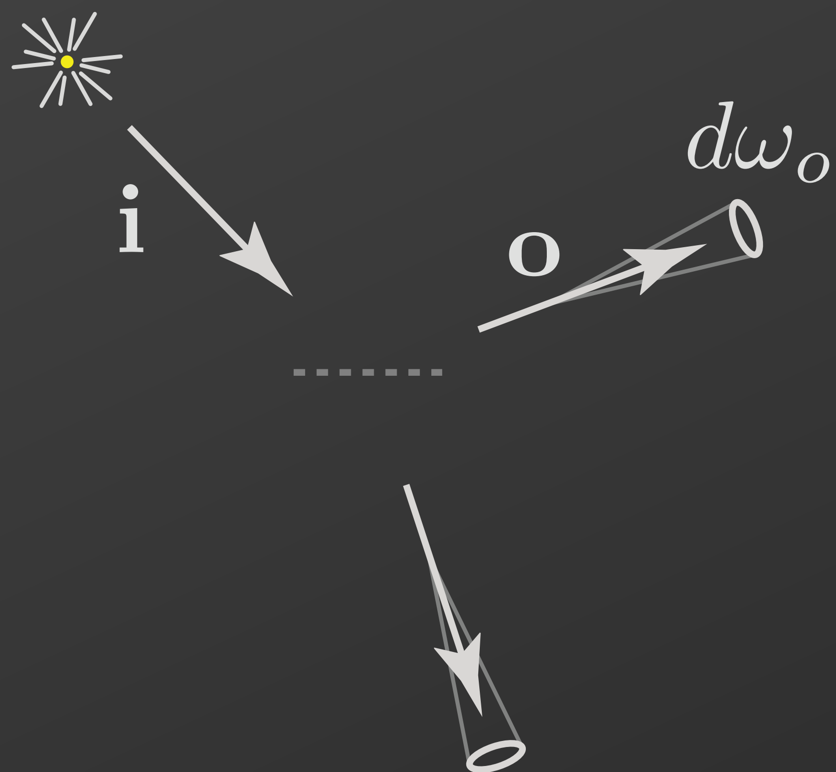
Incident irradiance E_i illuminates macrosurface area dA from direction $\mathbf{i}$.

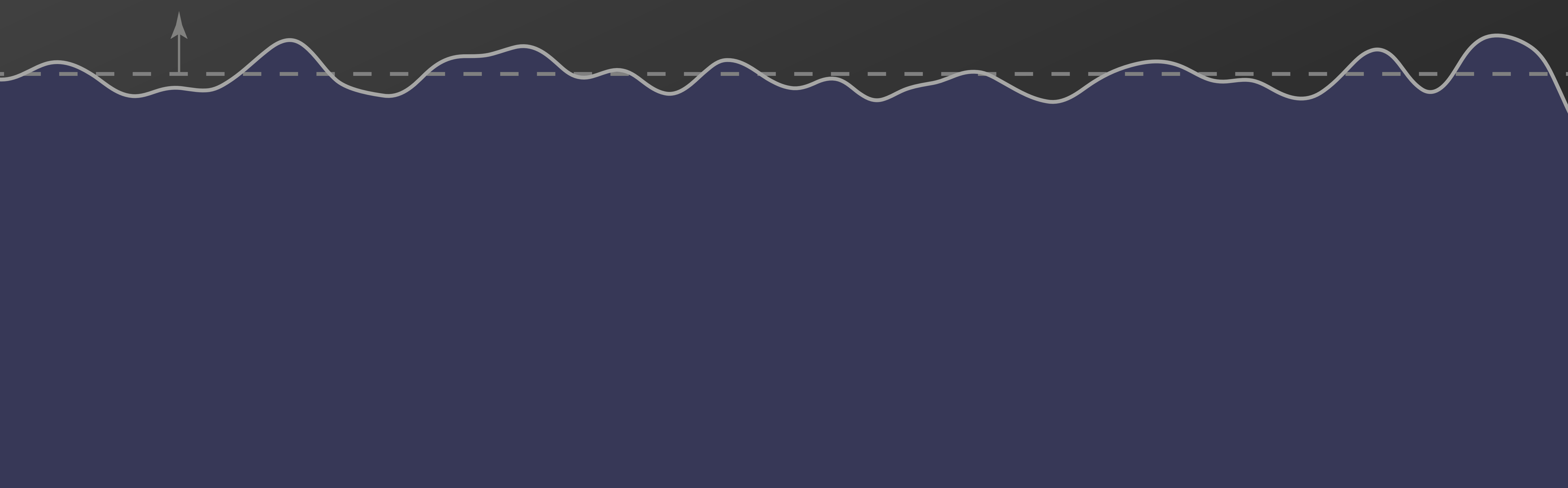


Microfacet scattering models

Incident irradiance E_i illuminates macrosurface area dA from direction **i**.

Scattered radiance L_r or L_t measured in direction **o** in solid angle $d\omega_o$.





Microfacet scattering models

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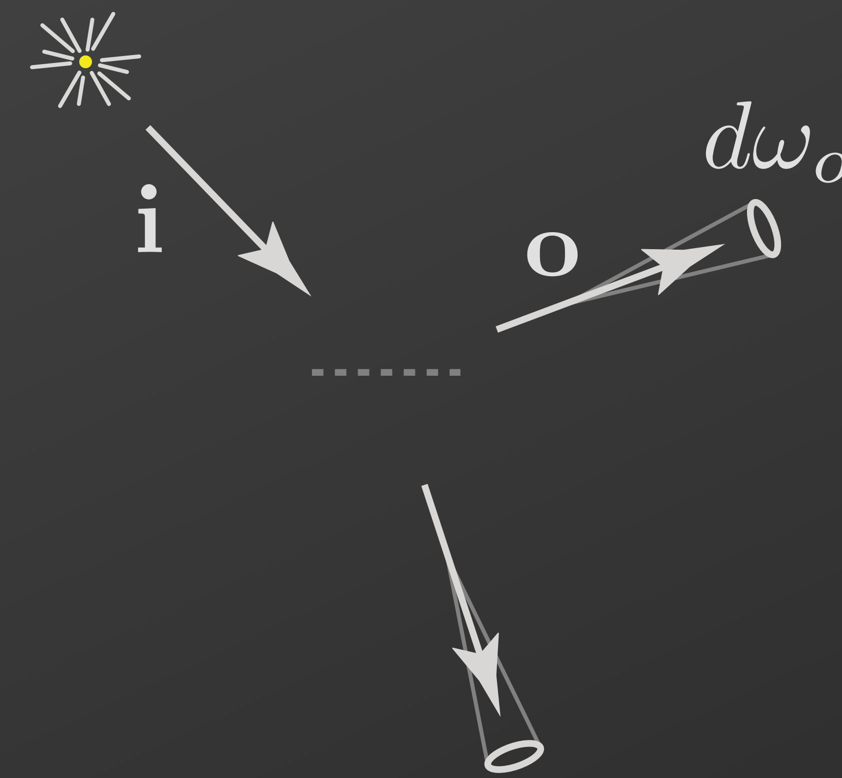
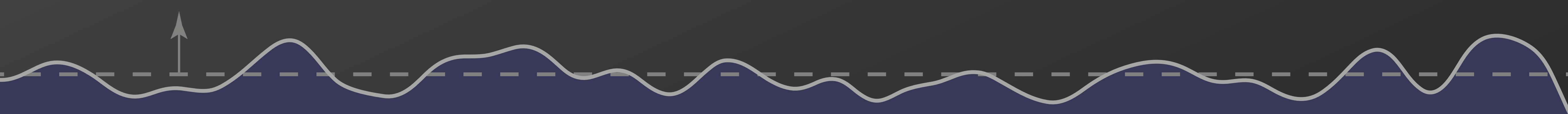




$$f_s(\mathbf{i}, \mathbf{o}) = \frac{L_{r,t}}{E_i}$$

Bidirectional **S**cattering **D**istribution **F**unction

“half-vector” function

$\mathbf{h}(\mathbf{i}, \mathbf{o})$

normal distribution

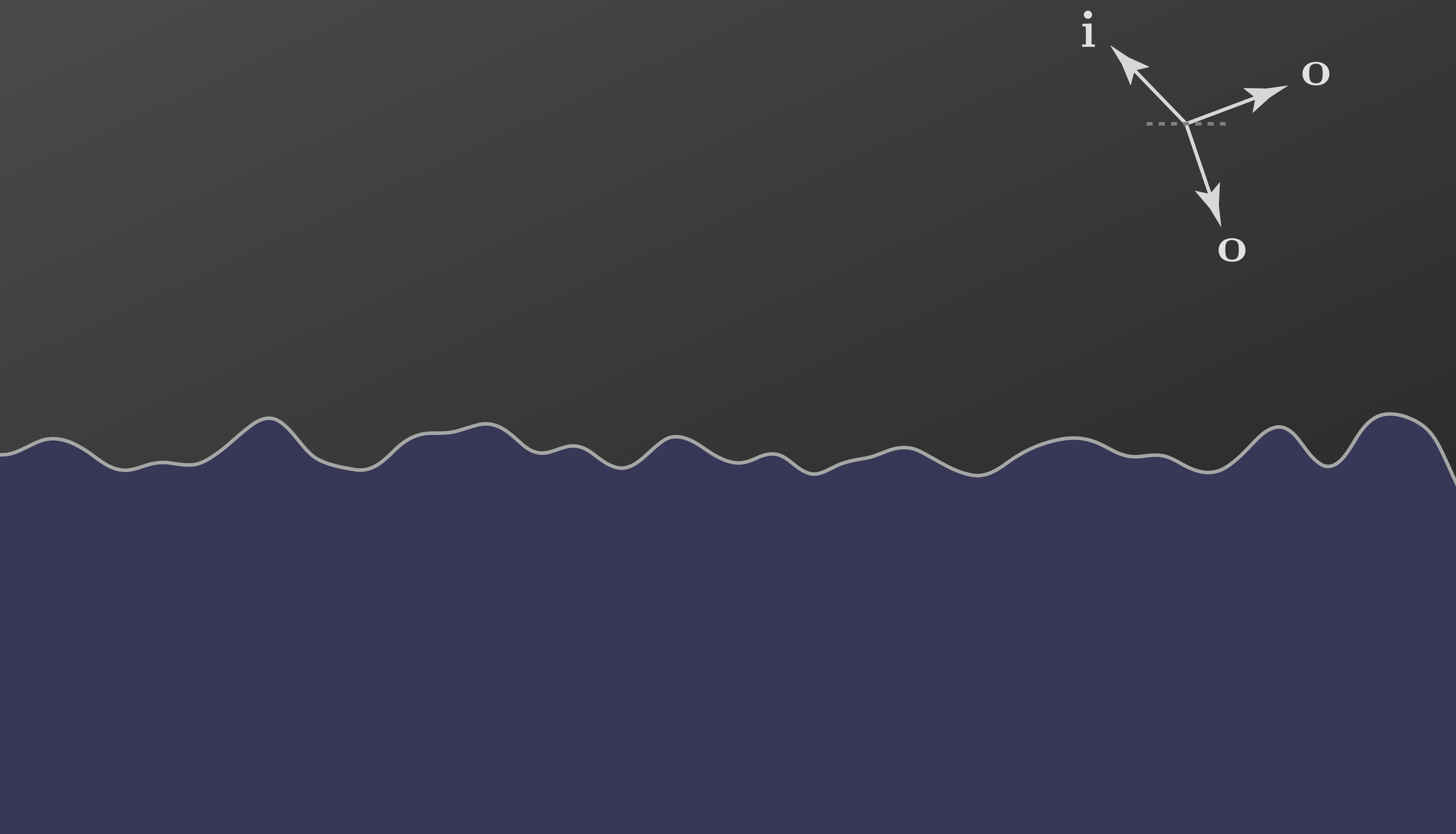
$D(\mathbf{m})$

shadowing–masking

$G(\mathbf{i}, \mathbf{o}, \mathbf{m})$

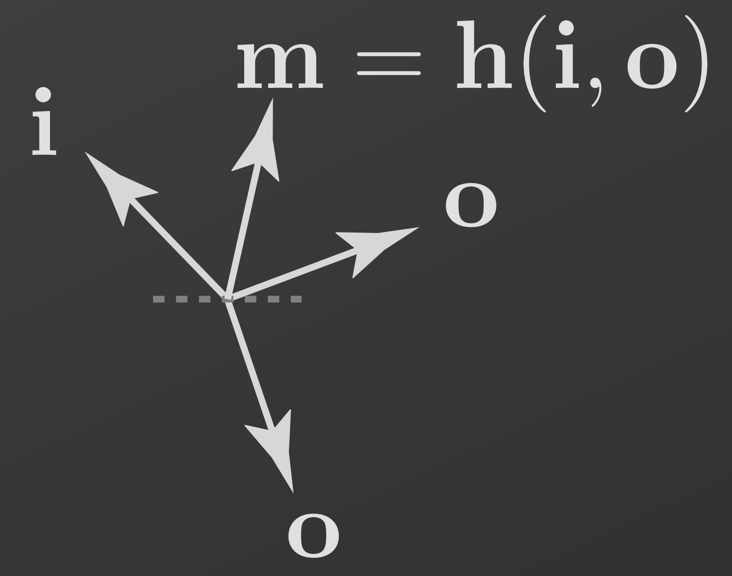
attenuation

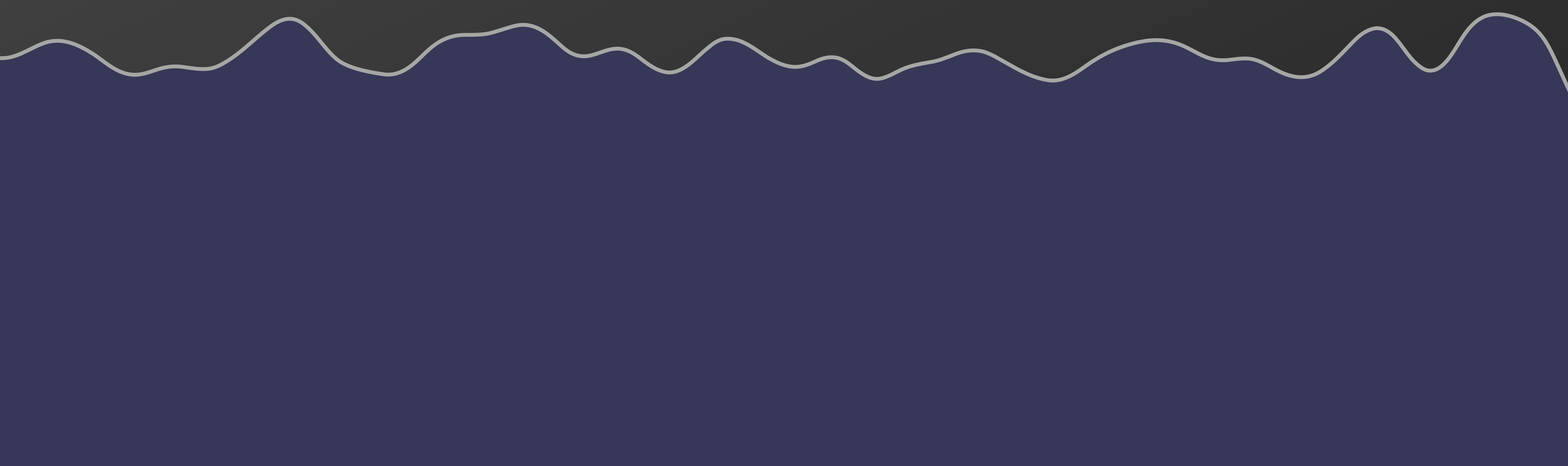
$\rho(\mathbf{i}, \mathbf{o})$



"half-vector" function	normal distribution	shadowing–masking	attenuation
$\mathbf{h}(\mathbf{i}, \mathbf{o})$	$D(\mathbf{m})$	$G(\mathbf{i}, \mathbf{o}, \mathbf{m})$	$\rho(\mathbf{i}, \mathbf{o})$

Gives the one microsurface normal **m** that will scatter light from **i** to **o**.





"half-vector" function $\mathbf{h}(\mathbf{i}, \mathbf{o})$

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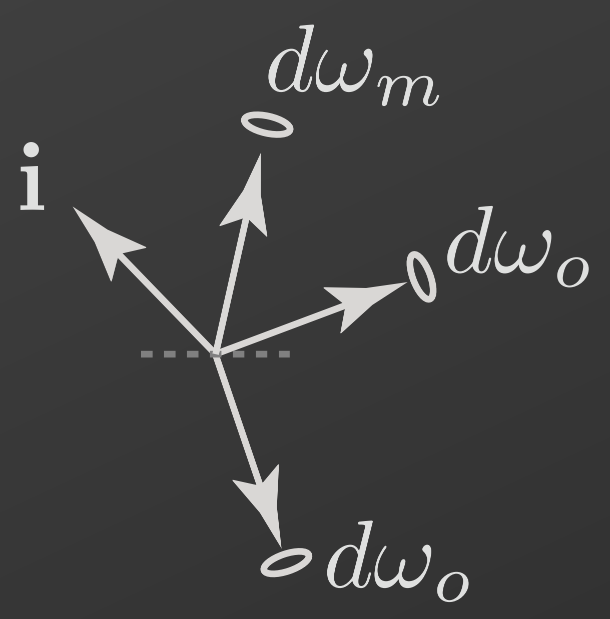
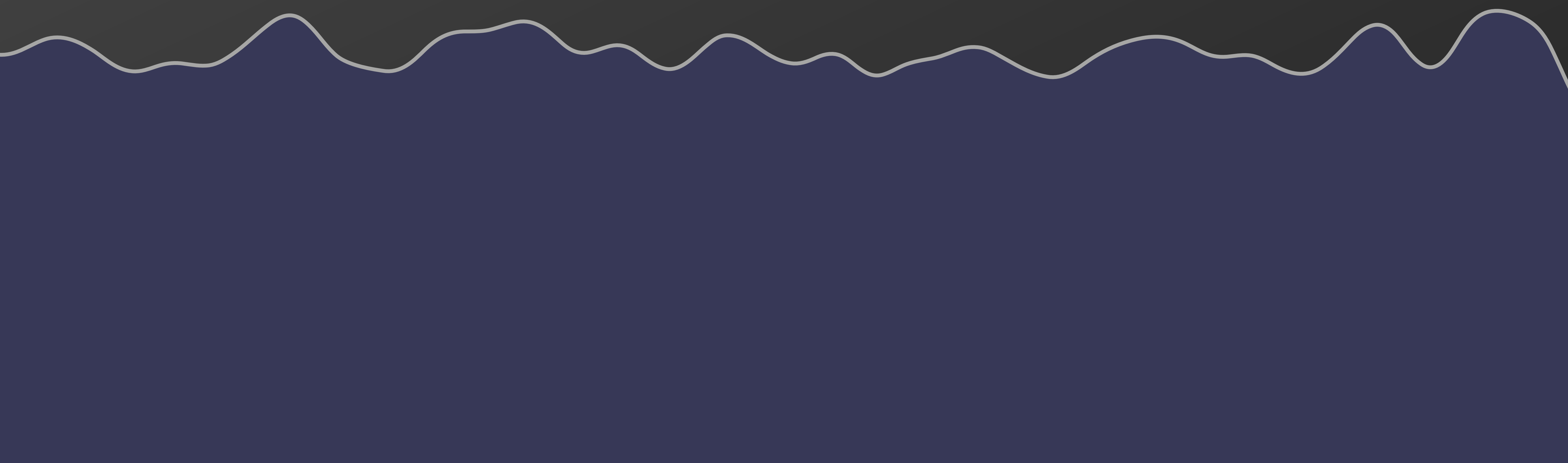
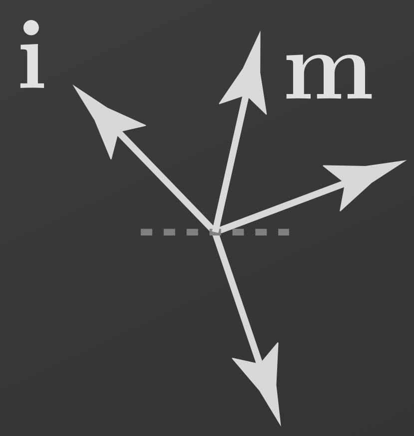


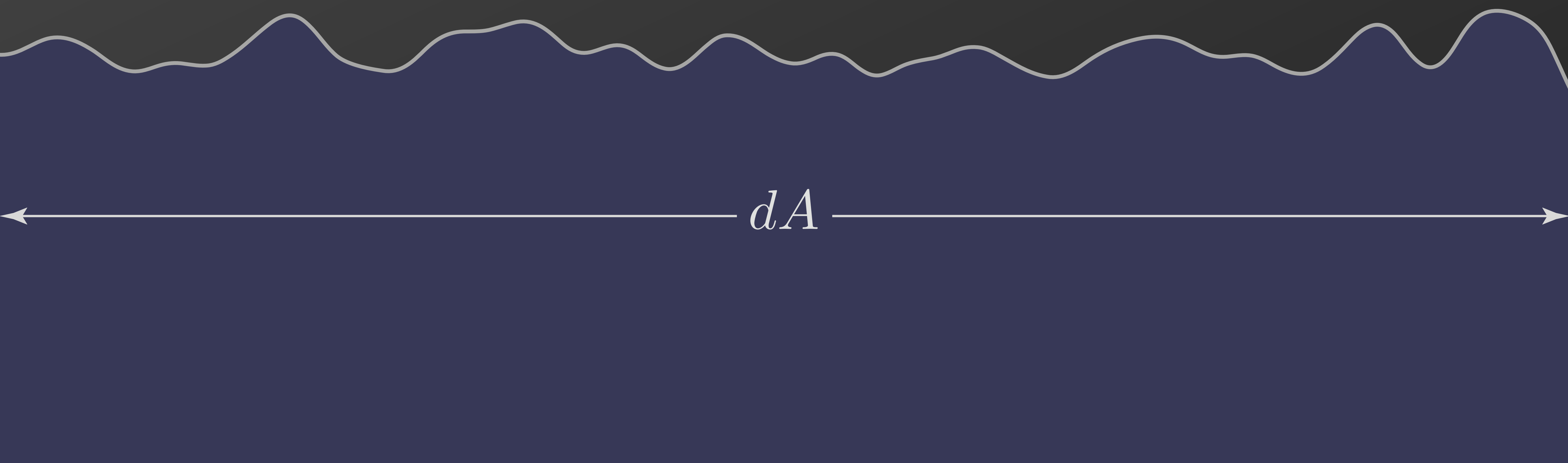
The size of the set of relevant normals $d\omega_m$ relative to the receiving solid angle $d\omega_o$ is determined by **h**.



"half-vector" function	normal distribution	shadowing–masking	attenuation
$\mathbf{h}(\mathbf{i}, \mathbf{o})$	$D(\mathbf{m})$	$G(\mathbf{i}, \mathbf{o}, \mathbf{m})$	$\rho(\mathbf{i}, \mathbf{o})$

Measures density of microsurface area with respect to microsurface normal.





“half-vector” function $\mathbf{h}(\mathbf{i},\mathbf{o})$

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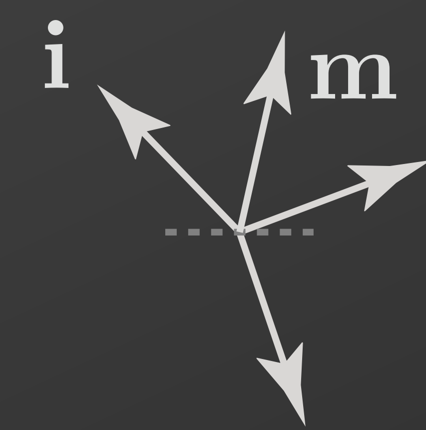

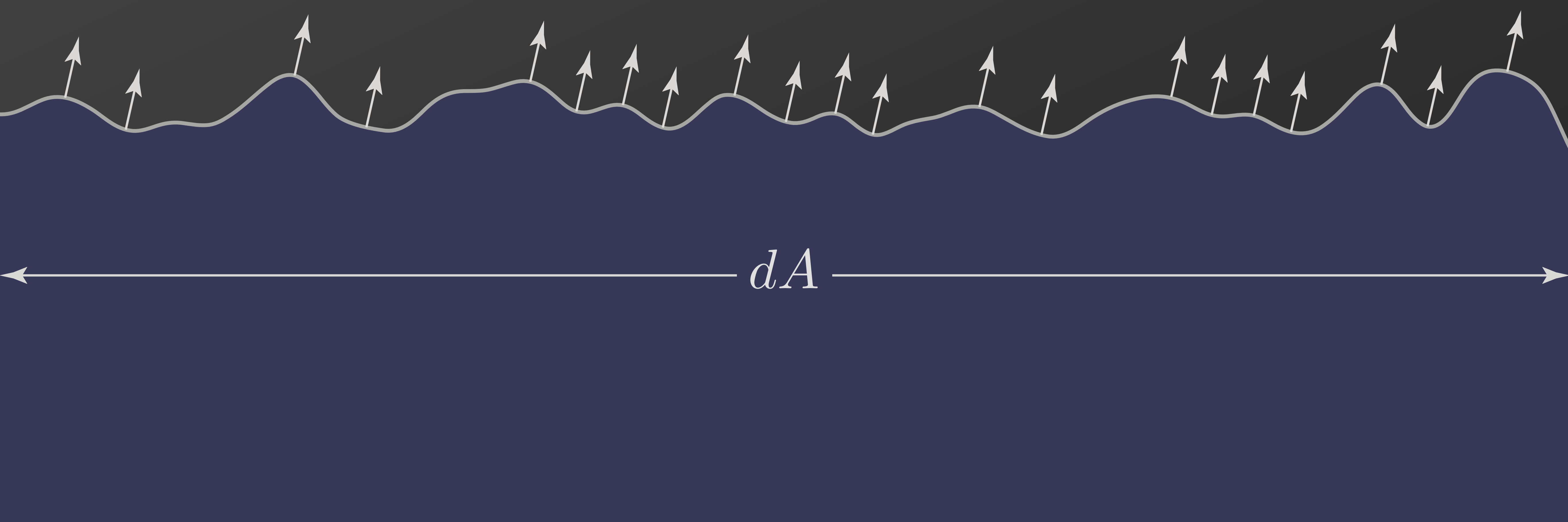

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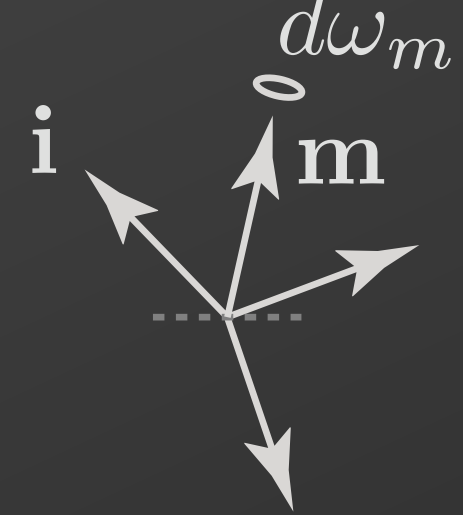
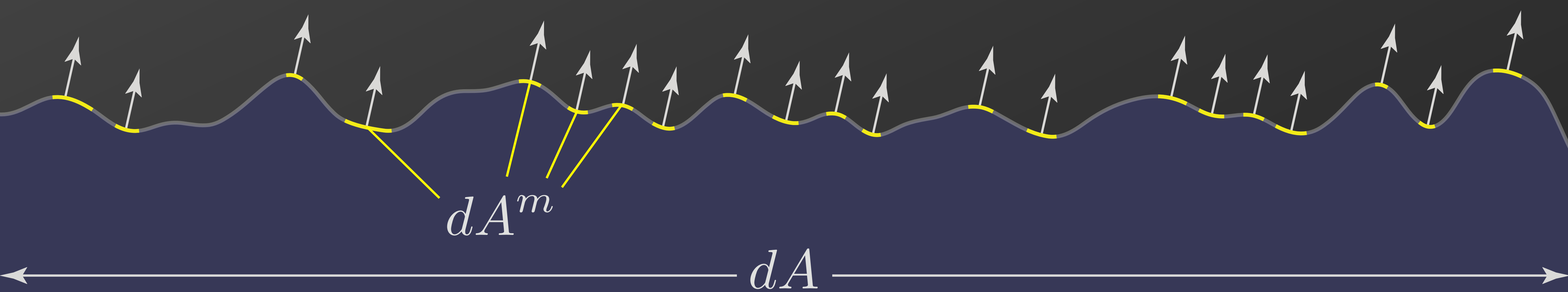


The ratio of relevant microsurface area dA^m to macrosurface area dA is $D(\mathbf{m})d\omega_m$.



$$dA^m = D(\mathbf{m})\, d\omega_m\, dA$$

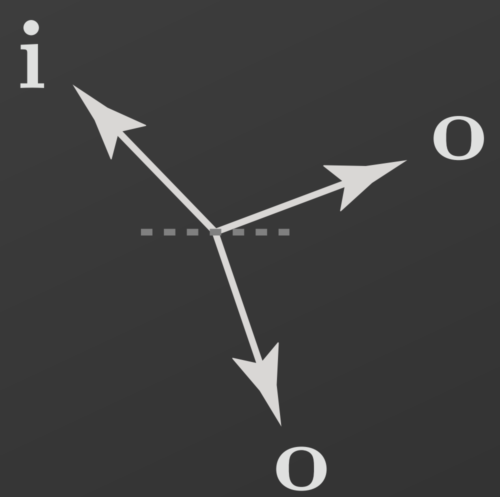
“half-vector” function $\mathbf{h}(\mathbf{i},\mathbf{o})$

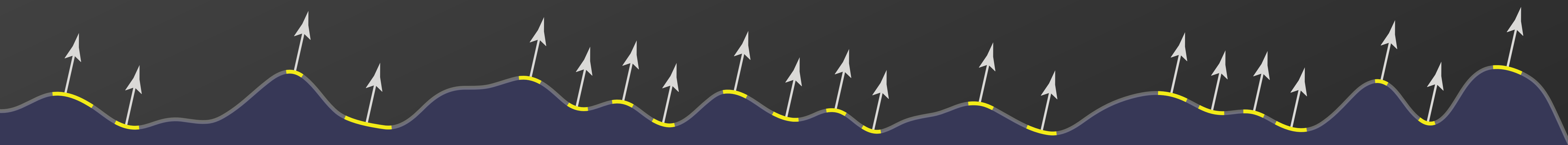
normal distribution $D(\mathbf{m})$

shadowing–masking $G(\mathbf{i},\mathbf{o},\mathbf{m})$

attenuation $\rho(\mathbf{i},\mathbf{o})$

Measures the fraction of points with microsurface normal **m** that are visible in directions **i** and **o**.





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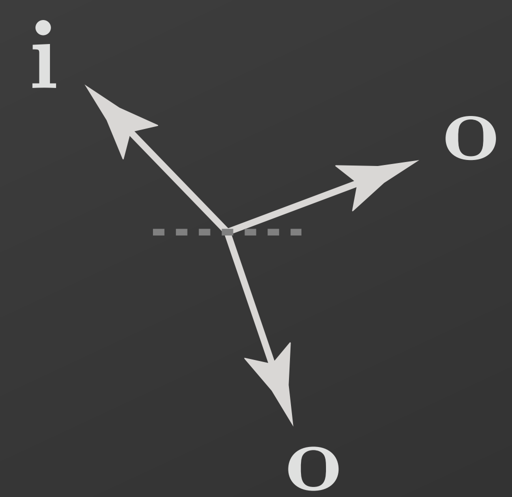

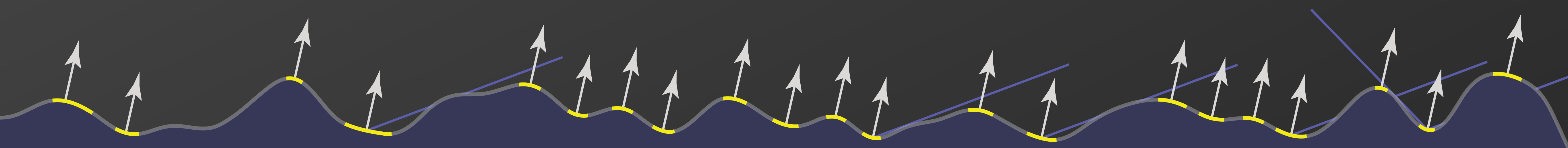

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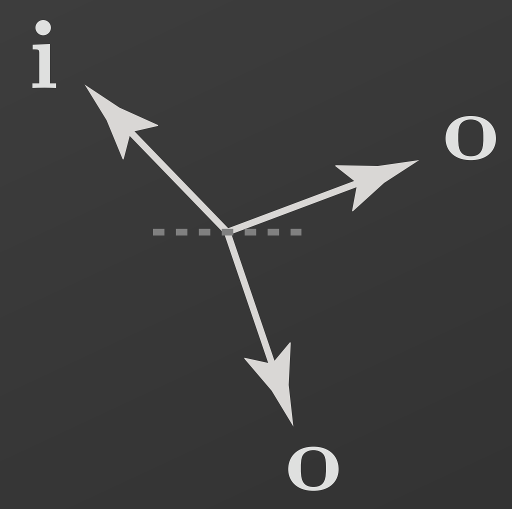

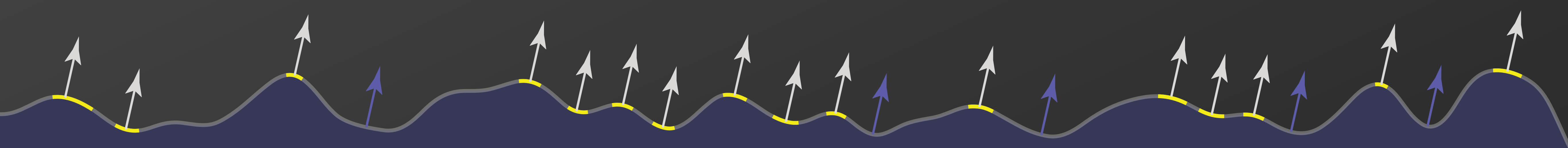

$$dA^m = D(\mathbf{m})\, G(\mathbf{i}, \mathbf{o}, \mathbf{m})\, d\omega_m\, dA$$

“half-vector” function
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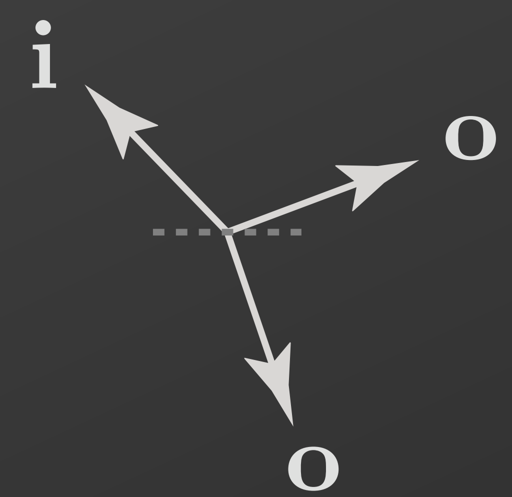
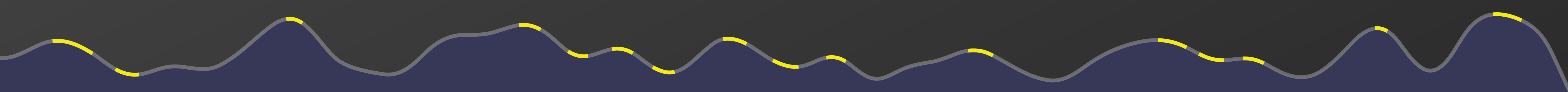


We now know the size of the scattering area, which determines how much light reflects.



$$dA^m = D(\mathbf{m})\, G(\mathbf{i}, \mathbf{o}, \mathbf{m})\, d\omega_m\, dA$$

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normal distribution
$D(\mathbf{m})$

shadowing–masking
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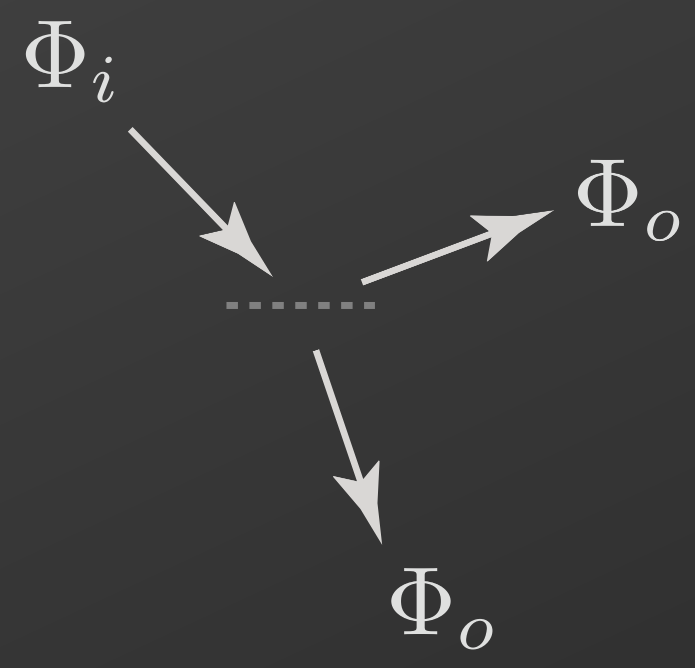
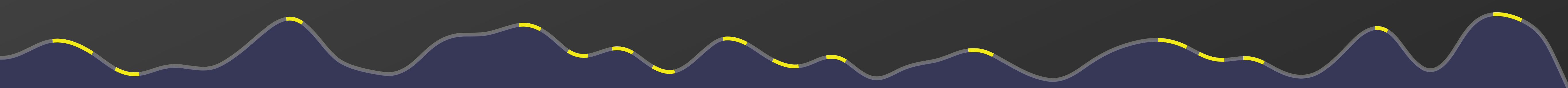
attenuation
$\rho(\mathbf{i}, \mathbf{o})$

Gives the fraction of the power incident on the scattering area dA^m that is scattered.





$$d\Phi_o^m = \frac{|\mathbf{i} \cdot \mathbf{m}|}{|\mathbf{i} \cdot \mathbf{n}|} \rho(\mathbf{i}, \mathbf{o})\, dA^m dE_i$$

$$dA^m = D(\mathbf{m})\, G(\mathbf{i}, \mathbf{o}, \mathbf{m})\, d\omega_m\, dA$$

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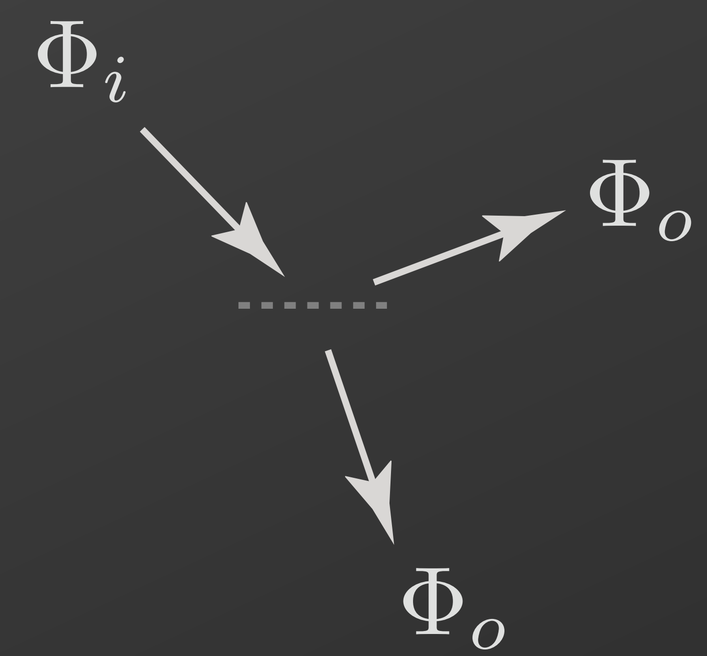
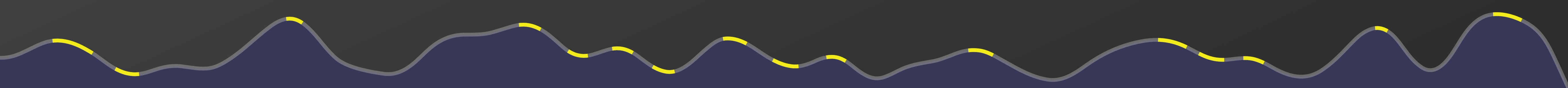
attenuation
$\rho(\mathbf{i}, \mathbf{o})$

Gives the fraction of the power incident on the scattering area dA^m that is scattered.

This scattered power is related to the incident irradiance by the attenuation and the scattering area, projected in the incident direction.





$$d\Phi_o^m = \frac{|\mathbf{i} \cdot \mathbf{m}|}{|\mathbf{i} \cdot \mathbf{n}|} \rho(\mathbf{i}, \mathbf{o}) \, dA^m dE_i$$

$$dA^m = D(\mathbf{m}) \, G(\mathbf{i}, \mathbf{o}, \mathbf{m}) \, d\omega_m \, dA$$

“half-vector” function $\mathbf{h}(\mathbf{i}, \mathbf{o})$

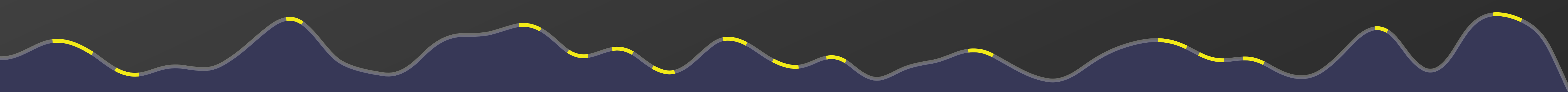
normal distribution $D(\mathbf{m})$

shadowing–masking $G(\mathbf{i}, \mathbf{o}, \mathbf{m})$

attenuation $\rho(\mathbf{i}, \mathbf{o})$

The BSDF is the ratio of scattered radiance to incident irradiance:

$$f_s(\mathbf{i}, \mathbf{o}) = \frac{dL_o}{dE_i} = \frac{d\Phi_o^m / (dA\, |\mathbf{o} \cdot \mathbf{n}|\, d\omega_o)}{dE_i}$$



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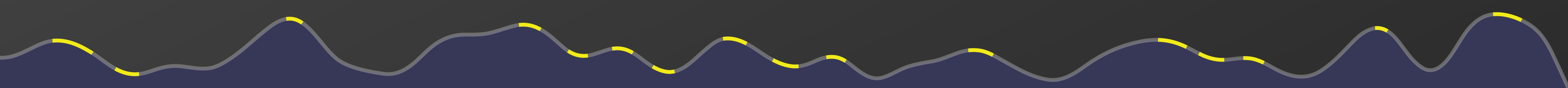


$$d\Phi_o^m = \frac{|\mathbf{i}\cdot\mathbf{m}|}{|\mathbf{i}\cdot\mathbf{n}|}\rho(\mathbf{i},\mathbf{o})\,dA^m dE_i$$

$$dA^m = D(\mathbf{m})\,G(\mathbf{i},\mathbf{o},\mathbf{m})\,d\omega_m\,dA$$

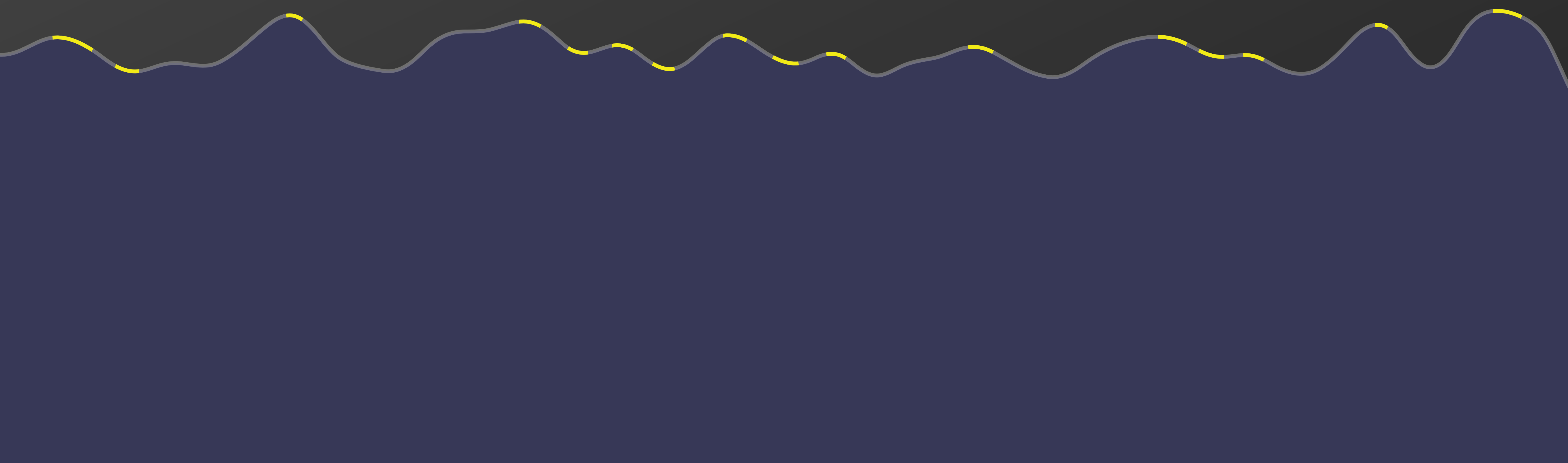
"half-vector" function $\mathbf{h}(\mathbf{i}, \mathbf{o})$

normal distribution $D(\mathbf{m})$

shadowing–masking $G(\mathbf{i}, \mathbf{o}, \mathbf{m})$

attenuation $\rho(\mathbf{i}, \mathbf{o})$

$$f_s(\mathbf{i}, \mathbf{o}) = \frac{|\mathbf{i} \cdot \mathbf{m}|}{|\mathbf{i} \cdot \mathbf{n}| \, |\mathbf{o} \cdot \mathbf{n}|} \, \rho(\mathbf{i}, \mathbf{o}) \, D(\mathbf{m}) \, G(\mathbf{i}, \mathbf{o}, \mathbf{m}) \, \frac{d\omega_m}{d\omega_o}$$



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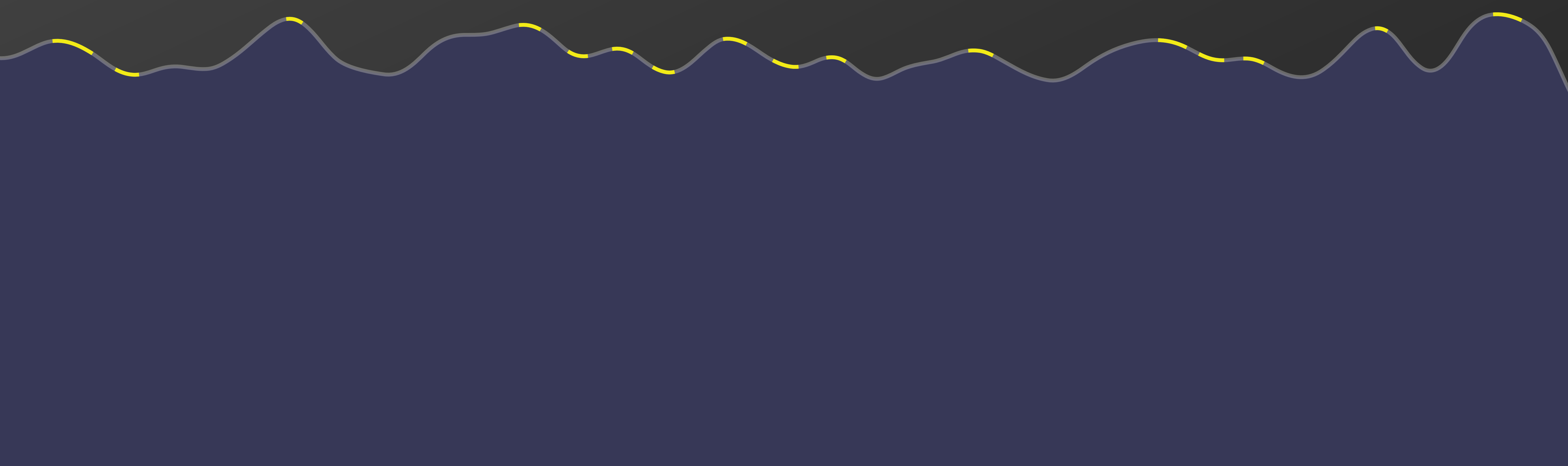
attenuation

$\rho(\mathbf{i}, \mathbf{o})$

Fresnel reflection

$$f_s(\mathbf{i}, \mathbf{o}) = \frac{|\mathbf{i} \cdot \mathbf{m}|}{|\mathbf{i} \cdot \mathbf{n}|\, |\mathbf{o} \cdot \mathbf{n}|} \rho(\mathbf{i}, \mathbf{o})\, D(\mathbf{m})\, G(\mathbf{i}, \mathbf{o}, \mathbf{m}) \frac{d\omega_m}{d\omega_o}$$



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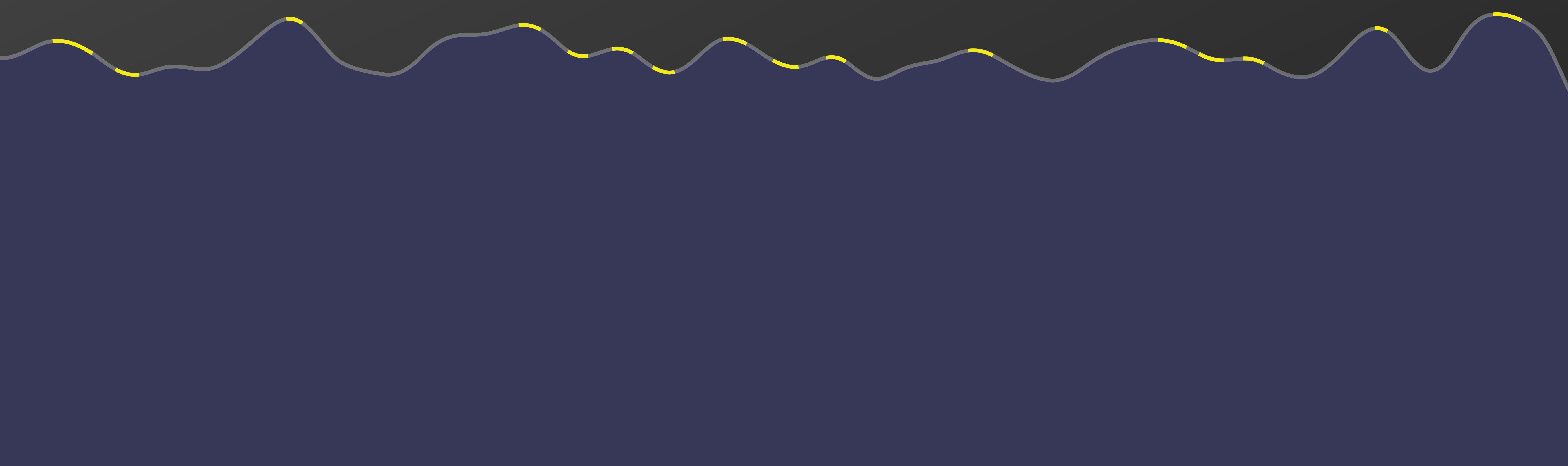
Fresnel reflection | surface roughness

$$f_s(\mathbf{i},\mathbf{o}) = \frac{|\mathbf{i}\cdot\mathbf{m}|}{|\mathbf{i}\cdot\mathbf{n}|\,|\mathbf{o}\cdot\mathbf{n}|}\,\rho(\mathbf{i},\mathbf{o})\,D(\mathbf{m})\,G(\mathbf{i},\mathbf{o},\mathbf{m})\,\frac{d\omega_m}{d\omega_o}$$



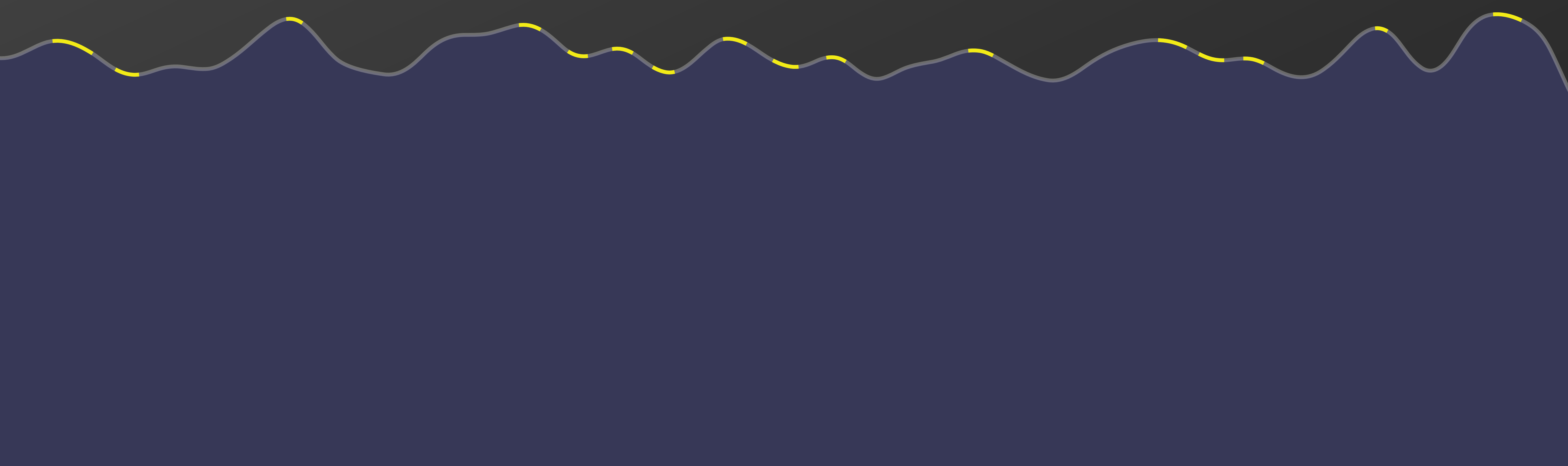
"half-vector" function	normal distribution	shadowing–masking	attenuation
$\mathbf{h}(\mathbf{i}, \mathbf{o})$	$D(\mathbf{m})$	$G(\mathbf{i}, \mathbf{o}, \mathbf{m})$	$\rho(\mathbf{i}, \mathbf{o})$

Fresnel reflection

surface roughness

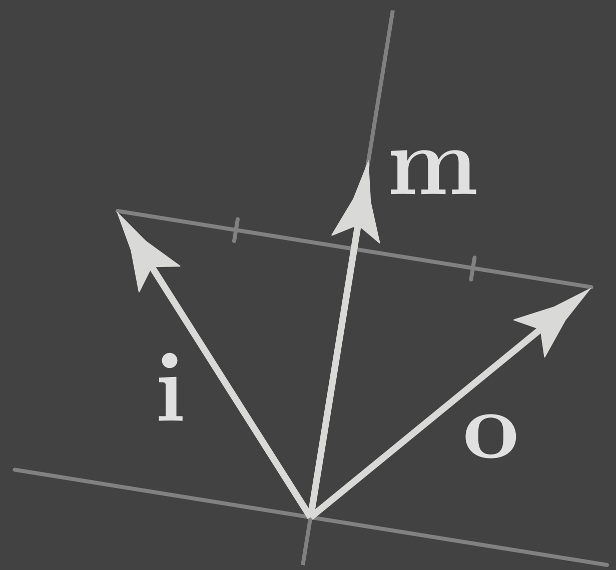
$$f_s(\mathbf{i}, \mathbf{o}) = \frac{|\mathbf{i} \cdot \mathbf{m}|}{|\mathbf{i} \cdot \mathbf{n}|\, |\mathbf{o} \cdot \mathbf{n}|} \rho(\mathbf{i}, \mathbf{o}) D(\mathbf{m}) G(\mathbf{i}, \mathbf{o}, \mathbf{m}) \frac{d\omega_m}{d\omega_o}$$

determined by geometry



Construction of half-vector

reflection



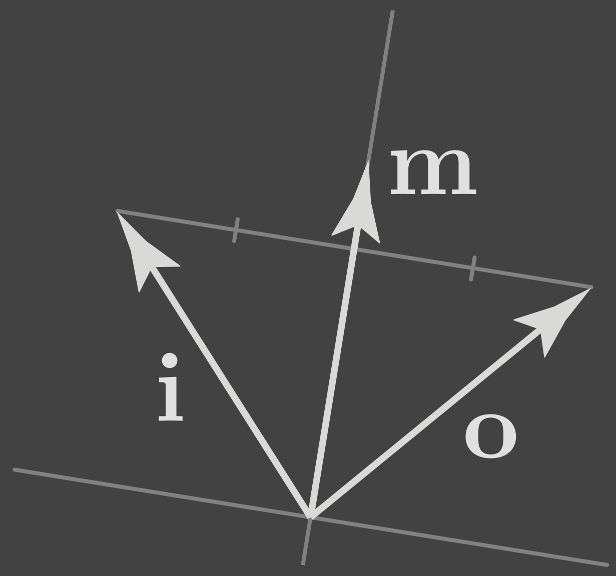
$\mathbf{i} + \mathbf{o}$ parallel to $\mathbf{m}$

refraction

Construction of half-vector

reflection

$$\mathbf{h}_r = \text{normalize}(\mathbf{i} + \mathbf{o})$$



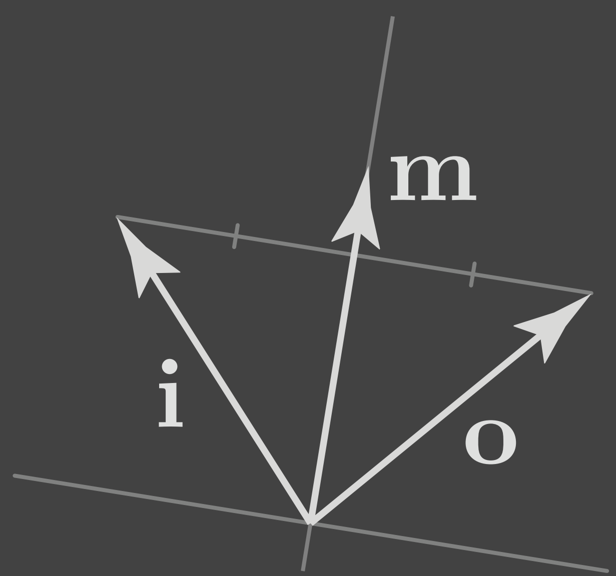
$\mathbf{i} + \mathbf{o}$ parallel to $\mathbf{m}$

refraction

Construction of half-vector

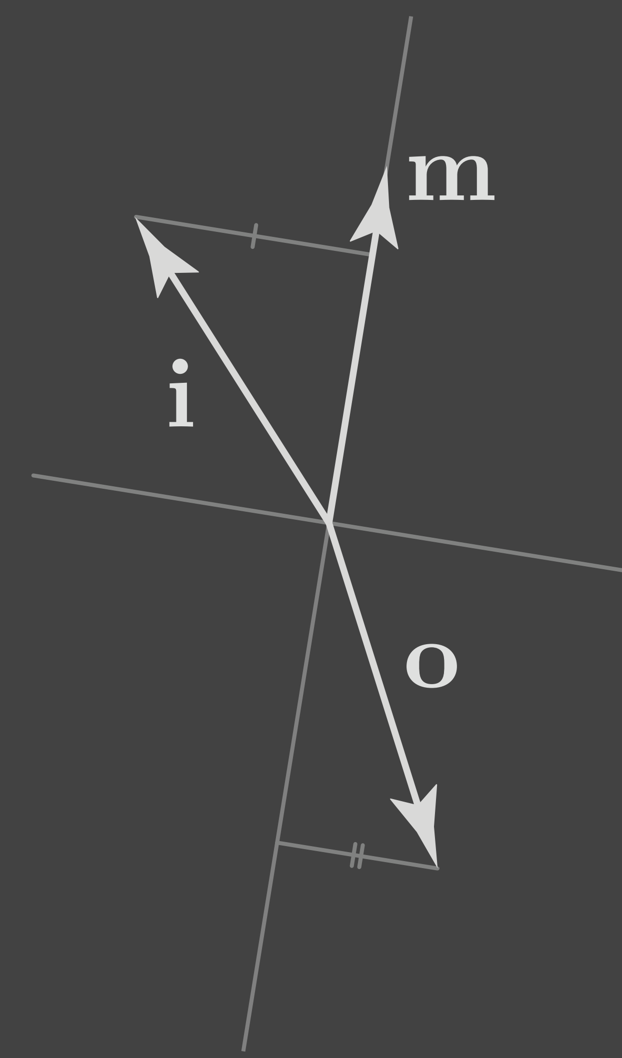
reflection

$$\mathbf{h}_r = \mathrm{normalize}(\mathbf{i} + \mathbf{o})$$



$\mathbf{i} + \mathbf{o}$ parallel to $\mathbf{m}$

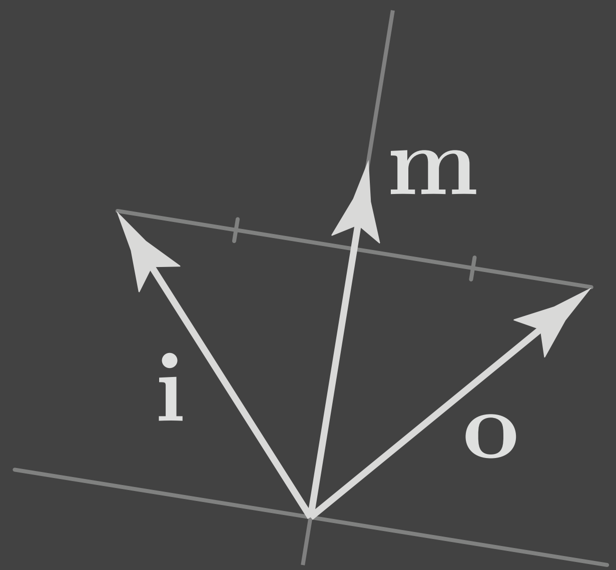
refraction



Construction of half-vector

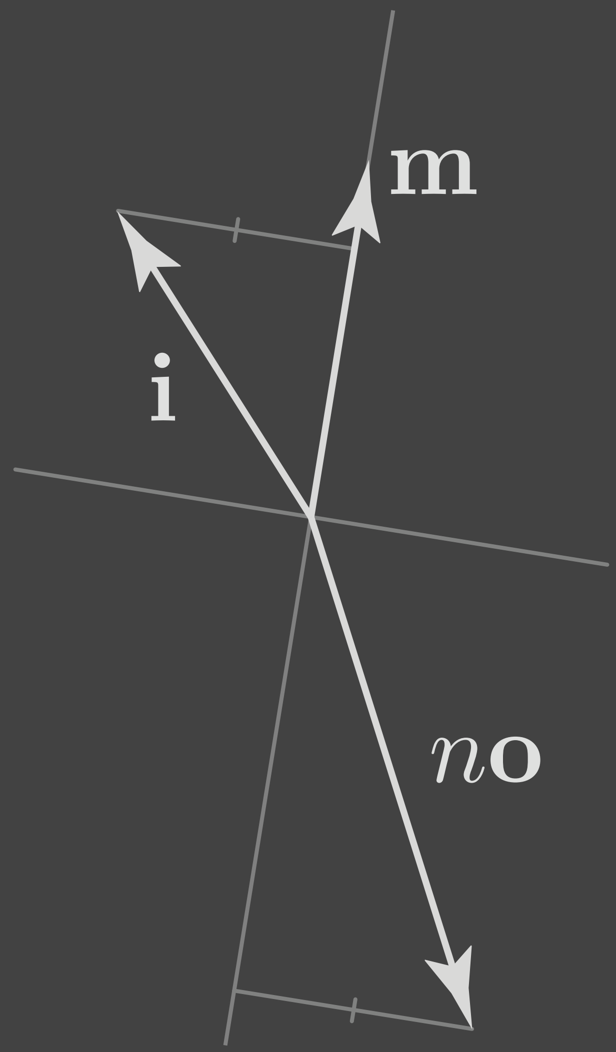
reflection

$$\mathbf{h}_r = \mathrm{normalize}(\mathbf{i} + \mathbf{o})$$



$\mathbf{i} + \mathbf{o}$ parallel to $\mathbf{m}$

refraction

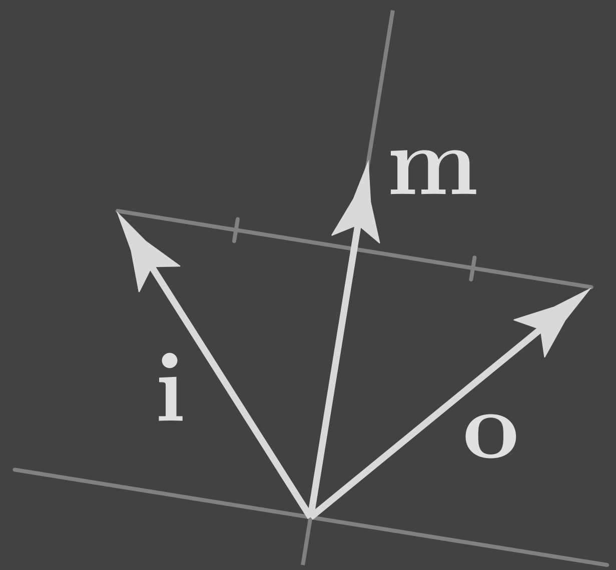


$\mathbf{i} + n\mathbf{o}$ parallel to $\mathbf{m}$

Construction of half-vector

reflection

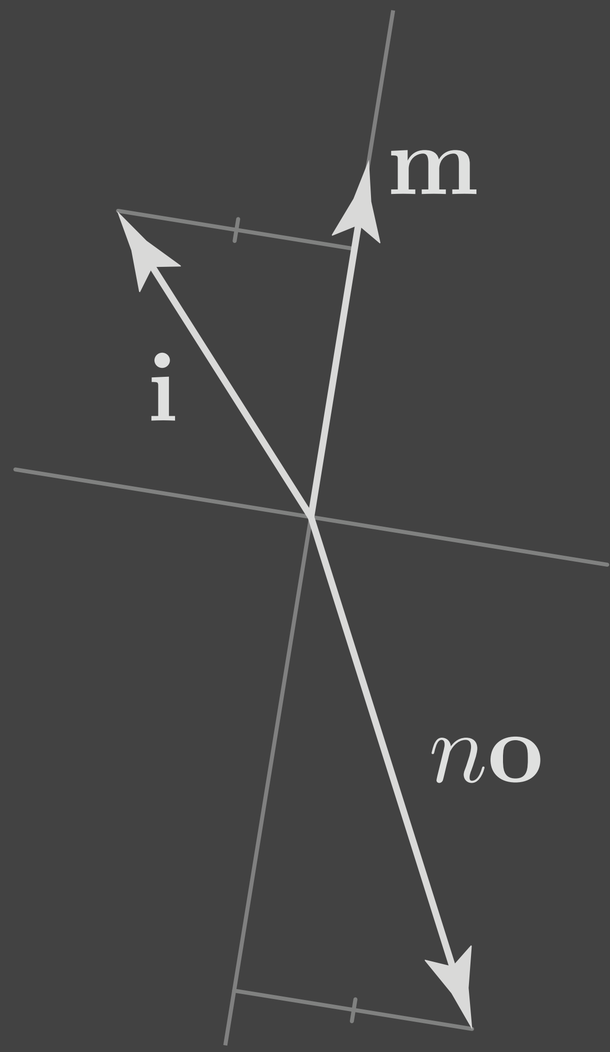
$$\mathbf{h}_r = \text{normalize}(\mathbf{i} + \mathbf{o})$$



$\mathbf{i} + \mathbf{o}$ parallel to $\mathbf{m}$

refraction

$$\mathbf{h}_t = -\text{normalize}(\mathbf{i} + n\mathbf{o})$$

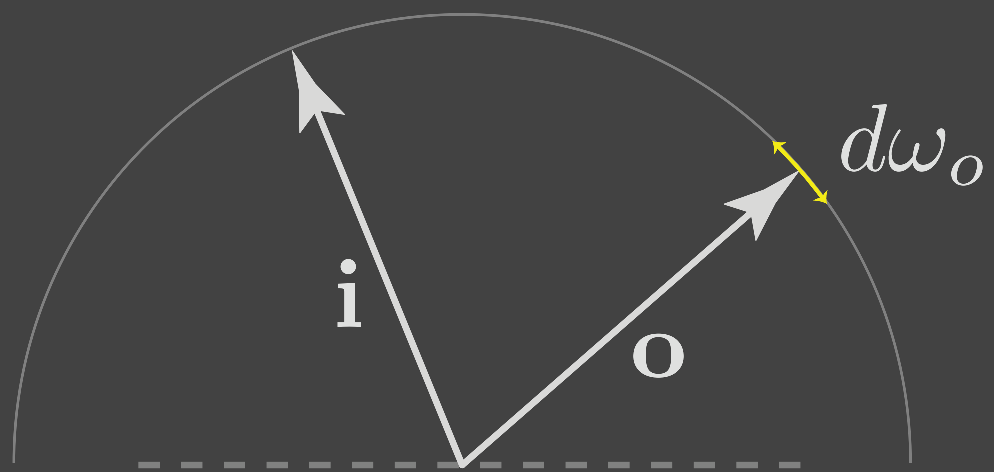


$\mathbf{i} + n\mathbf{o}$ parallel to $\mathbf{m}$

Construction of half-vector solid angle

reflection

$$\mathbf{h}_r = \mathrm{normalize}(\mathbf{i} + \mathbf{o})$$



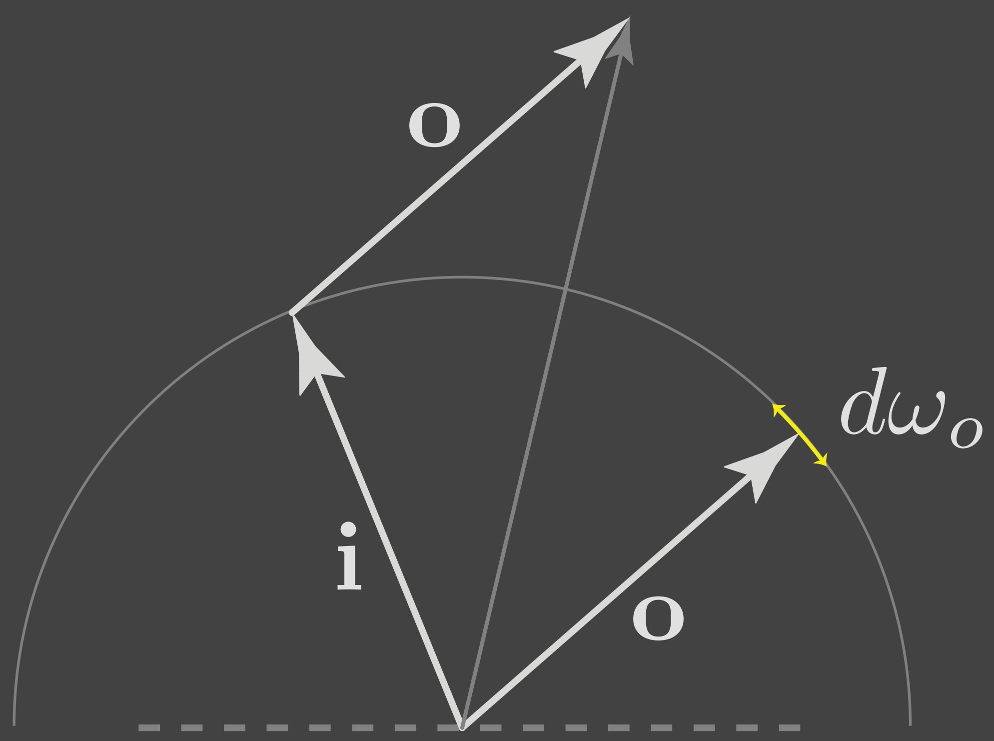
refraction

$$\mathbf{h}_t = -\mathrm{normalize}(\mathbf{i} + n\mathbf{o})$$

Construction of half-vector solid angle

reflection

$$\mathbf{h}_r = \text{normalize}(\mathbf{i} + \mathbf{o})$$



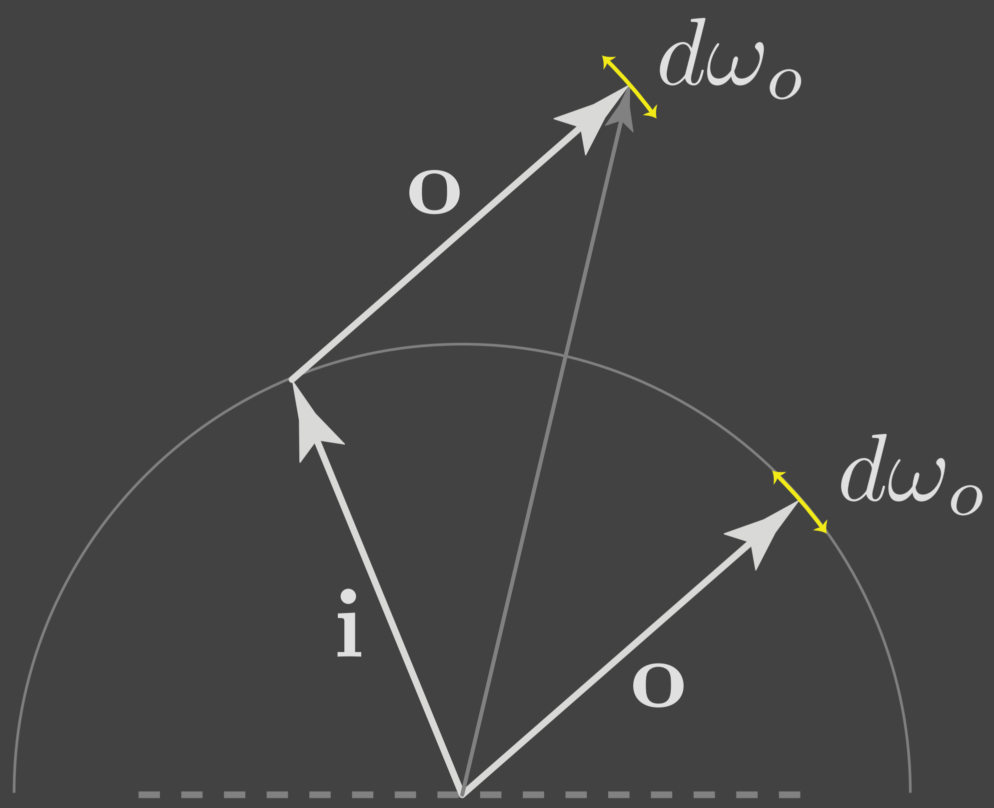
refraction

$$\mathbf{h}_t = -\text{normalize}(\mathbf{i} + n\mathbf{o})$$

Construction of half-vector solid angle

reflection

$$\mathbf{h}_r = \text{normalize}(\mathbf{i} + \mathbf{o})$$



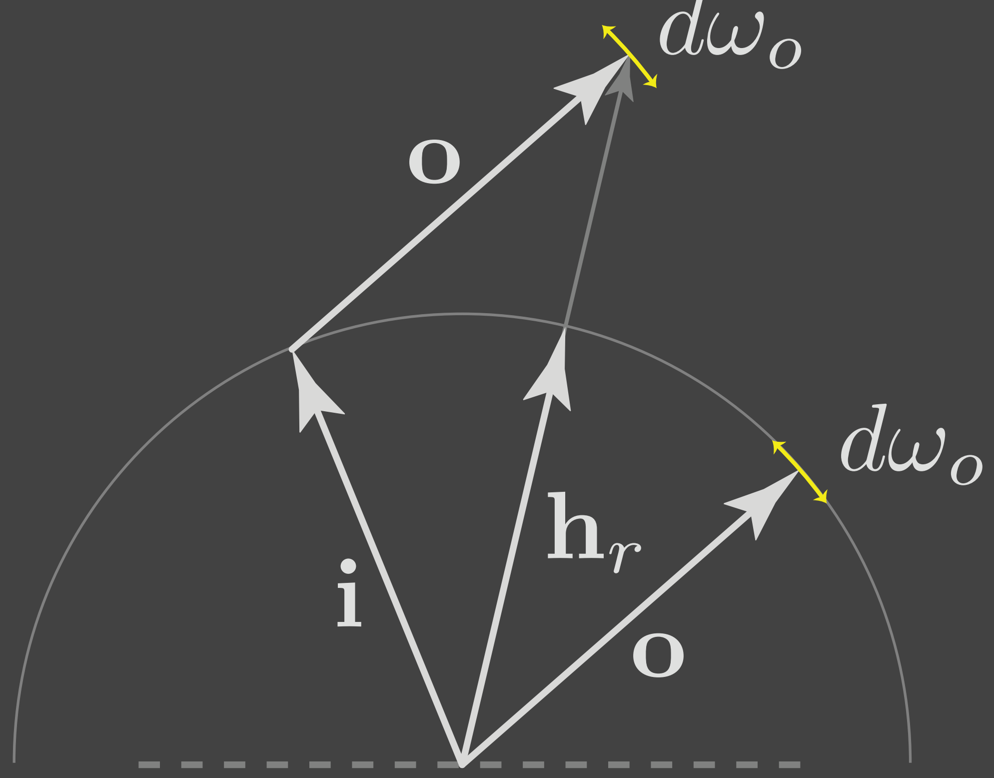
refraction

$$\mathbf{h}_t = -\text{normalize}(\mathbf{i} + n\mathbf{o})$$

Construction of half-vector solid angle

reflection

$$\mathbf{h}_r = \mathrm{normalize}(\mathbf{i} + \mathbf{o})$$



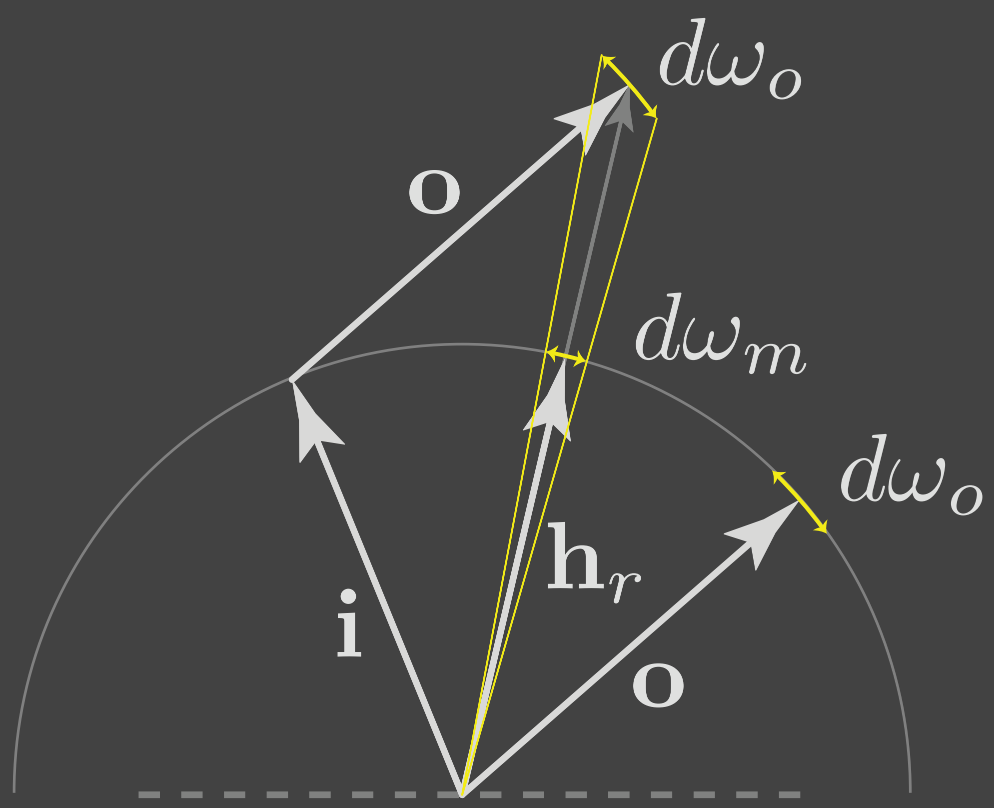
refraction

$$\mathbf{h}_t = -\mathrm{normalize}(\mathbf{i} + n\mathbf{o})$$

Construction of half-vector solid angle

reflection

$$\mathbf{h}_r = \text{normalize}(\mathbf{i} + \mathbf{o})$$



$$d\omega_m = \frac{|\mathbf{o} \cdot \mathbf{h}_r|}{\|\mathbf{i} + \mathbf{o}\|^2} \, d\omega_o$$

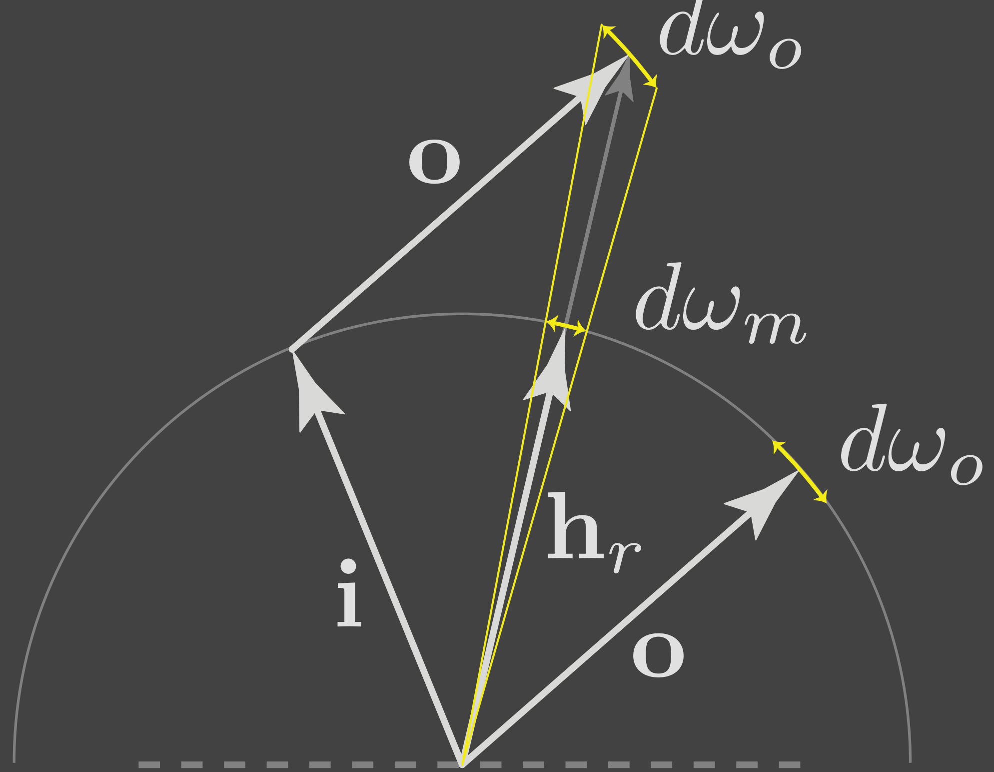
refraction

$$\mathbf{h}_t = -\text{normalize}(\mathbf{i} + n\mathbf{o})$$

Construction of half-vector solid angle

reflection

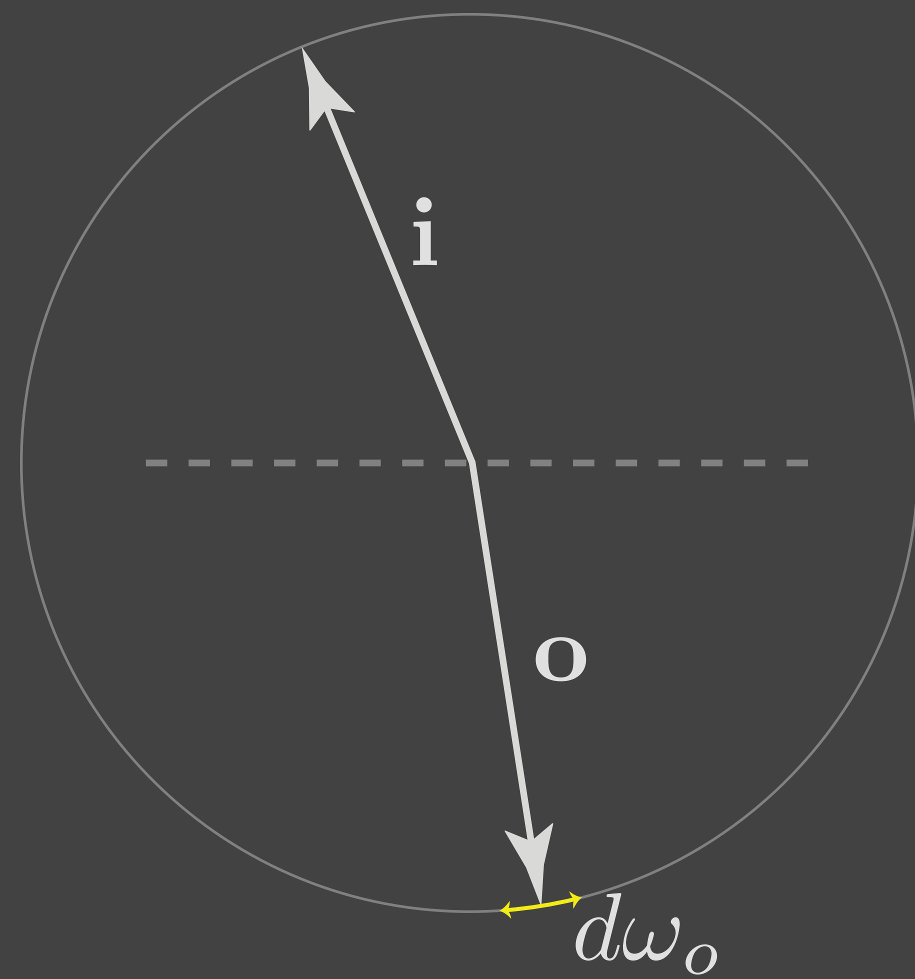
$$\mathbf{h}_r = \text{normalize}(\mathbf{i} + \mathbf{o})$$



$$d\omega_m = \frac{|\mathbf{o} \cdot \mathbf{h}_r|}{\|\mathbf{i} + \mathbf{o}\|^2} \, d\omega_o$$

refraction

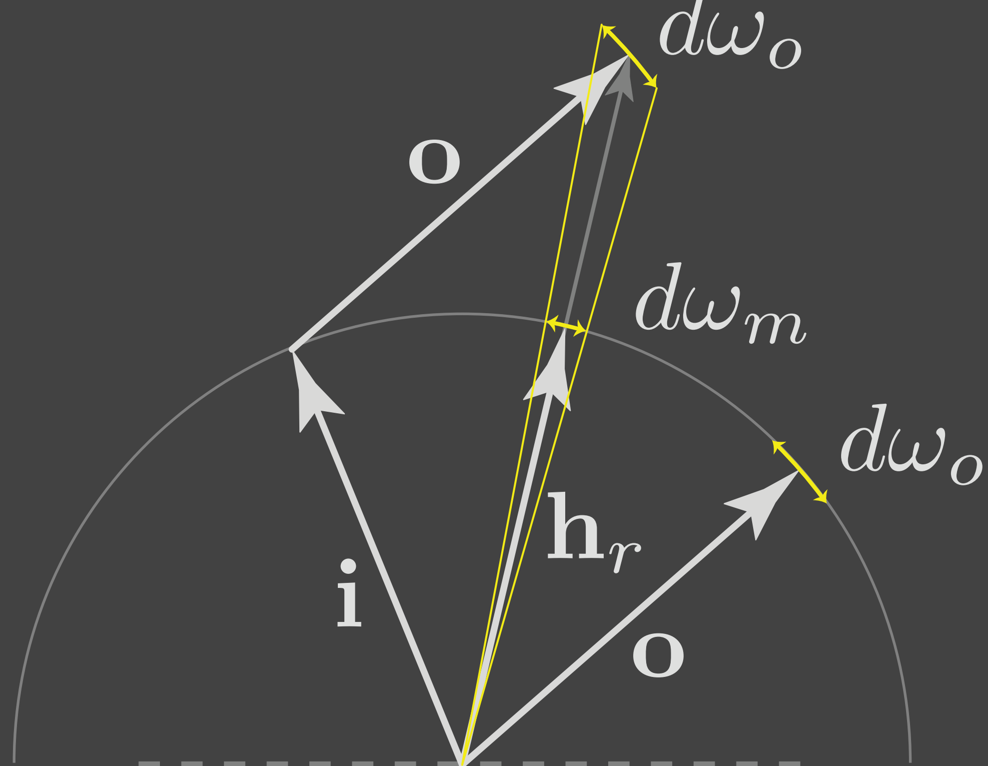
$$\mathbf{h}_t = -\text{normalize}(\mathbf{i} + n\mathbf{o})$$



Construction of half-vector solid angle

reflection

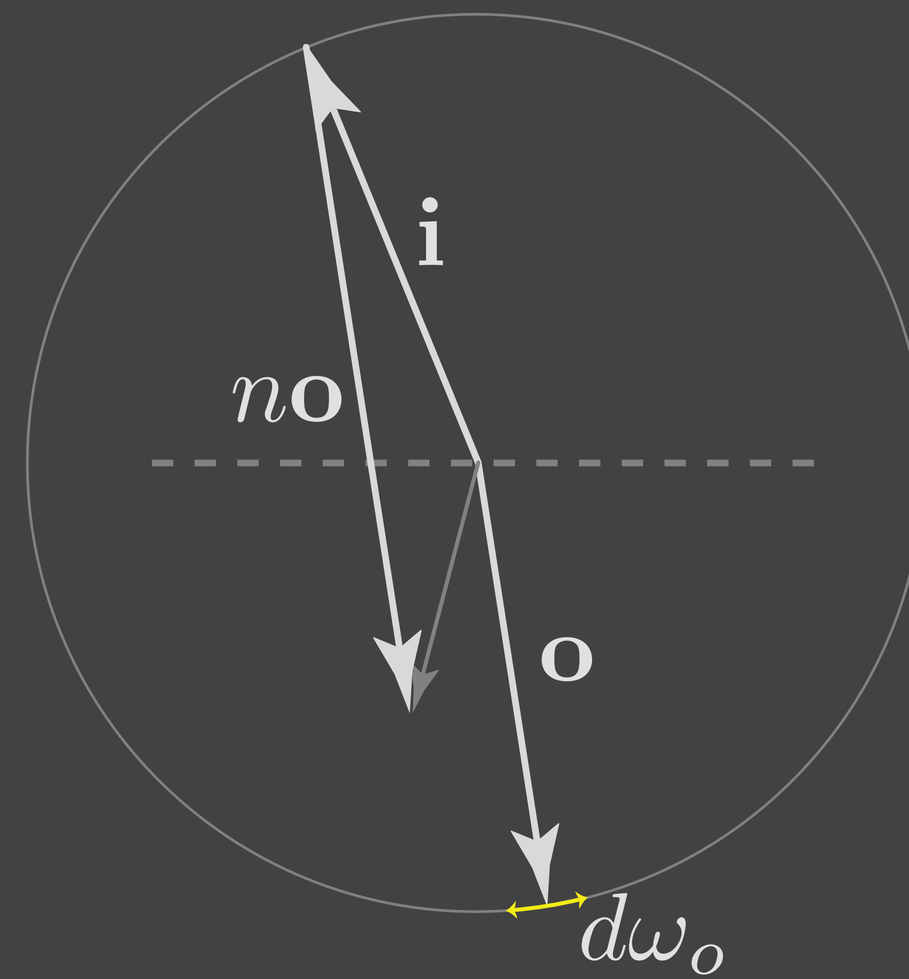
$$\mathbf{h}_r = \text{normalize}(\mathbf{i} + \mathbf{o})$$



$$d\omega_m = \frac{|\mathbf{o} \cdot \mathbf{h}_r|}{\|\mathbf{i} + \mathbf{o}\|^2} \, d\omega_o$$

refraction

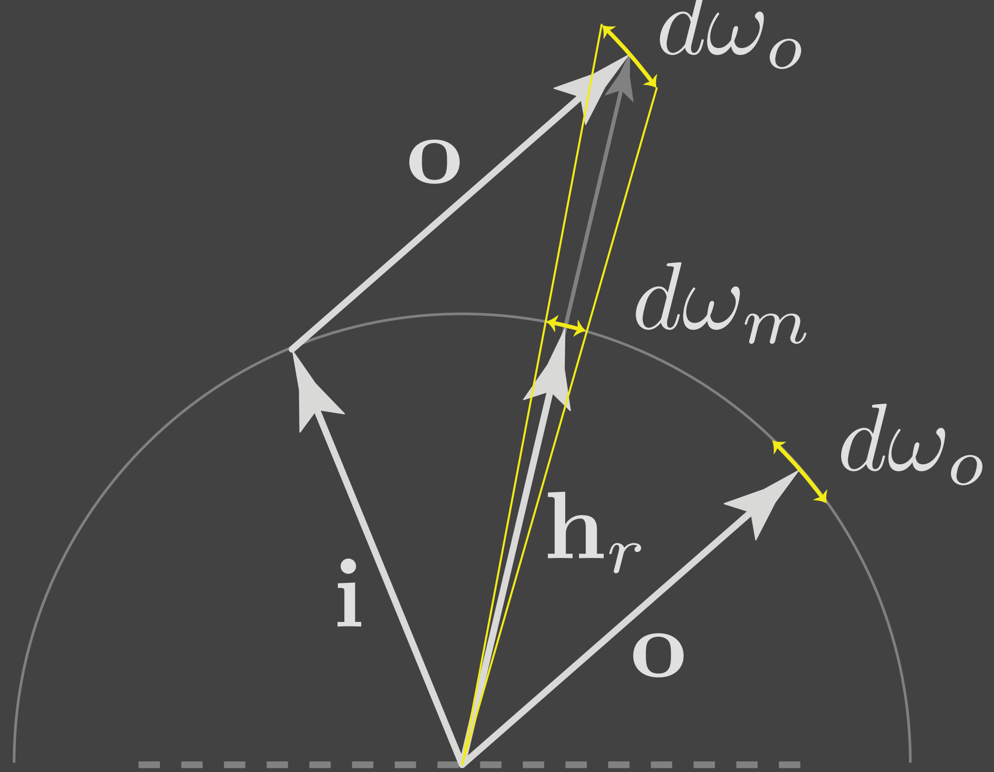
$$\mathbf{h}_t = -\text{normalize}(\mathbf{i} + n\mathbf{o})$$



Construction of half-vector solid angle

reflection

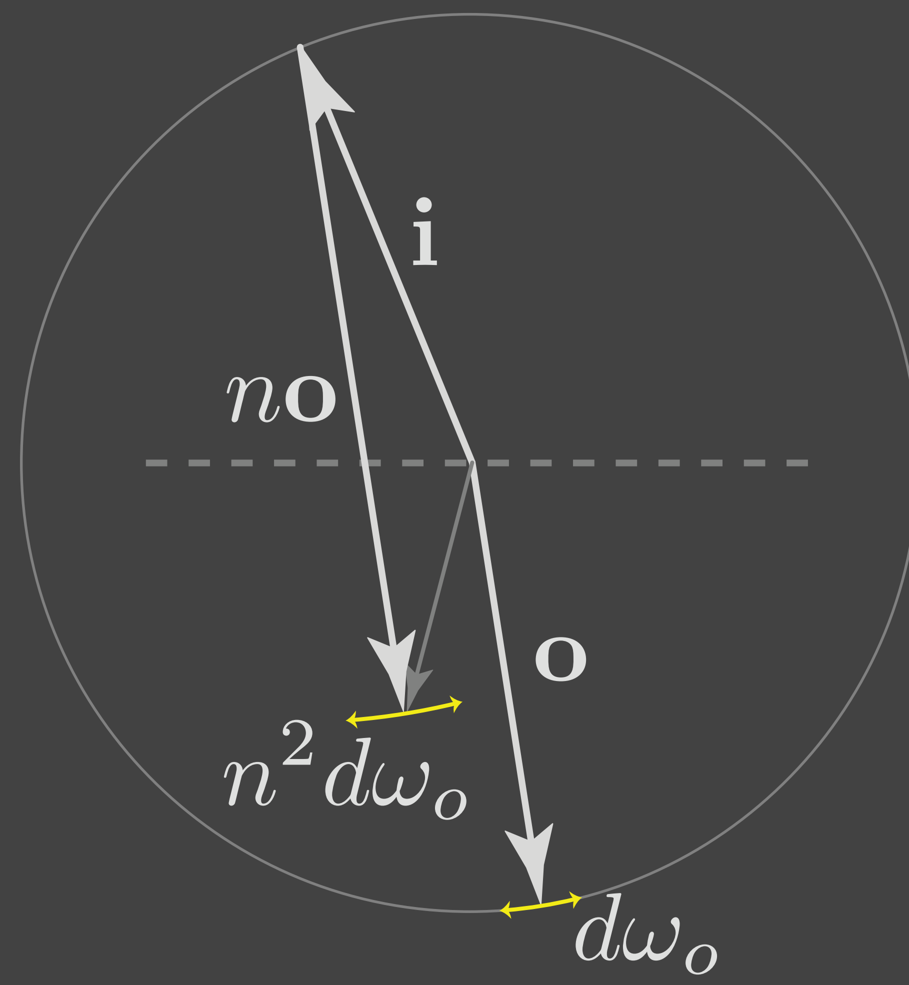
$$\mathbf{h}_r = \text{normalize}(\mathbf{i} + \mathbf{o})$$



$$d\omega_m = \frac{|\mathbf{o} \cdot \mathbf{h}_r|}{\|\mathbf{i} + \mathbf{o}\|^2} \, d\omega_o$$

refraction

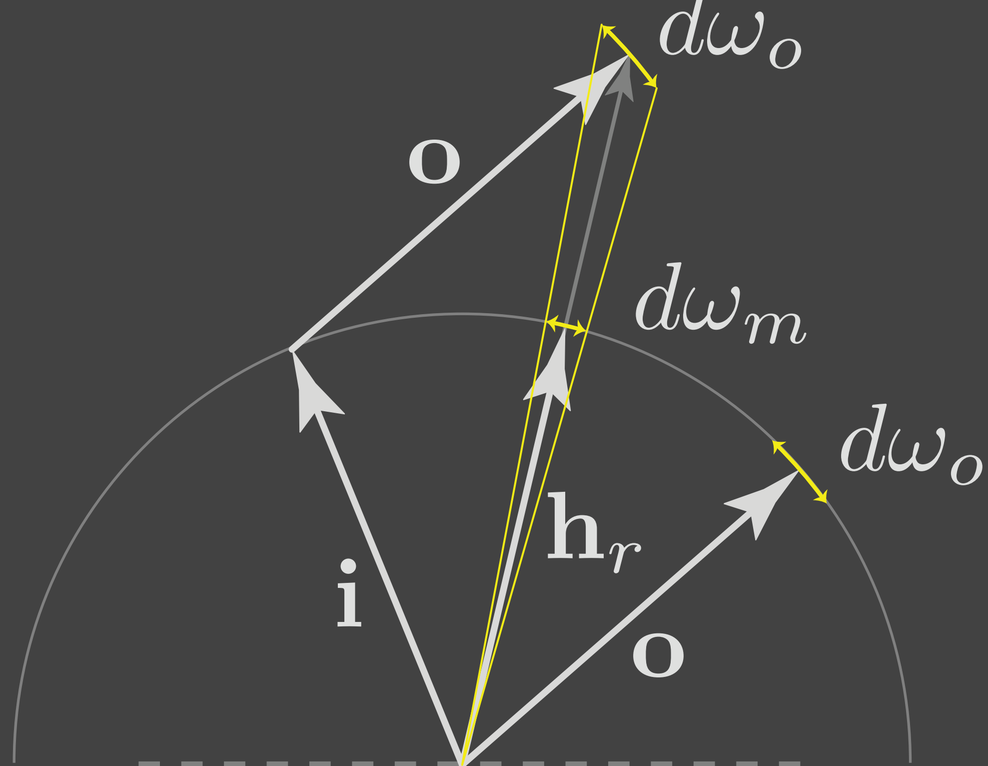
$$\mathbf{h}_t = -\text{normalize}(\mathbf{i} + n\mathbf{o})$$



Construction of half-vector solid angle

reflection

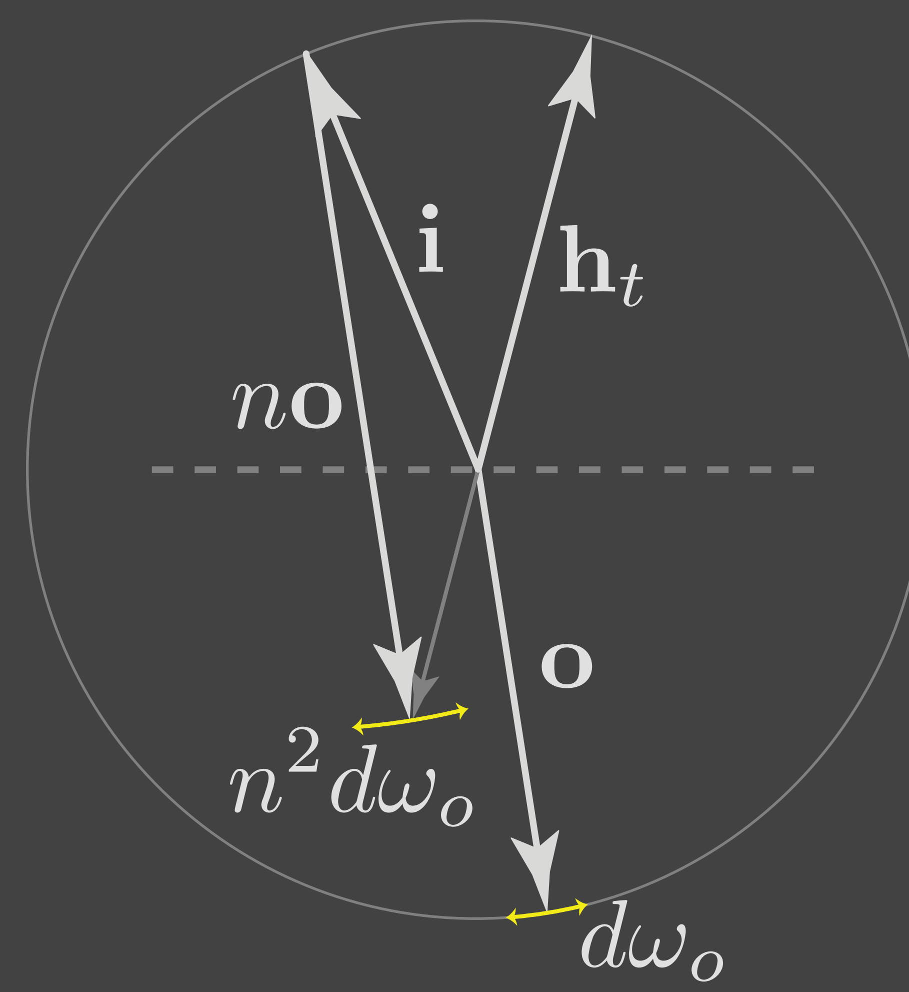
$$\mathbf{h}_r = \text{normalize}(\mathbf{i} + \mathbf{o})$$



$$d\omega_m = \frac{|\mathbf{o} \cdot \mathbf{h}_r|}{\|\mathbf{i} + \mathbf{o}\|^2} \, d\omega_o$$

refraction

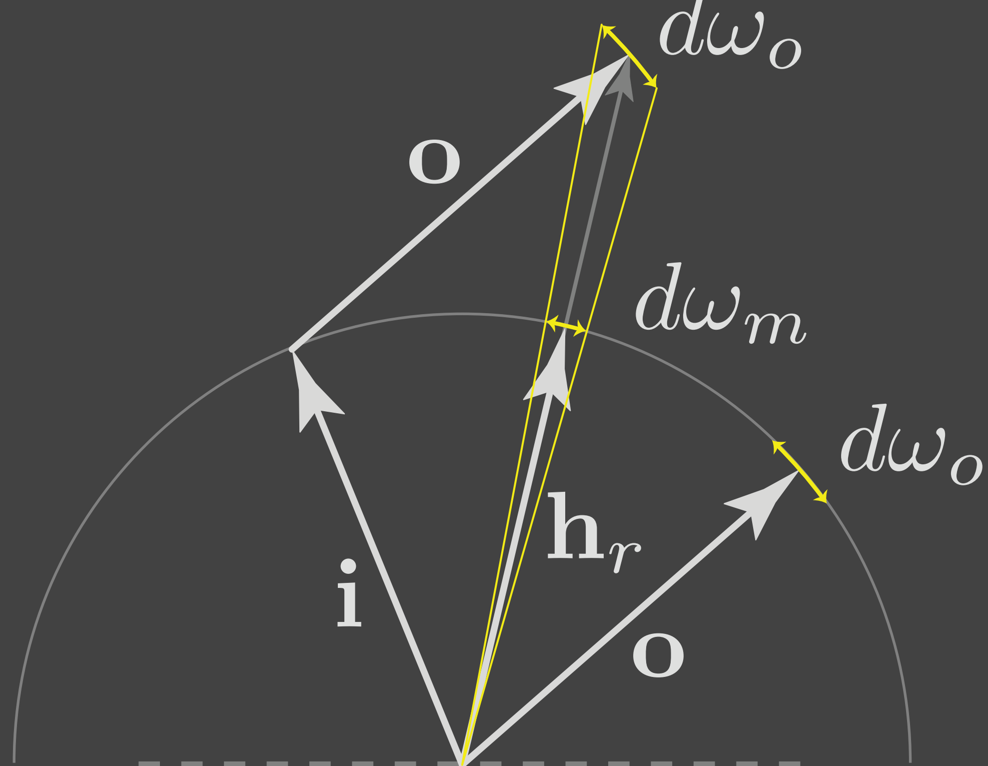
$$\mathbf{h}_t = -\text{normalize}(\mathbf{i} + n\mathbf{o})$$



Construction of half-vector solid angle

reflection

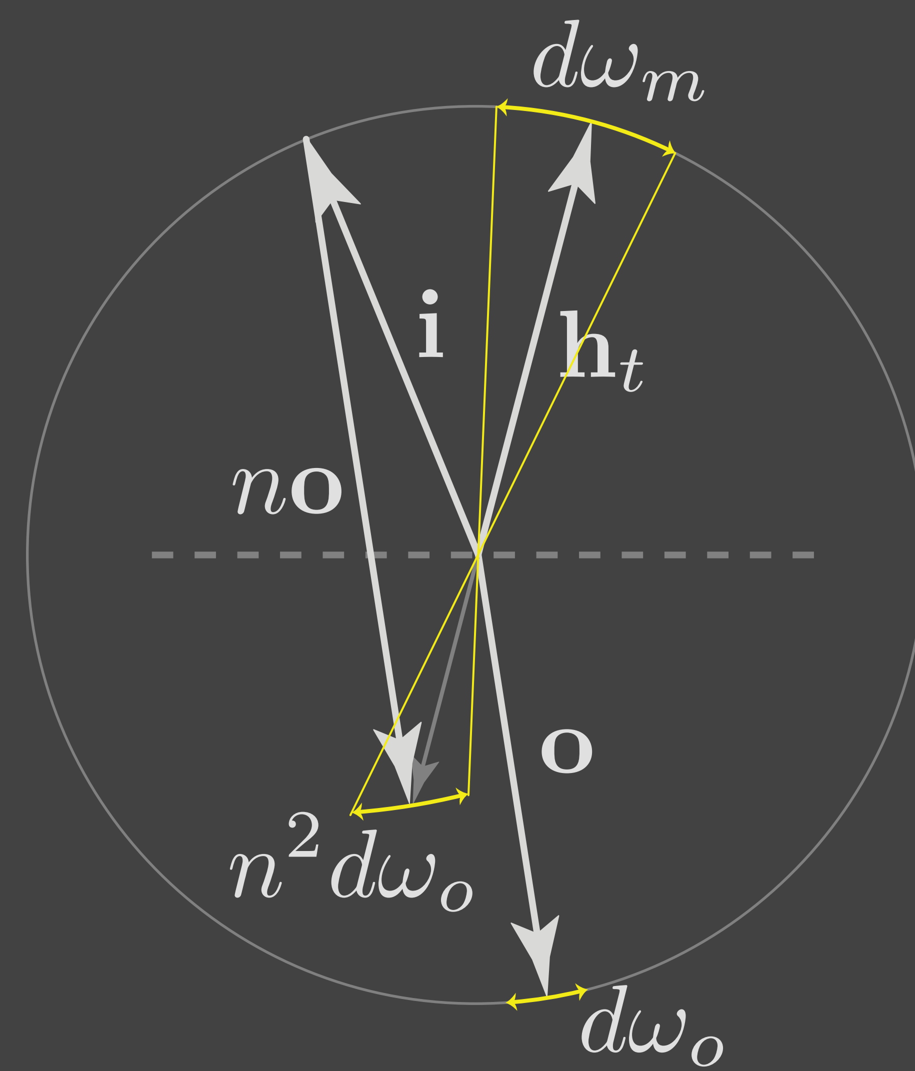
$$\mathbf{h}_r = \text{normalize}(\mathbf{i} + \mathbf{o})$$



$$d\omega_m = \frac{|\mathbf{o} \cdot \mathbf{h}_r|}{\|\mathbf{i} + \mathbf{o}\|^2} \, d\omega_o$$

refraction

$$\mathbf{h}_t = -\text{normalize}(\mathbf{i} + n\mathbf{o})$$



$$d\omega_m = \frac{|\mathbf{o} \cdot \mathbf{h}_t|}{\|\mathbf{i} + n\mathbf{o}\|^2} \, n^2 d\omega_o$$

Result: scattering functions

reflection

$$f_s(\mathbf{i},\mathbf{o}) = \frac{|\mathbf{i}\cdot\mathbf{m}|}{|\mathbf{i}\cdot\mathbf{n}|\,|\mathbf{o}\cdot\mathbf{n}|}\,\rho(\mathbf{i},\mathbf{o})\,D(\mathbf{m})\,G(\mathbf{i},\mathbf{o},\mathbf{m})\,\frac{d\omega_m}{d\omega_o}$$

transmission

$$f_s(\mathbf{i},\mathbf{o}) = \frac{|\mathbf{i}\cdot\mathbf{m}|}{|\mathbf{i}\cdot\mathbf{n}|\,|\mathbf{o}\cdot\mathbf{n}|}\,\rho(\mathbf{i},\mathbf{o})\,D(\mathbf{m})\,G(\mathbf{i},\mathbf{o},\mathbf{m})\,\frac{d\omega_m}{d\omega_o}$$

Result: scattering functions

reflection

$$f_r(\mathbf{i}, \mathbf{o}) = \frac{|\mathbf{i} \cdot \mathbf{m}|}{|\mathbf{i} \cdot \mathbf{n}| \, |\mathbf{o} \cdot \mathbf{n}|} \, F(\mathbf{i}, \mathbf{m}) \, D(\mathbf{m}) \, G(\mathbf{i}, \mathbf{o}, \mathbf{m}) \, \frac{|\mathbf{o} \cdot \mathbf{m}|}{\|\mathbf{i} + \mathbf{o}\|^2}$$

transmission

$$f_s(\mathbf{i}, \mathbf{o}) = \frac{|\mathbf{i} \cdot \mathbf{m}|}{|\mathbf{i} \cdot \mathbf{n}| \, |\mathbf{o} \cdot \mathbf{n}|} \, \rho(\mathbf{i}, \mathbf{o}) \, D(\mathbf{m}) \, G(\mathbf{i}, \mathbf{o}, \mathbf{m}) \, \frac{d\omega_m}{d\omega_o}$$

Result: scattering functions

reflection

$$f_r(\mathbf{i}, \mathbf{o}) = \frac{|\mathbf{i} \cdot \mathbf{m}|}{|\mathbf{i} \cdot \mathbf{n}| \, |\mathbf{o} \cdot \mathbf{n}|} \, F(\mathbf{i}, \mathbf{m}) \, D(\mathbf{m}) \, G(\mathbf{i}, \mathbf{o}, \mathbf{m}) \, \frac{|\mathbf{o} \cdot \mathbf{m}|}{\|\mathbf{i} + \mathbf{o}\|^2}$$

transmission

$$f_t(\mathbf{i}, \mathbf{o}) = \frac{|\mathbf{i} \cdot \mathbf{m}|}{|\mathbf{i} \cdot \mathbf{n}| \, |\mathbf{o} \cdot \mathbf{n}|} \, (1 - F(\mathbf{i}, \mathbf{m})) \, D(\mathbf{m}) \, G(\mathbf{i}, \mathbf{o}, \mathbf{m}) \, \frac{n^2 |\mathbf{o} \cdot \mathbf{m}|}{\|\mathbf{i} + n\mathbf{o}\|^2}$$

Result: scattering functions

reflection

$$f_r(\mathbf{i}, \mathbf{o}) = \frac{|\mathbf{i} \cdot \mathbf{m}|\, |\mathbf{o} \cdot \mathbf{m}|}{|\mathbf{i} \cdot \mathbf{n}|\, |\mathbf{o} \cdot \mathbf{n}|} \, \frac{F(\mathbf{i}, \mathbf{m})\, D(\mathbf{m})\, G(\mathbf{i}, \mathbf{o}, \mathbf{m})}{\|\mathbf{i} + \mathbf{o}\|^2}$$

transmission

$$f_t(\mathbf{i}, \mathbf{o}) = \frac{|\mathbf{i} \cdot \mathbf{m}|}{|\mathbf{i} \cdot \mathbf{n}|\, |\mathbf{o} \cdot \mathbf{n}|} \, (1 - F(\mathbf{i}, \mathbf{m}))\, D(\mathbf{m})\, G(\mathbf{i}, \mathbf{o}, \mathbf{m}) \, \frac{n^2 |\mathbf{o} \cdot \mathbf{m}|}{\|\mathbf{i} + n\mathbf{o}\|^2}$$

Result: scattering functions

reflection

$$f_r(\mathbf{i}, \mathbf{o}) = \frac{|\mathbf{i} \cdot \mathbf{m}| \, |\mathbf{o} \cdot \mathbf{m}|}{|\mathbf{i} \cdot \mathbf{n}| \, |\mathbf{o} \cdot \mathbf{n}|} \, \frac{F(\mathbf{i}, \mathbf{m}) \, D(\mathbf{m}) \, G(\mathbf{i}, \mathbf{o}, \mathbf{m})}{\|\mathbf{i} + \mathbf{o}\|^2}$$

transmission

$$f_t(\mathbf{i}, \mathbf{o}) = \frac{|\mathbf{i} \cdot \mathbf{m}| \, |\mathbf{o} \cdot \mathbf{m}|}{|\mathbf{i} \cdot \mathbf{n}| \, |\mathbf{o} \cdot \mathbf{n}|} \, \frac{n^2 (1 - F(\mathbf{i}, \mathbf{m})) \, D(\mathbf{m}) \, G(\mathbf{i}, \mathbf{o}, \mathbf{m})}{\|\mathbf{i} + n\mathbf{o}\|^2}$$

Result: scattering functions

reflection

$$f_r(\mathbf{i},\mathbf{o}) = \frac{1}{|\mathbf{i}\cdot\mathbf{n}|\,|\mathbf{o}\cdot\mathbf{n}|}\,\frac{F(\mathbf{i},\mathbf{m})\,D(\mathbf{m})\,G(\mathbf{i},\mathbf{o},\mathbf{m})}{4}$$

transmission

$$f_t(\mathbf{i},\mathbf{o}) = \frac{|\mathbf{i}\cdot\mathbf{m}|\,|\mathbf{o}\cdot\mathbf{m}|}{|\mathbf{i}\cdot\mathbf{n}|\,|\mathbf{o}\cdot\mathbf{n}|}\,\frac{n^2(1-F(\mathbf{i},\mathbf{m}))\,D(\mathbf{m})\,G(\mathbf{i},\mathbf{o},\mathbf{m})}{\|\mathbf{i}+n\mathbf{o}\|^2}$$

Result: scattering functions

reflection

$$f_r(\mathbf{i}, \mathbf{o}) = \frac{F(\mathbf{i}, \mathbf{m})\, D(\mathbf{m})\, G(\mathbf{i}, \mathbf{o}, \mathbf{m})}{4|\mathbf{i} \cdot \mathbf{n}|\, |\mathbf{o} \cdot \mathbf{n}|}$$

transmission

$$f_t(\mathbf{i}, \mathbf{o}) = \frac{|\mathbf{i} \cdot \mathbf{m}|\, |\mathbf{o} \cdot \mathbf{m}|}{|\mathbf{i} \cdot \mathbf{n}|\, |\mathbf{o} \cdot \mathbf{n}|} \frac{n^2 (1 - F(\mathbf{i}, \mathbf{m}))\, D(\mathbf{m})\, G(\mathbf{i}, \mathbf{o}, \mathbf{m})}{\|\mathbf{i} + n\mathbf{o}\|^2}$$

Fresnel reflectance

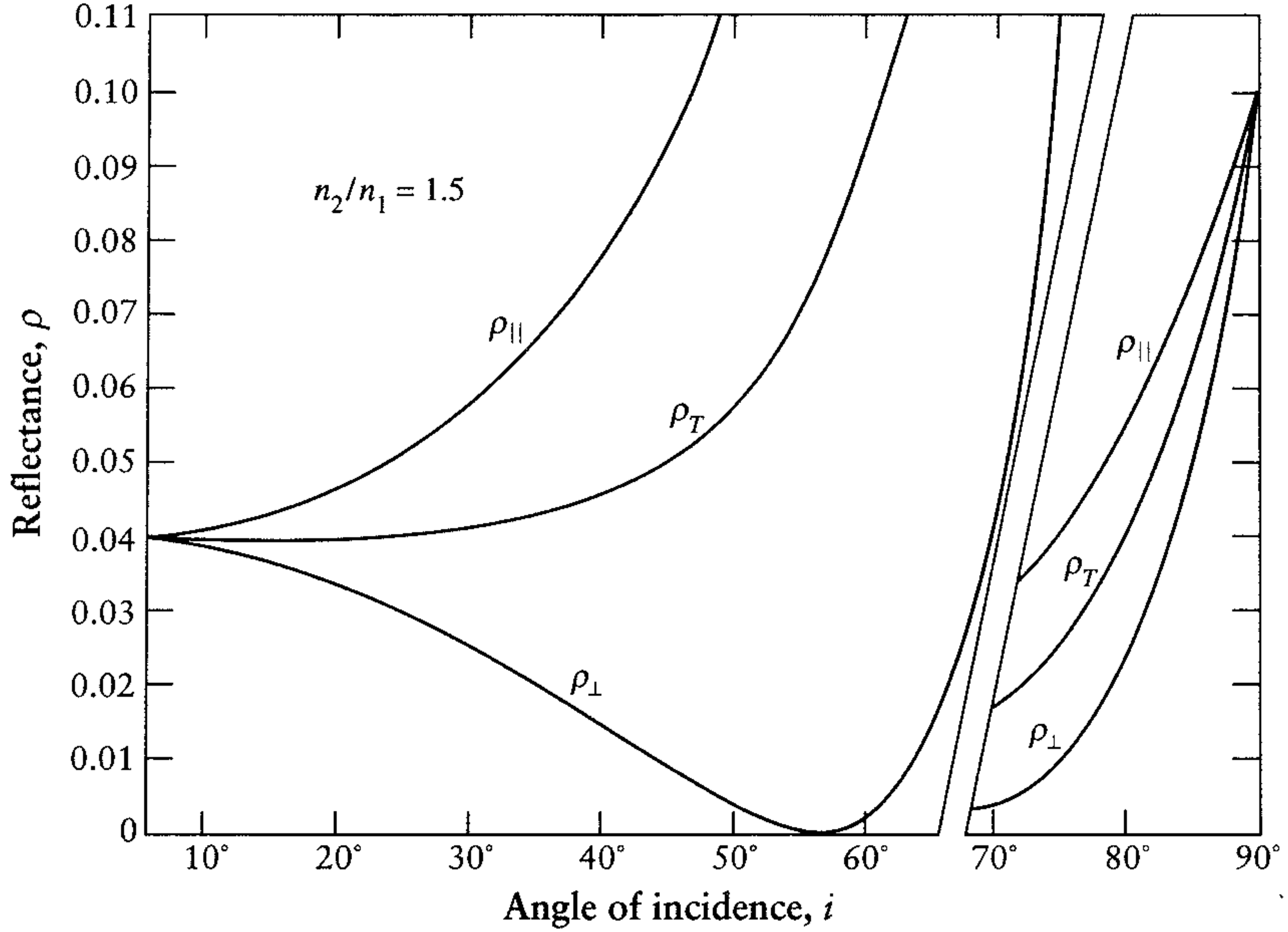


FIGURE 15.8

The Fresnel reflectance for an air-glass boundary with index of refraction 1.5. We show the two polarized components and the term for unpolarized light. Redrawn from Judd and Wyszecki, *Color in Business, Science and Industry*, fig. 3.2, p. 400.

Glassner, *Principles of Digital Image Synthesis*

Fresnel reflectance

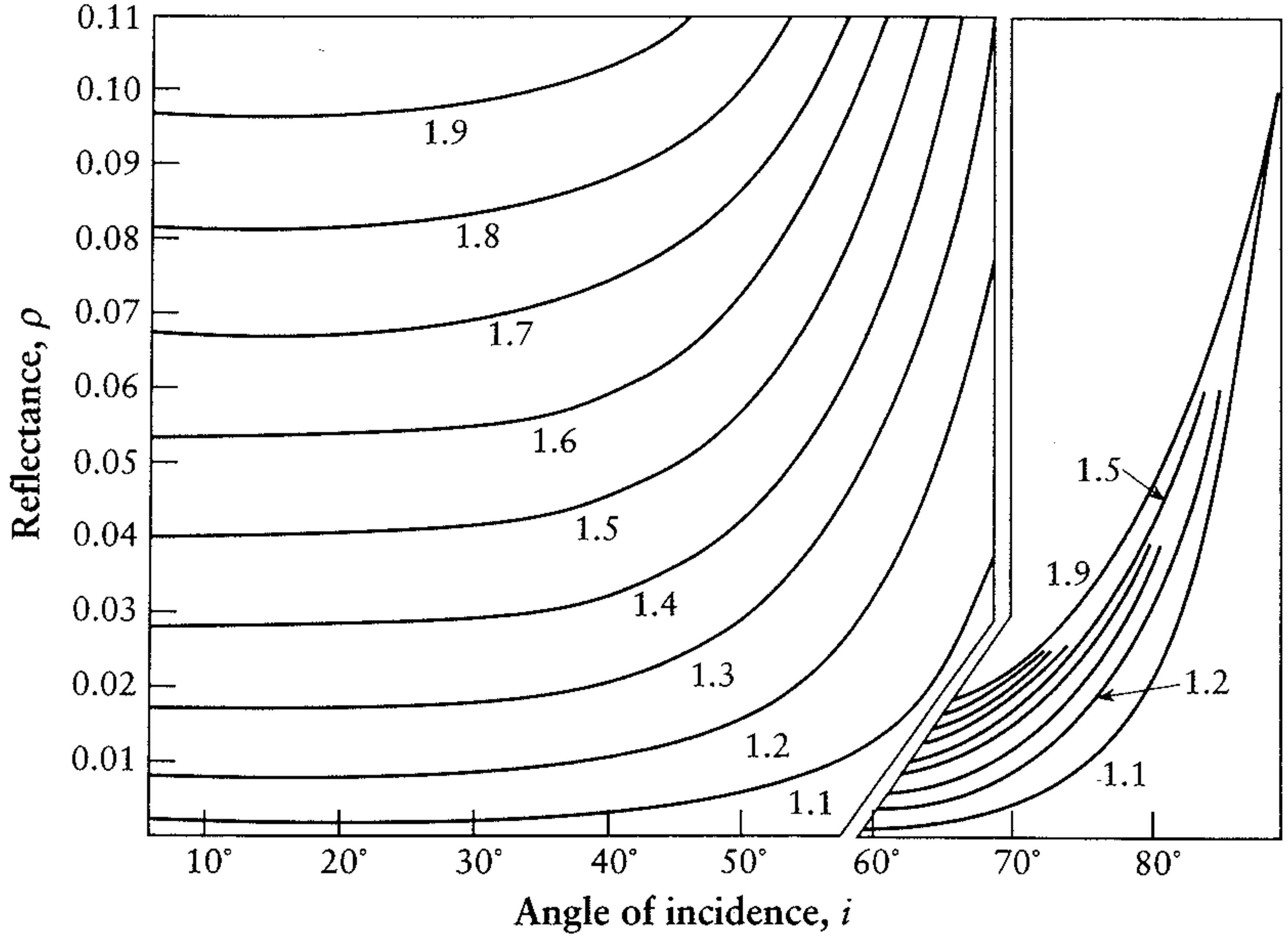


FIGURE 15.9

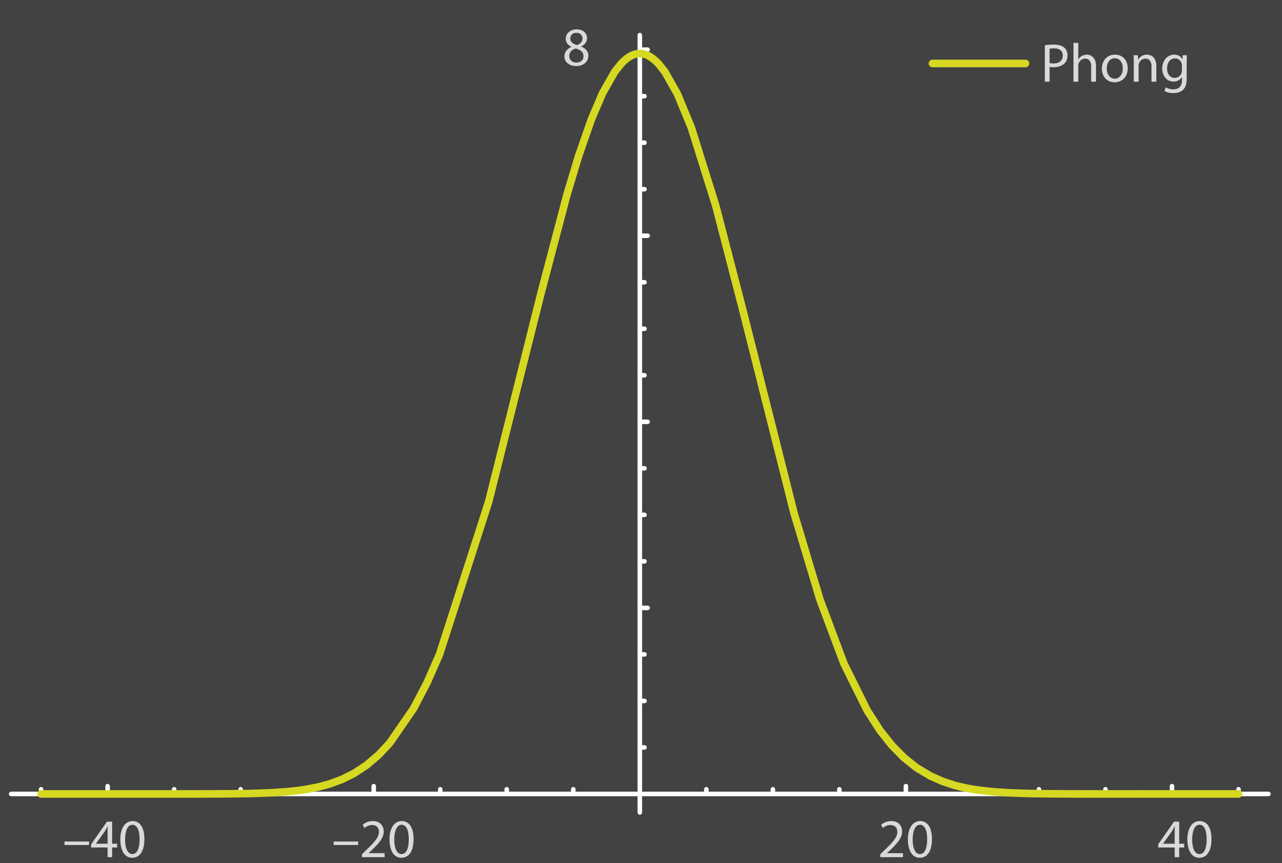
The Fresnel reflection for unpolarized light for different indices of refraction. Redrawn from Judd and Wyszecki, *Color in Business, Science and Industry*, fig. 3.3, p. 401.

Glassner, *Principles of Digital Image Synthesis*

Normal distributions

Choice of distribution is determined by surface

- Phong, Beckman are popular choices
- “GGX” distribution is another option
- [Smith 67] gives a way to produce smooth Gs



$$D(\theta_m)$$

Normal distributions

Choice of distribution is determined by surface

- Phong, Beckman are popular choices
- “GGX” distribution is another option
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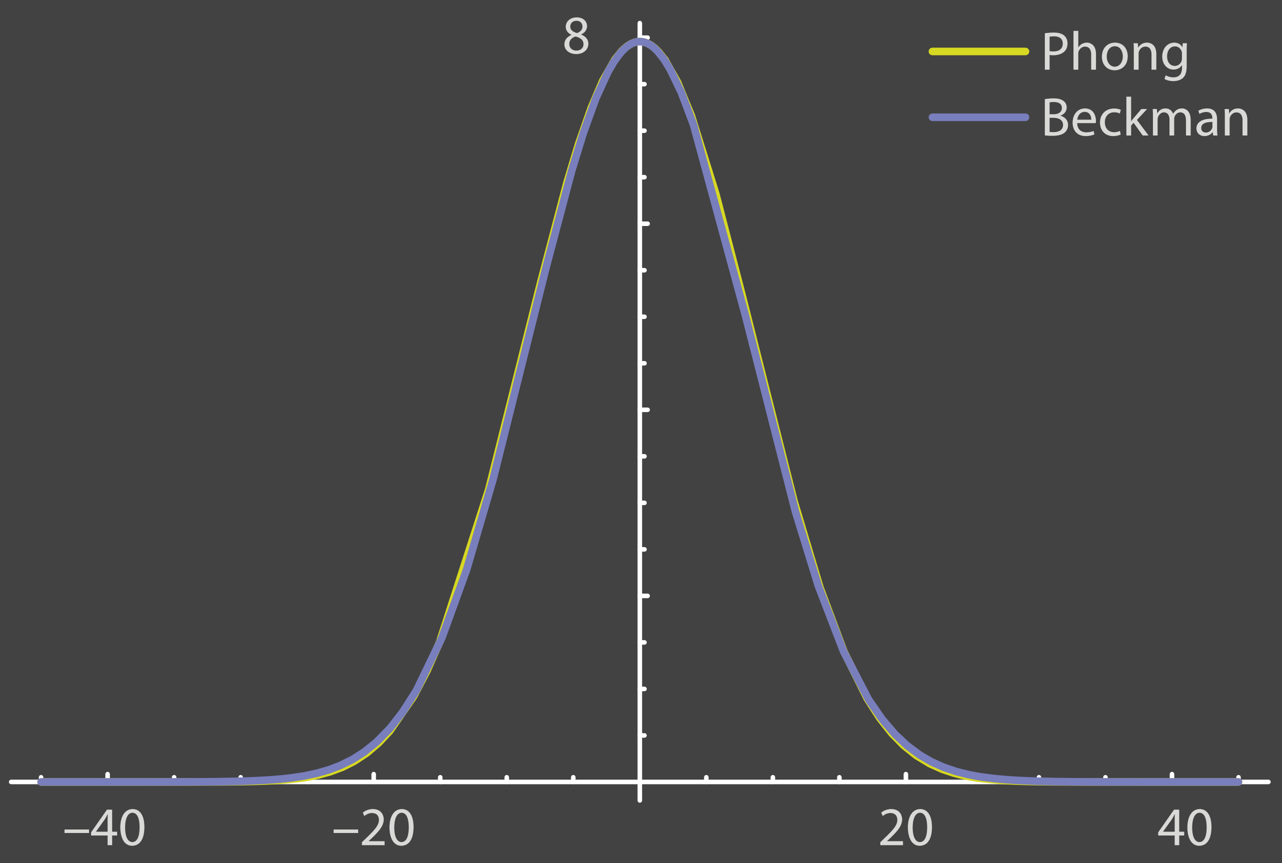

$$D(\theta_m)$$

Normal distributions

Choice of distribution is determined by surface

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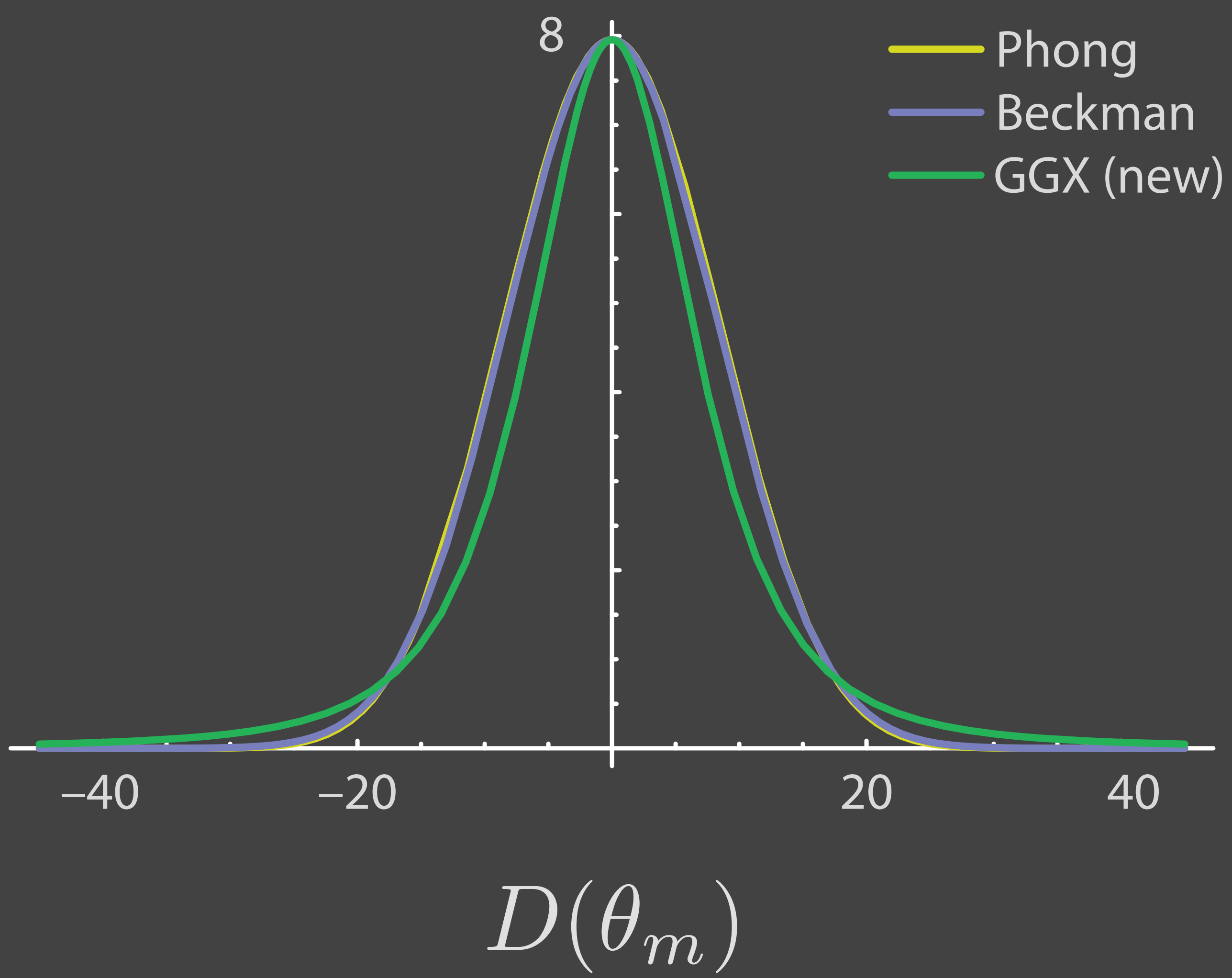

Normal distributions

Choice of distribution is determined by surface

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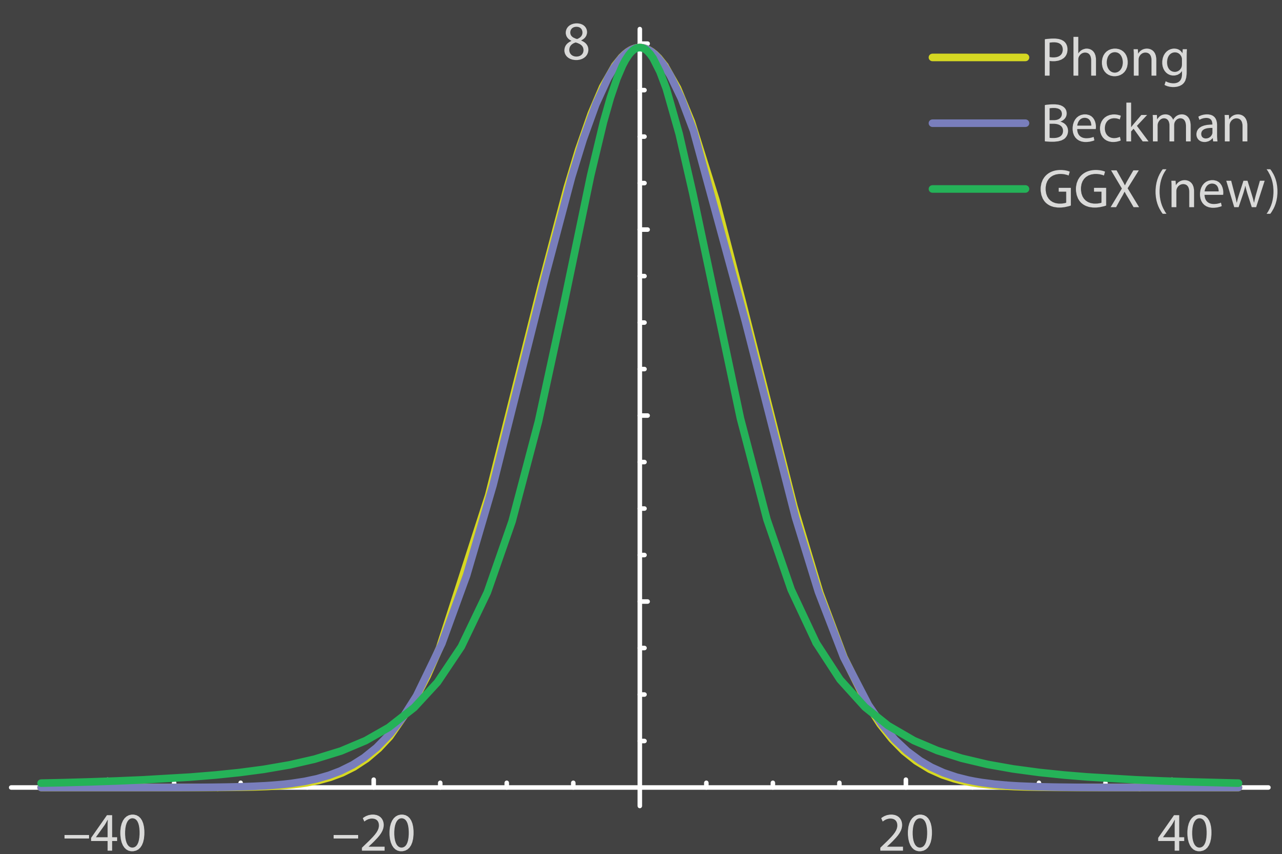
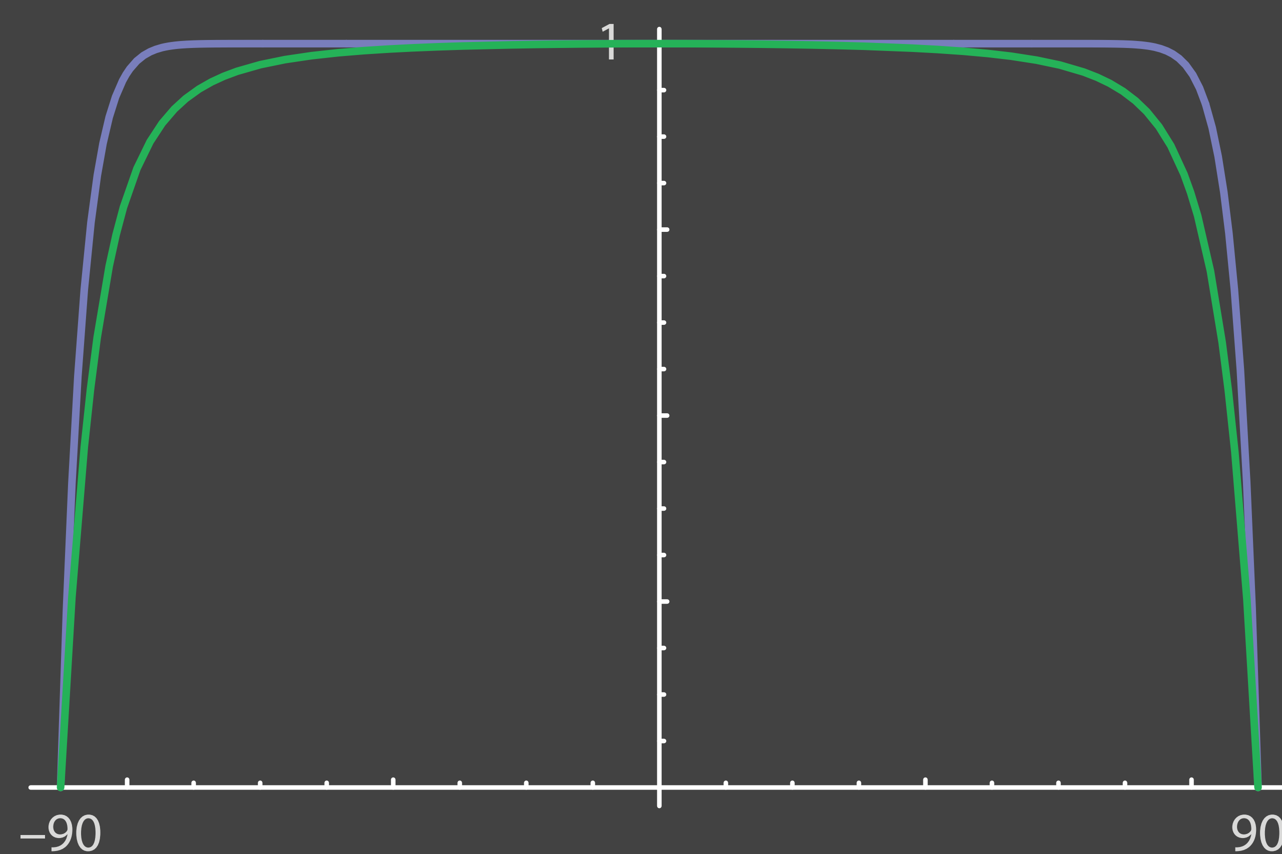
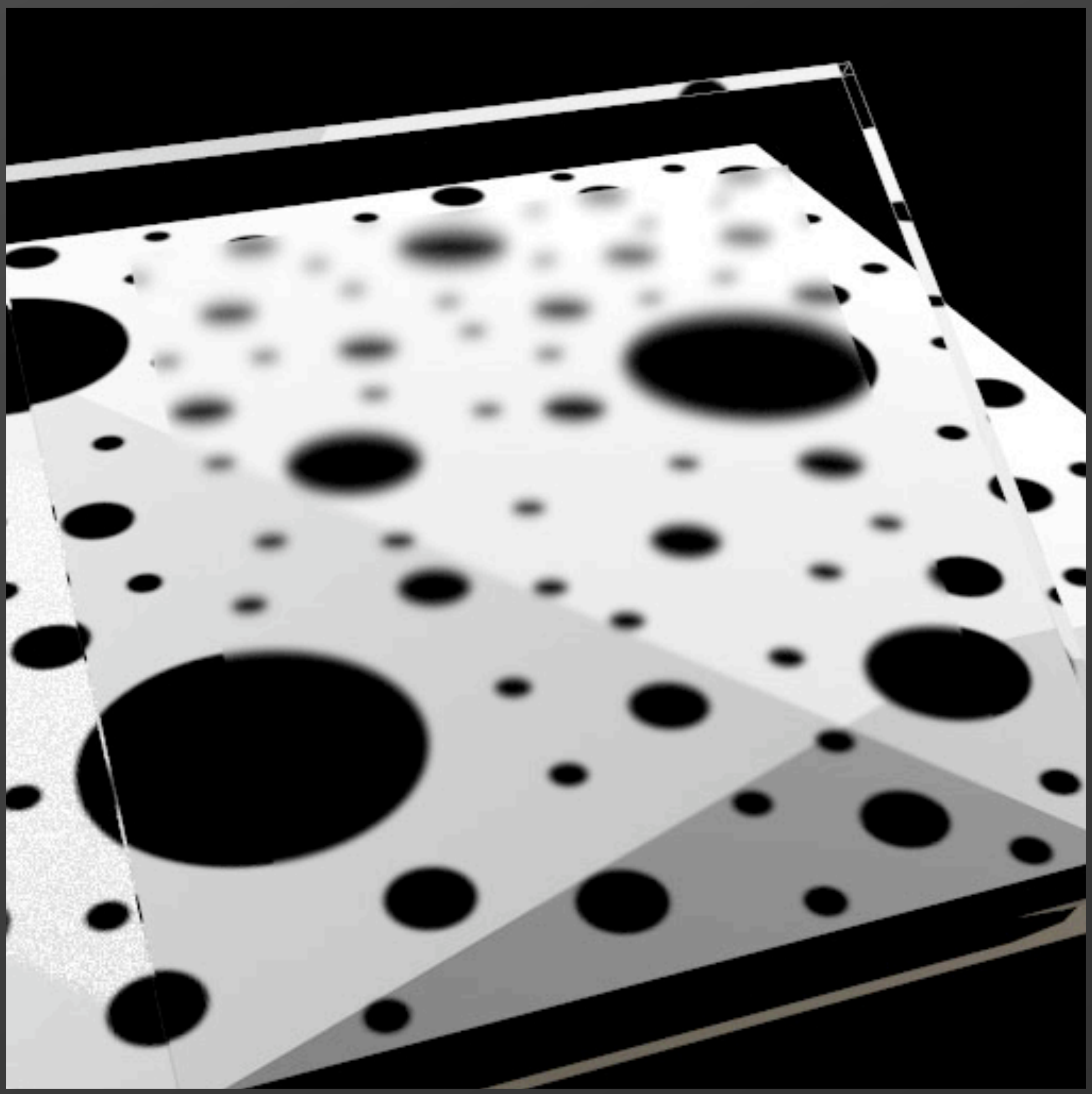


$D(\theta_m)$

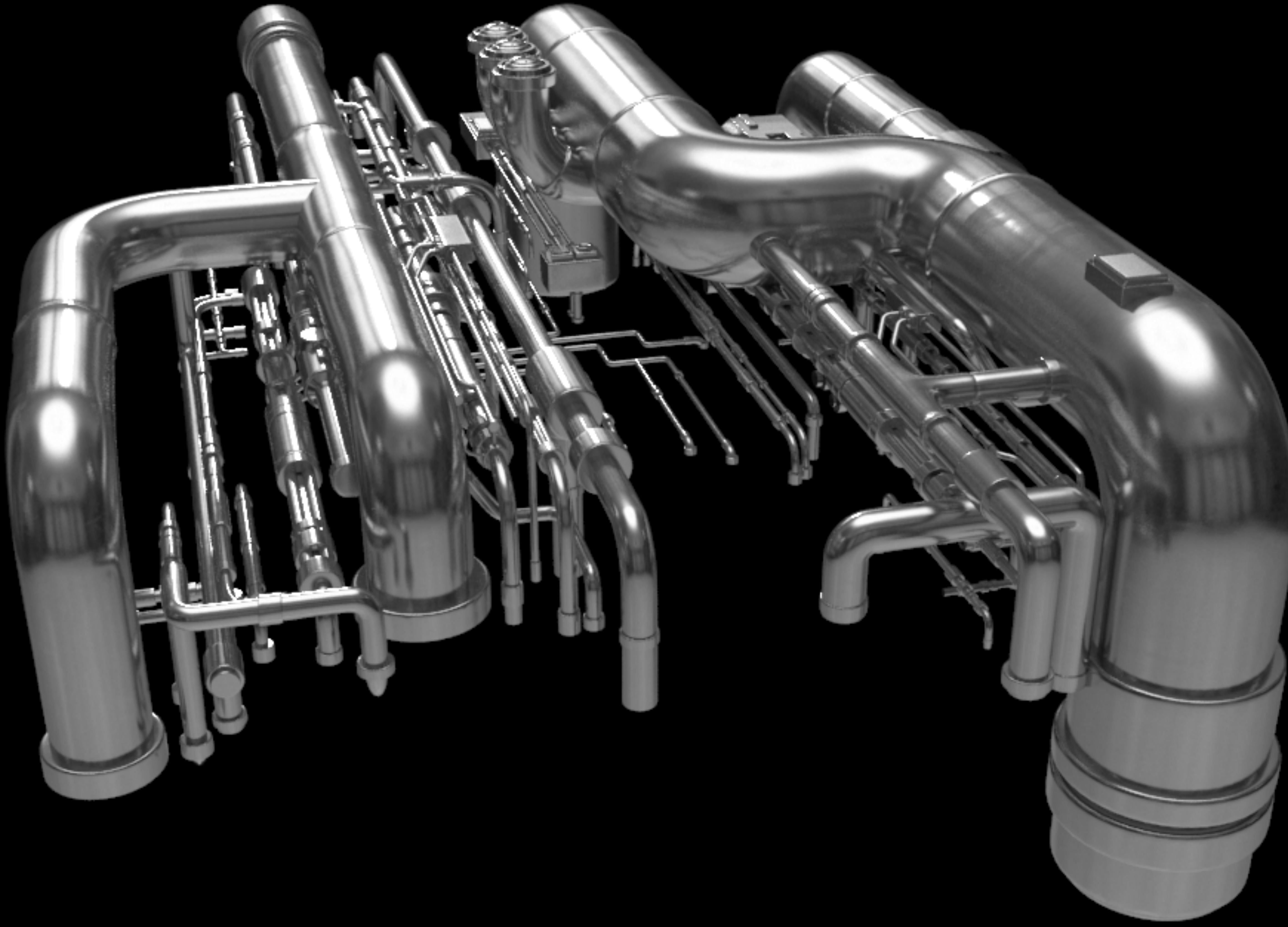


$G_1(\theta_{i,o})$

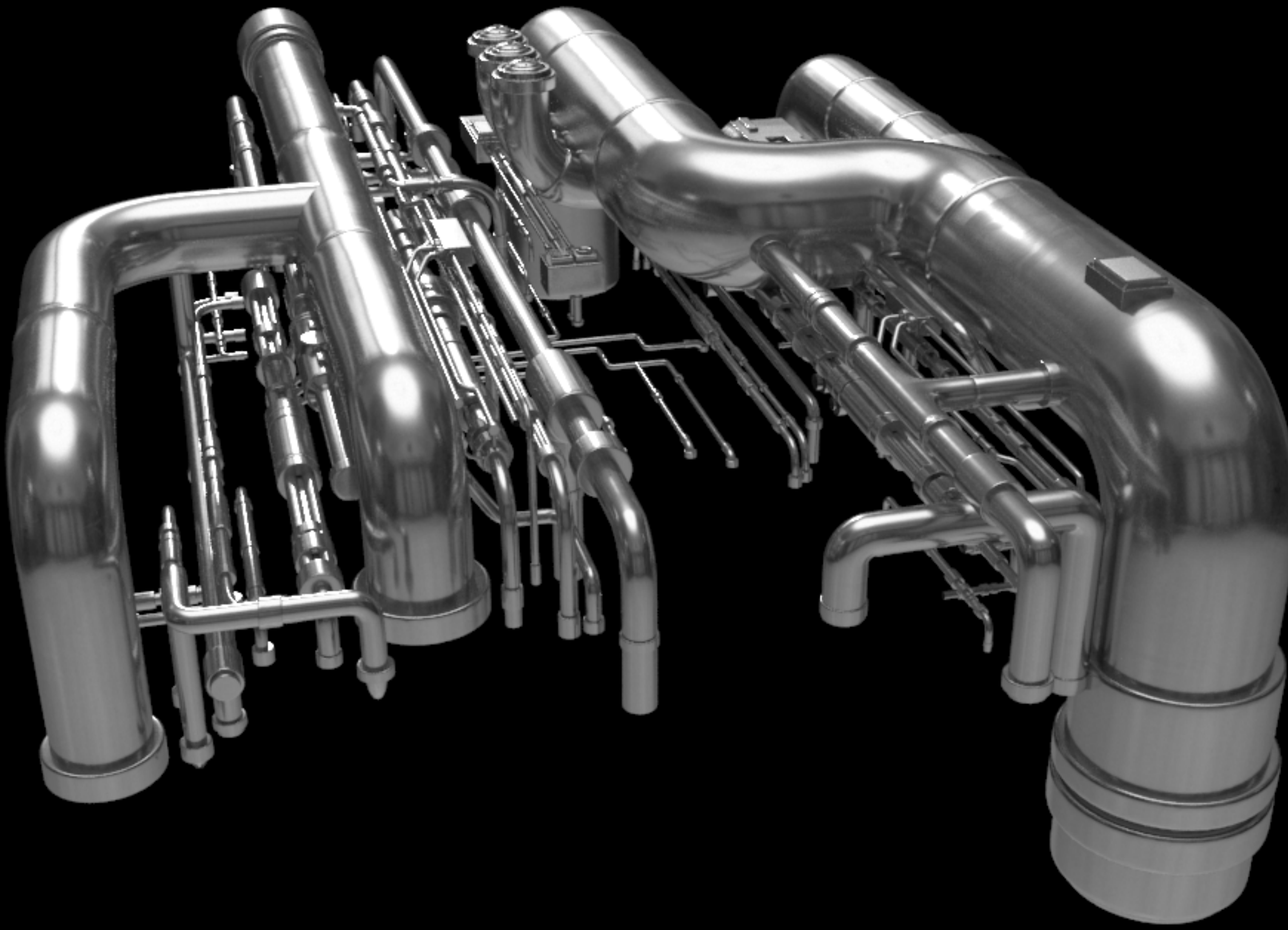
Result (transmission)



[Walter et al. EGSR 2007]



Gaussian
(Beckmann)



GGX
(Trowbridge-Reitz)

Neil Blevins | neilblevins.com