CS5630: Physically Based Realistic Rendering

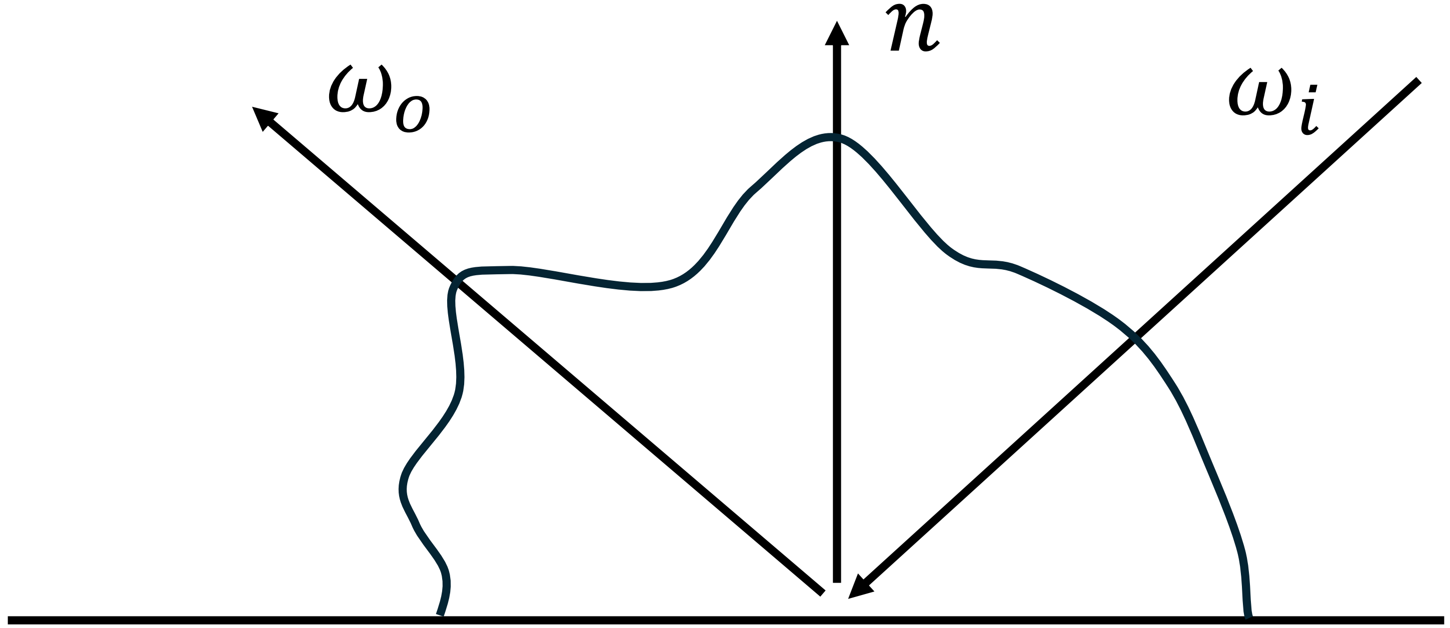
Mariia Soroka

08: Illumination Integrals

BRDFs Recap

$$f_r(\omega, \omega_o) = \frac{dL_o(\omega_o)}{L_i(\omega)(\omega \cdot n)d\omega} = \frac{dL_o(\omega_o)}{dE(\omega)}$$

BRDF – bidirectional reflectance distribution function

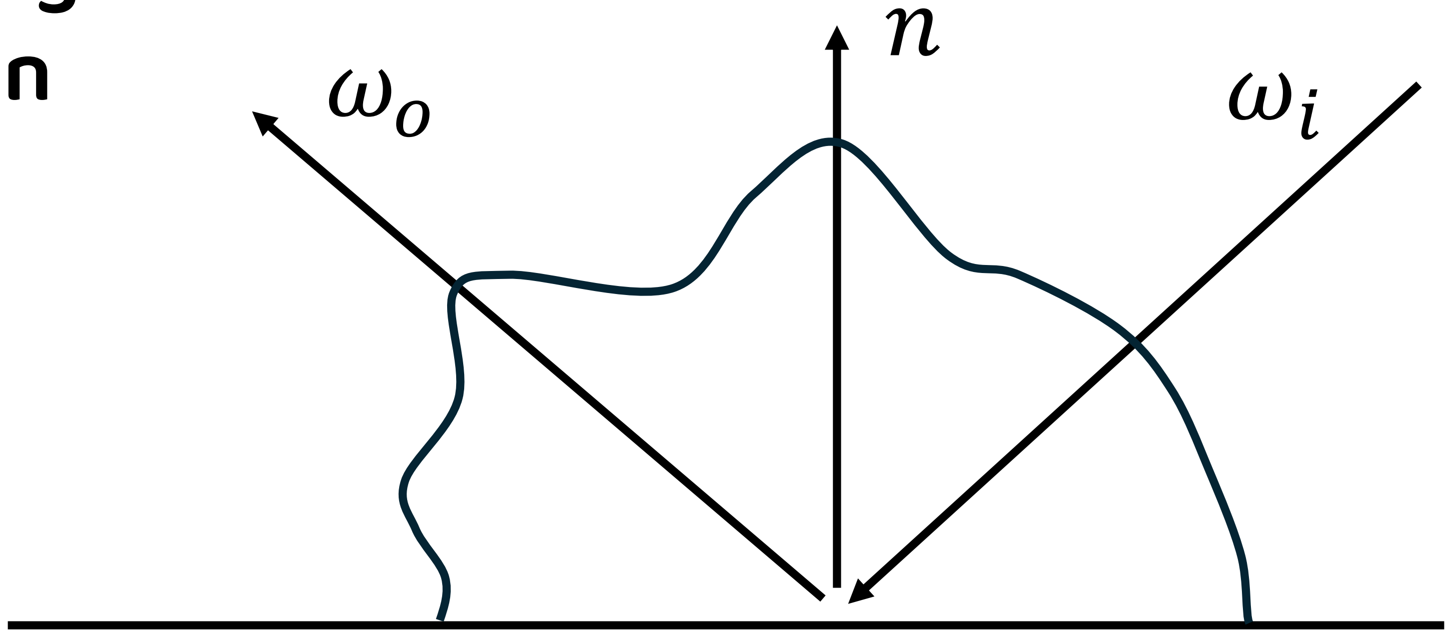


BRDFs Recap

$$L_o(\omega_o) = \int_{H^2} f_r(\omega, \omega_o) L_i(\omega)\, (\omega \cdot n) d\omega$$

Scattering equation

BRDF – bidirectional reflectance distribution function



Scattering Equation

$$L_o(\omega_o) = \int_{H^2} f_r(\omega, \omega_o) L_i(\omega)\,(\omega \cdot n) d\omega$$

$$\approx \frac{1}{N} \sum_{k=1}^{N} f_r(\omega_k, \omega_o) L_i(\omega_k)(\omega_k \cdot n) \frac{1}{p(\omega_k)}$$

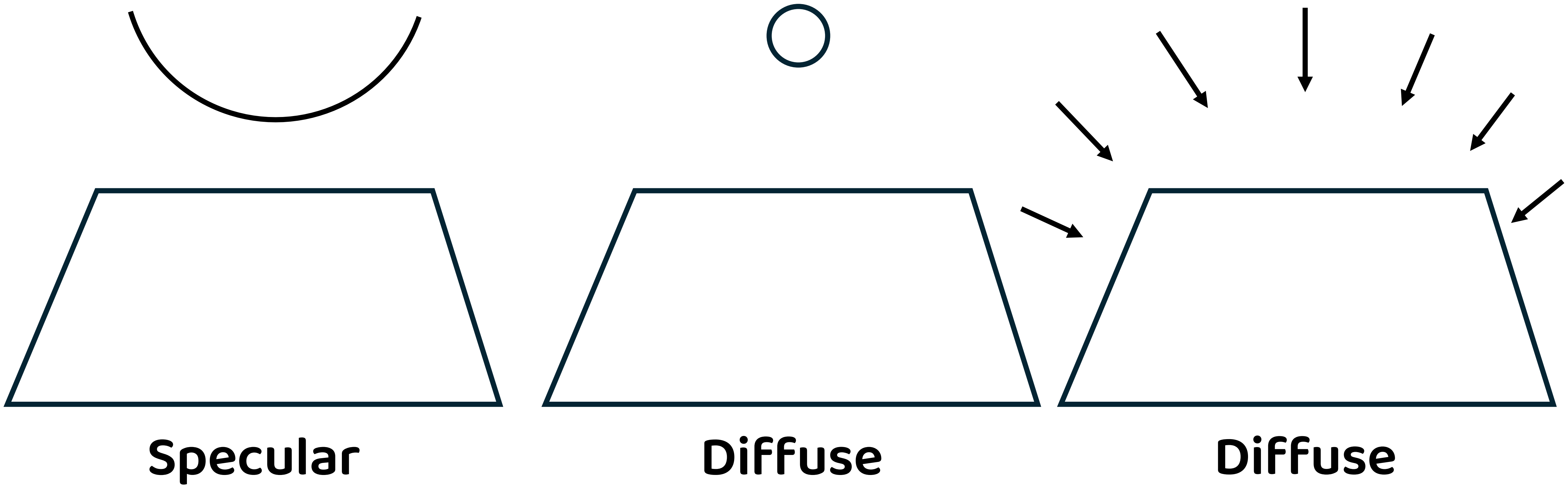
Sampling density on H^2

Uniform Sampling

$$\overline{L_o^{unif}}(\omega_o) \approx \frac{1}{N}\sum_{k=1}^{N} f_r(\omega_k, \omega_o) L_i(\omega_k)(\omega_k \cdot n)(2\pi)$$

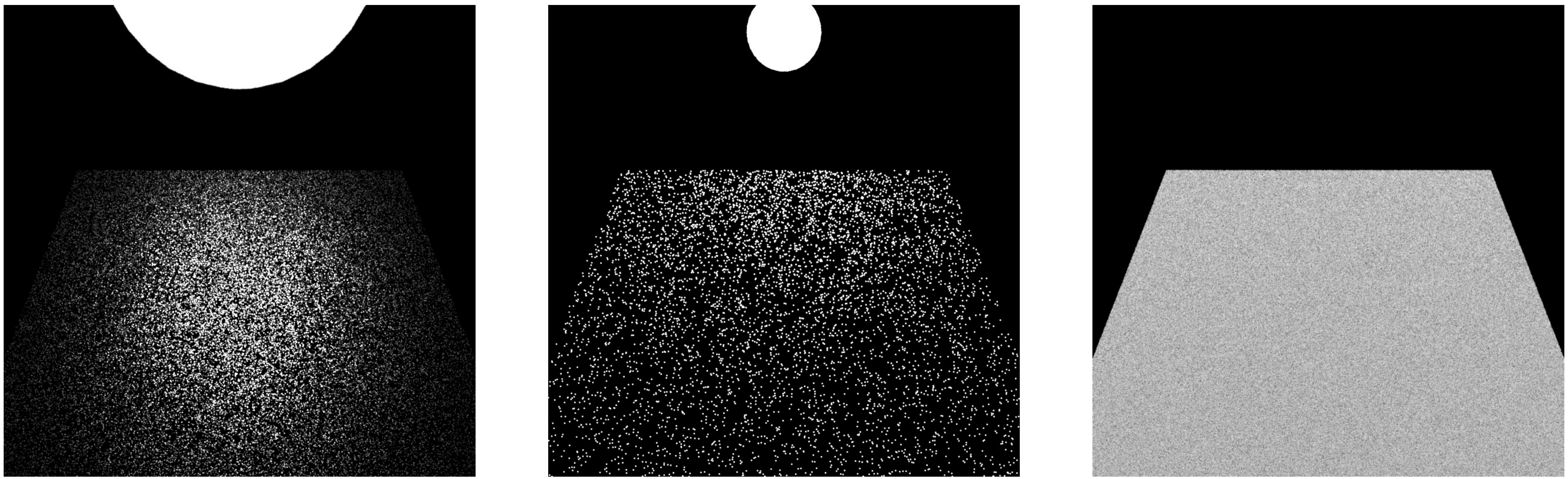
Uniform Sampling

$$\overline{L_o^{unif}}(\omega_o) \approx \frac{1}{N}\sum_{k=1}^{N} f_r(\omega_k, \omega_o) L_i(\omega_k)(\omega_k \cdot n)(2\pi)$$



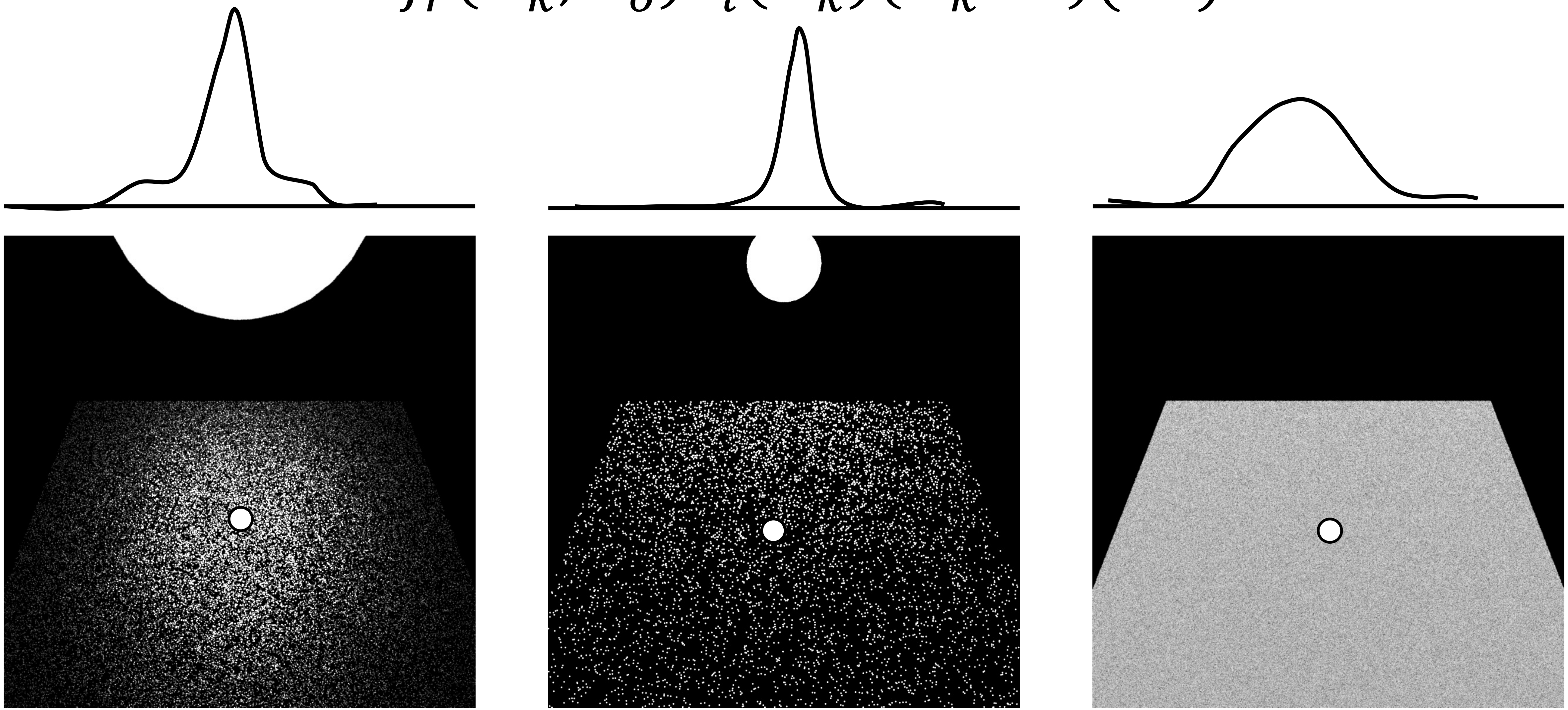
Uniform Sampling

$$\overline{L_o^{unif}}(\omega_o) \approx \frac{1}{N}\sum_{k=1}^{N} f_r(\omega_k, \omega_o) L_i(\omega_k)(\omega_k \cdot n)(2\pi)$$



Uniform Sampling

$$f_r(\omega_k, \omega_o) L_i(\omega_k)(\omega_k \cdot n)(2\pi)$$



Importance Sampling Reminder

$$I = \int_D f(x)dx$$

$$\bar{I} = \frac{1}{N}\sum_{k=1}^{N}\frac{f(X_k)}{p(X_k)}$$

X_k **i.i.d. samples. Their distribution has density** $\boldsymbol{p}$**.**

$$Var(\bar{I}) = \frac{1}{N}Var\left(\frac{f(X_k)}{p(X_k)}\right)$$

Scattering Equation

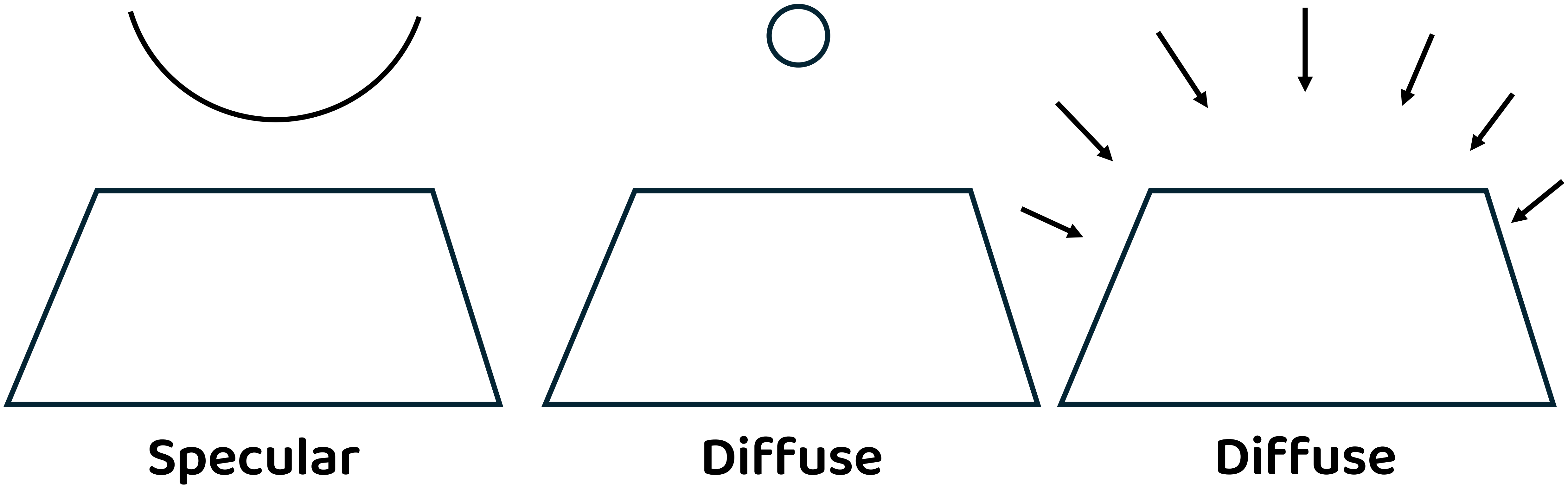
$$L_o(\omega_o) = \int_{H^2} f_r(\omega, \omega_o) L_i(\omega)\,(\omega \cdot n) d\omega$$

$$\approx \frac{1}{N} \sum_{k=1}^{N} f_r(\omega_k, \omega_o) L_i(\omega_k)(\omega_k \cdot n) \frac{1}{p(\omega_k)}$$

Sampling density on H^2

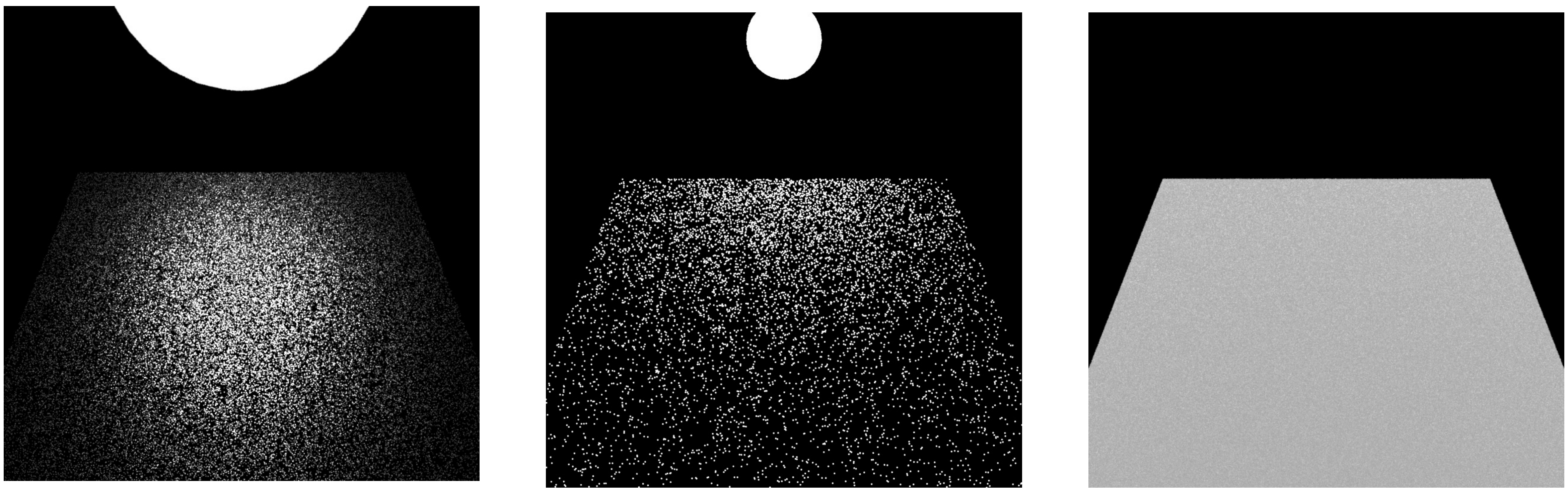
Cosine Sampling

$$\overline{L_o^{cos}}(\omega_o) \approx \frac{1}{N}\sum_{k=1}^{N} f_r(\omega_k, \omega_o) L_i(\omega_k)(\omega_k \cdot n)(\frac{\pi}{(\omega_k \cdot n)})$$



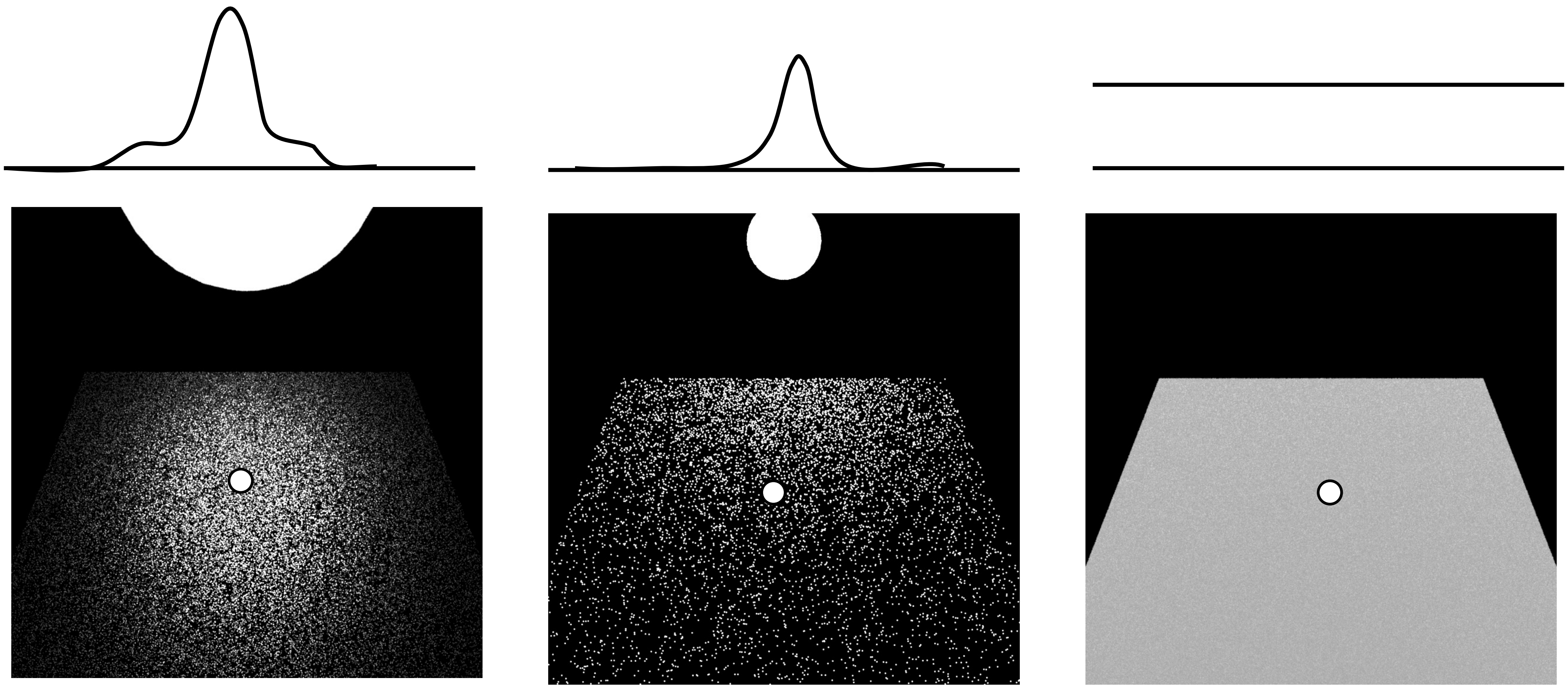
Cosine Sampling

$$\overline{L_o^{cos}}(\omega_o) \approx \frac{1}{N}\sum_{k=1}^{N} f_r(\omega_k, \omega_o) L_i(\omega_k)(\omega_k \cdot n)\left(\frac{\pi}{(\omega_k \cdot n)}\right)$$



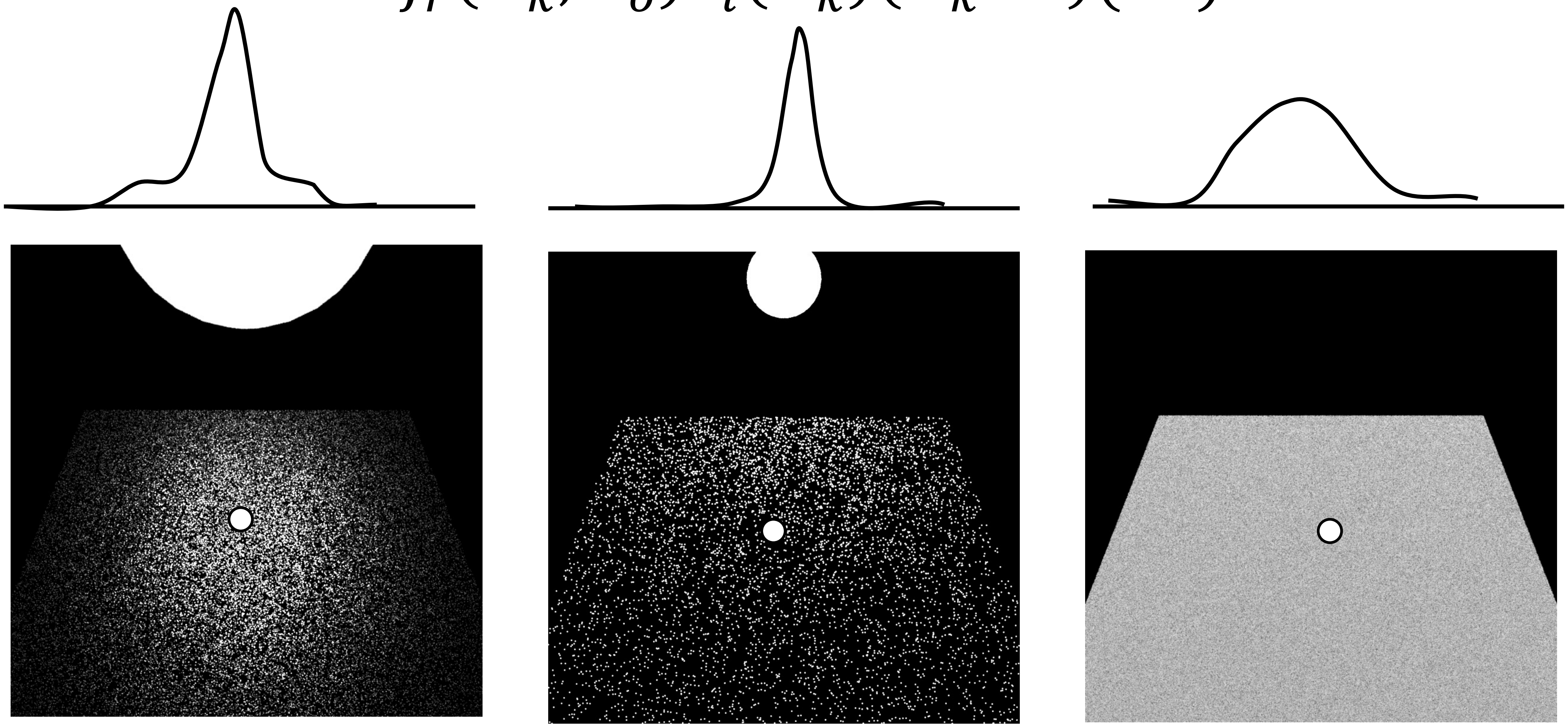
Cosine Sampling

$$f_r(\omega_k, \omega_o) L_i(\omega_k) \pi$$



Uniform Sampling

$$f_r(\omega_k, \omega_o) L_i(\omega_k)(\omega_k \cdot n)(2\pi)$$



Scattering Equation

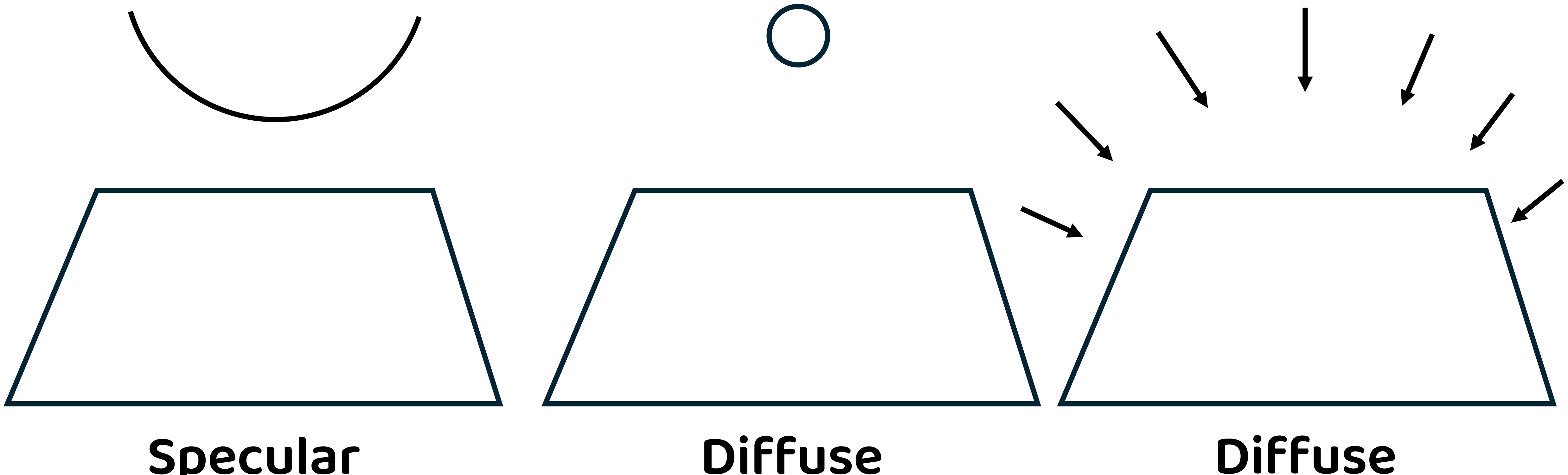
$$L_o(\omega_o) = \int_{H^2} f_r(\omega, \omega_o) L_i(\omega)\,(\omega \cdot n) d\omega$$

$$\approx \frac{1}{N} \sum_{k=1}^{N} f_r(\omega_k, \omega_o) L_i(\omega_k)(\omega_k \cdot n) \frac{1}{p(\omega_k)}$$

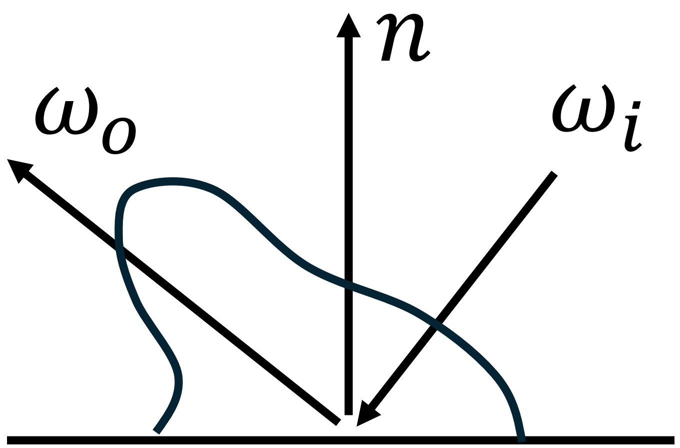
Sampling density on H^2

BRDF Sampling

$$\overline{L_o^{brdf}}(\omega_o) \approx \frac{1}{N}\sum_{k=1}^{N} f_r(\omega_k,\omega_o)L_i(\omega_k)(\omega_k \cdot n)(\frac{C}{\widetilde{f_r}(\omega_k,\omega_o)})$$

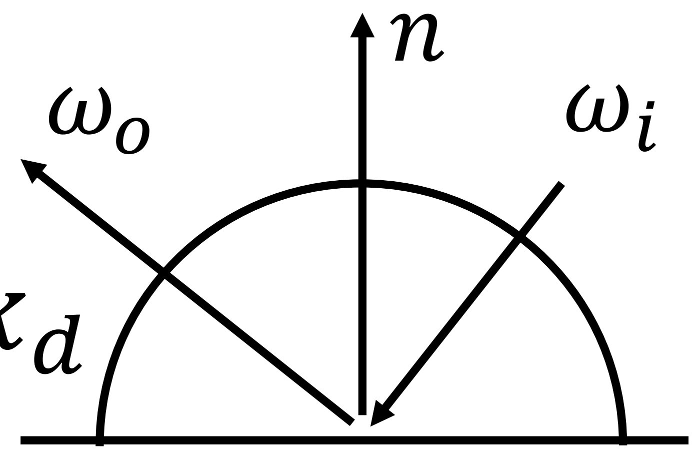


How to sample BRDFs?



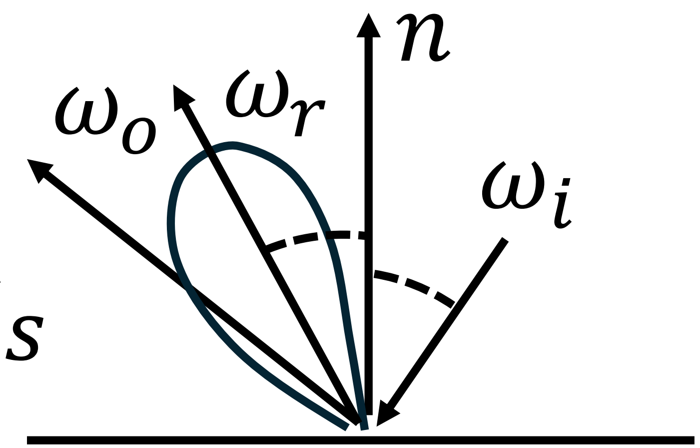
$= k_d$

Lambertian diffuse component



$+ k_s$

Specular lobe



$$f_r^{Phong}(\omega_i, \omega_o) = \frac{k_d}{\pi} + k_s \frac{(\alpha + 1)(\omega_o, \omega_r)^{\alpha}}{2\,\pi}$$

How to sample BRDFs?

$$f_r^{Phong}(\omega_i, \omega_o) = \frac{k_d}{\pi} + k_s \frac{(\alpha + 1)(\omega_o, \omega_r)^\alpha}{2\pi}$$

Bernoulli random variable → **Sample chosen lobe**

$$Pr(diffuse) = \frac{k_d}{k_d + k_s}$$

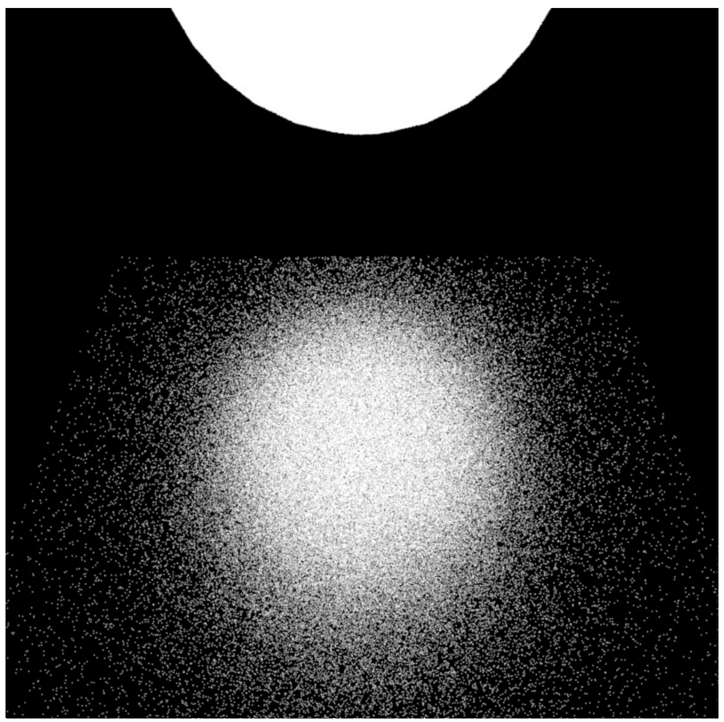
$$Pr(specular) = \frac{k_s}{k_d + k_s}$$

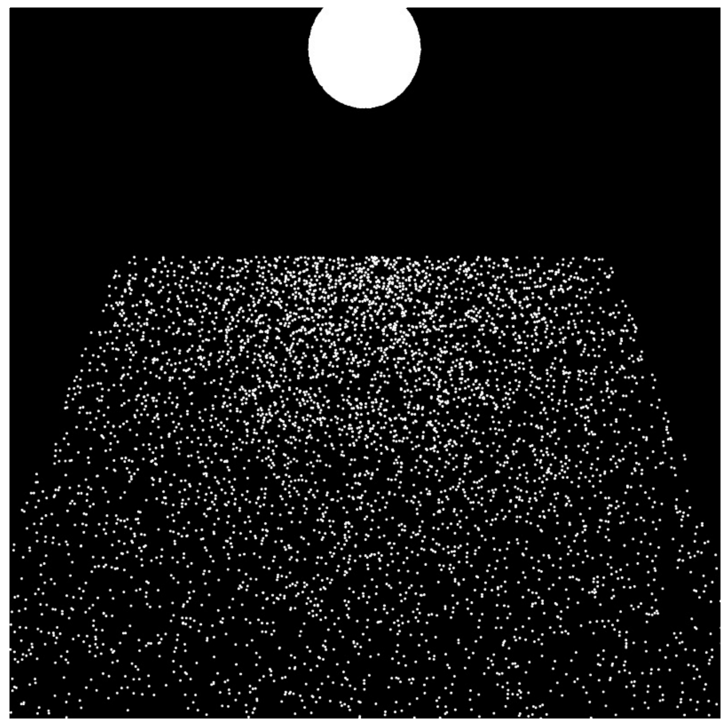
Technical detail

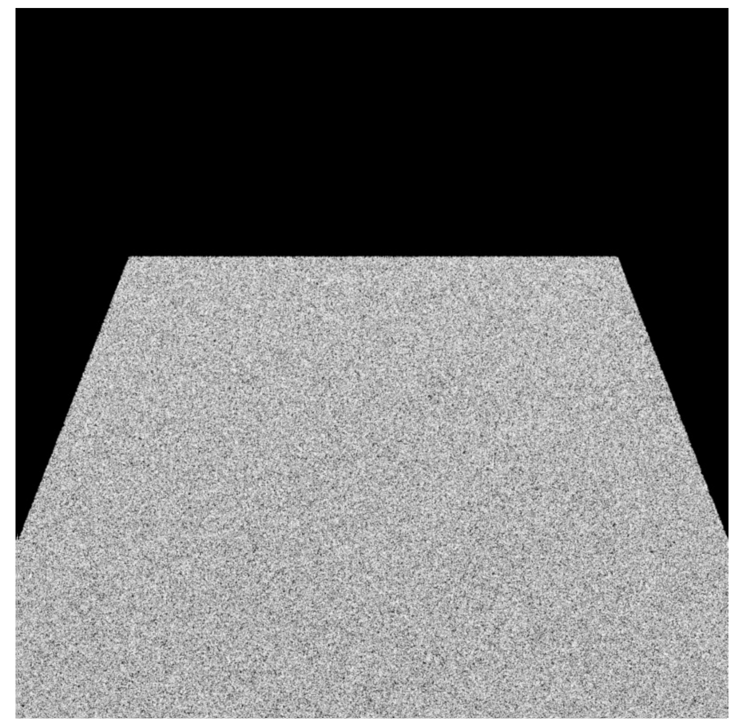
How many random samples are needed?

BRDF Sampling

$$\overline{L_o^{brdf}}(\omega_o) \approx \frac{1}{N}\sum_{k=1}^{N} f_r(\omega_k,\omega_o)L_i(\omega_k)(\omega_k \cdot n)(\frac{C}{\widetilde{f_r}(\omega_k,\omega_o)})$$

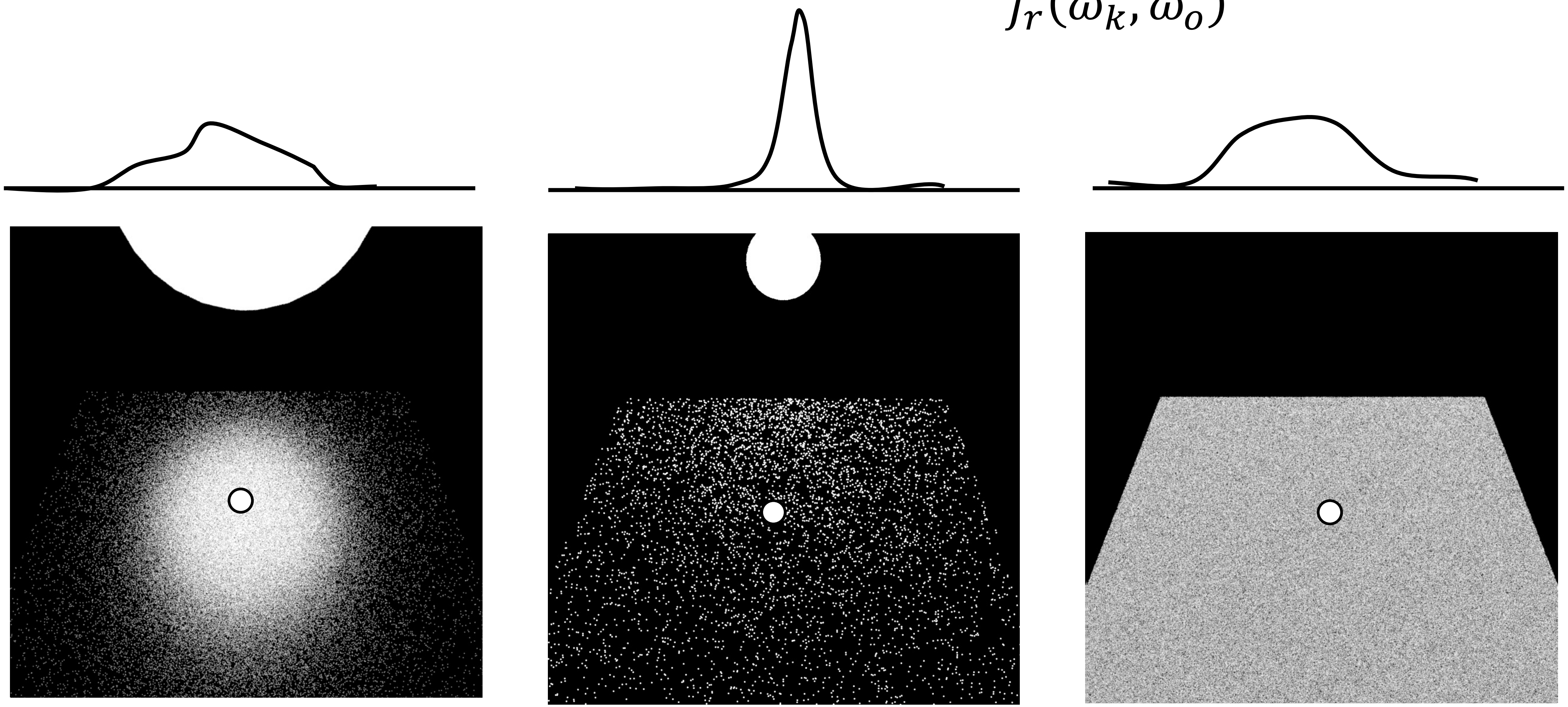






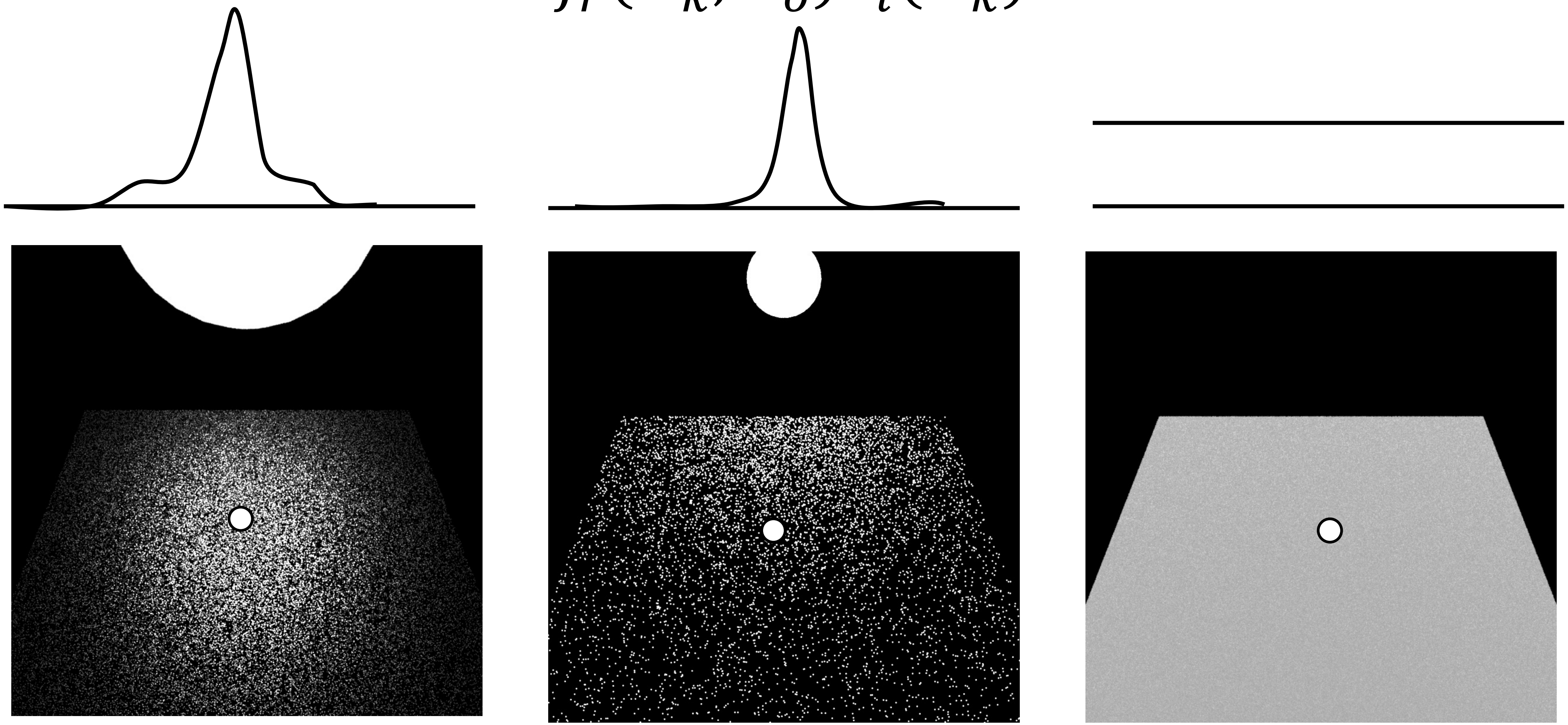
BRDF Sampling

$$f_r(\omega_k, \omega_o) L_i(\omega_k)(\omega_k \cdot n)\left(\frac{C}{\widetilde{f_r}(\omega_k, \omega_o)}\right)$$

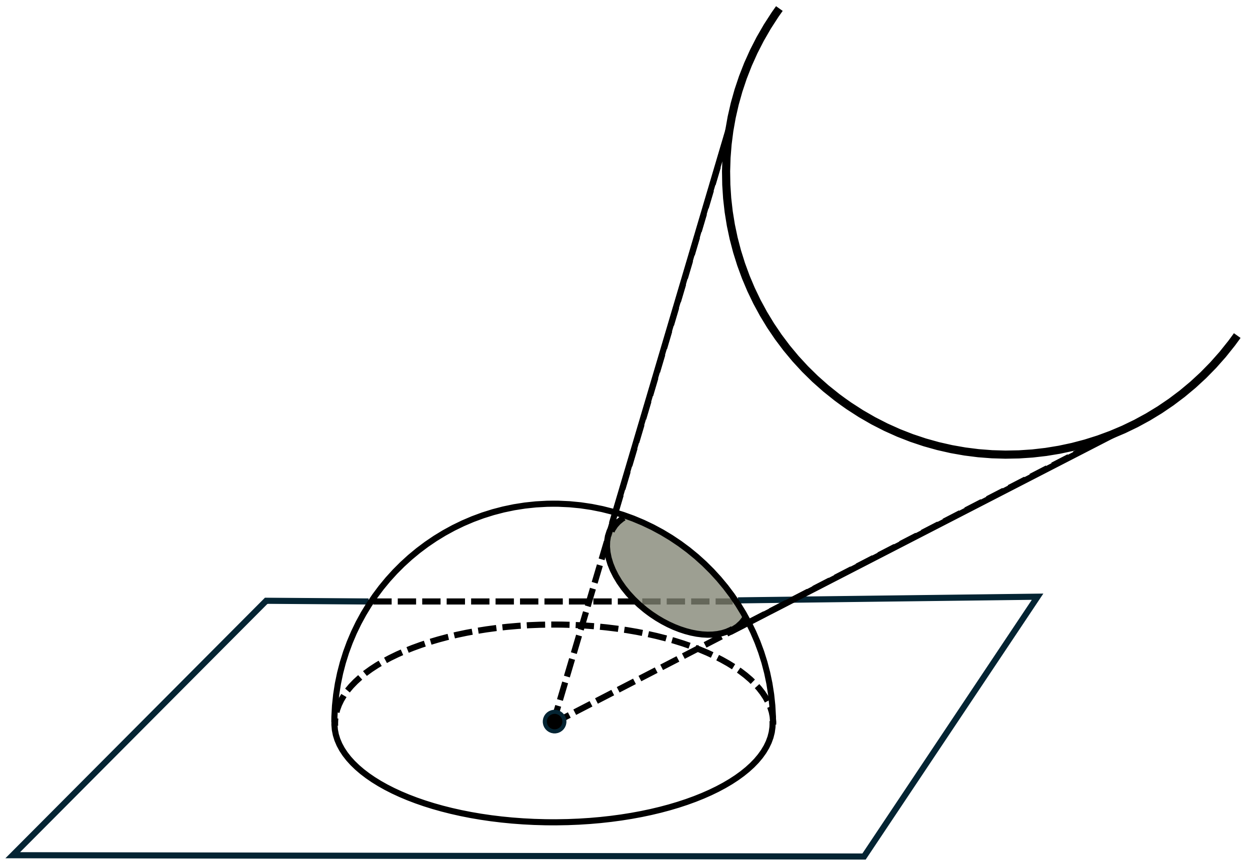


Cosine Sampling

$$f_r(\omega_k, \omega_o) L_i(\omega_k) \pi$$

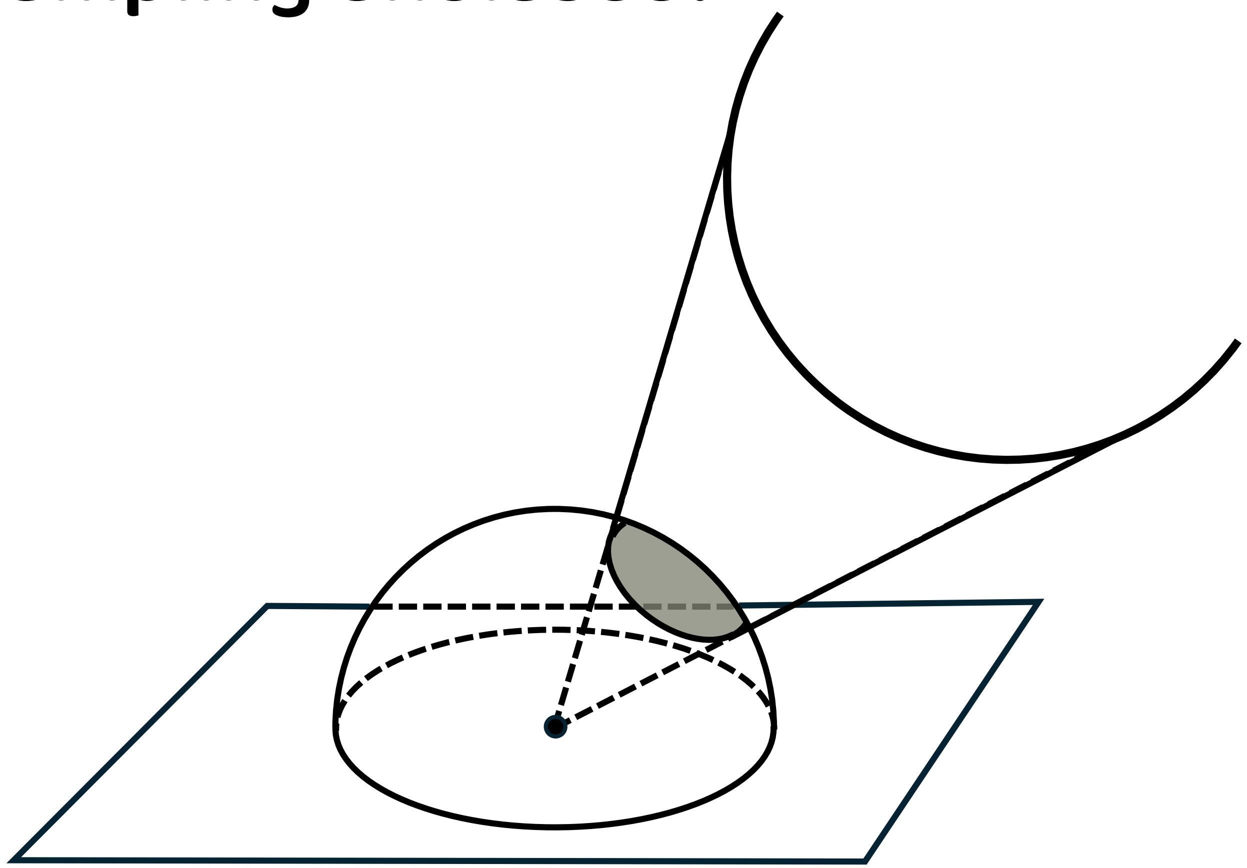


Emitter Sampling



Emitter Sampling

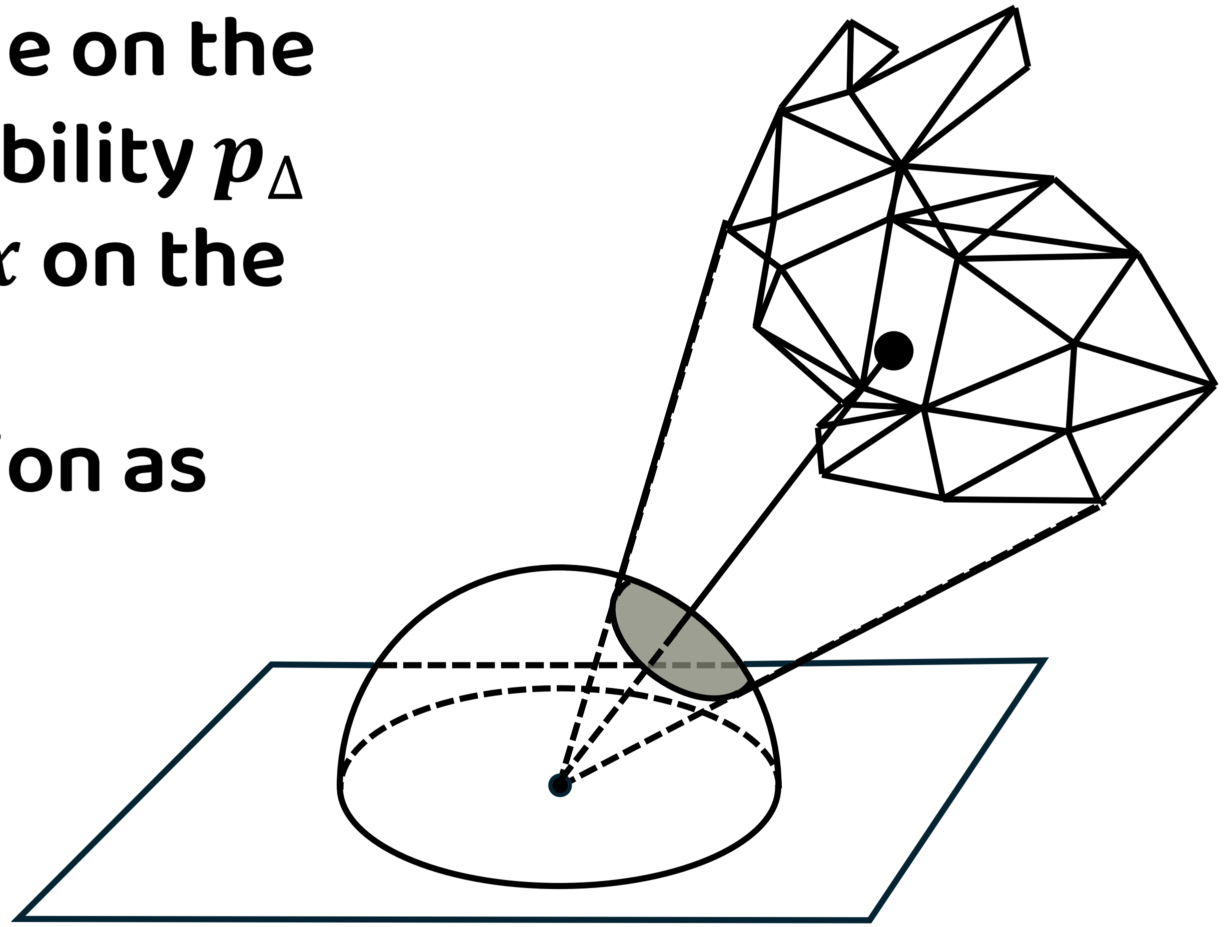
When is such sampling unbiased?



Emitter Sampling: Area Lights

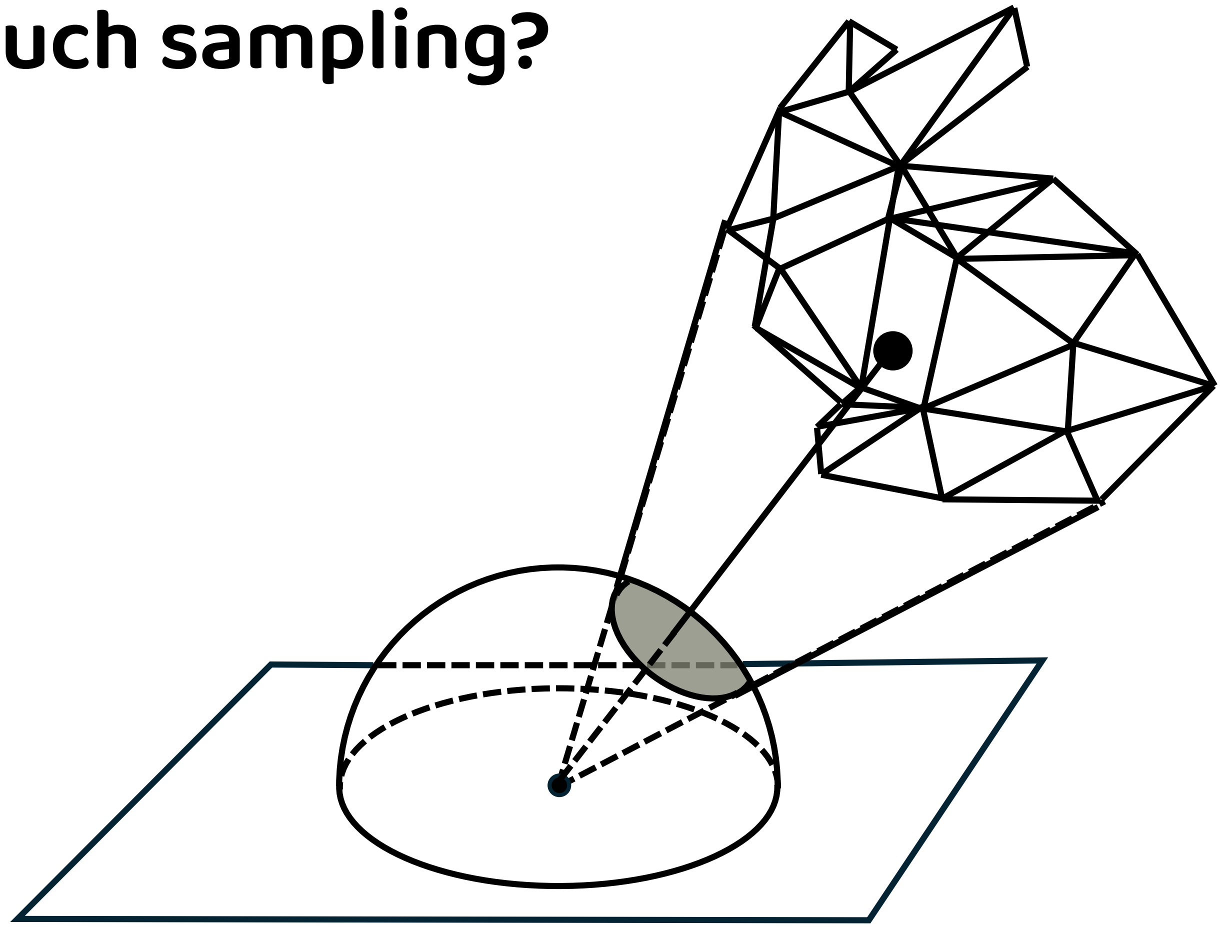
1. Sample a triangle on the light with probability p_Δ
2. Sample a point x on the triangle
3. Compute direction as

$$\omega = \frac{(x - p)}{\|x - p\|}$$



Emitter Sampling: Area Lights

What is the pdf of such sampling?

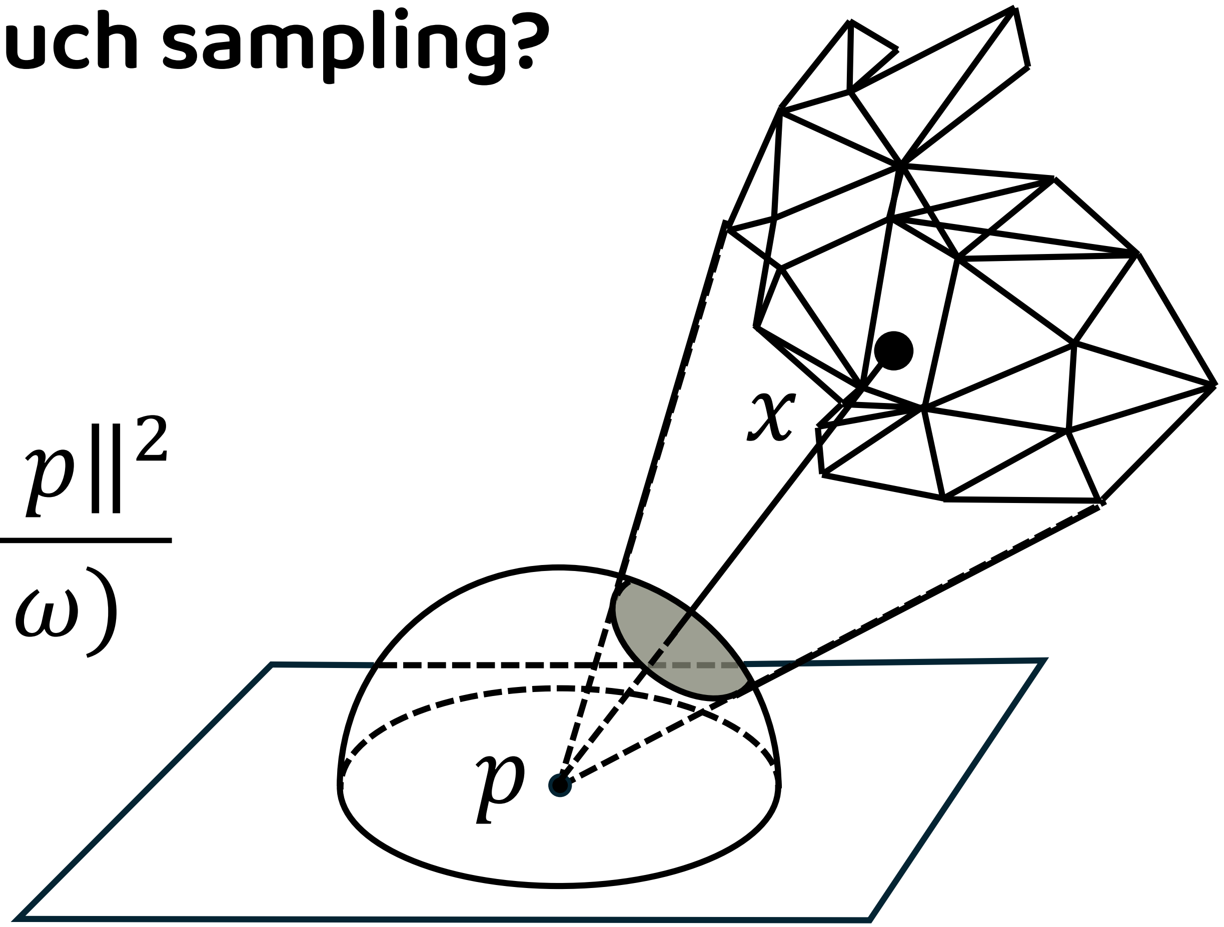


Emitter Sampling: Area Lights

What is the pdf of such sampling?

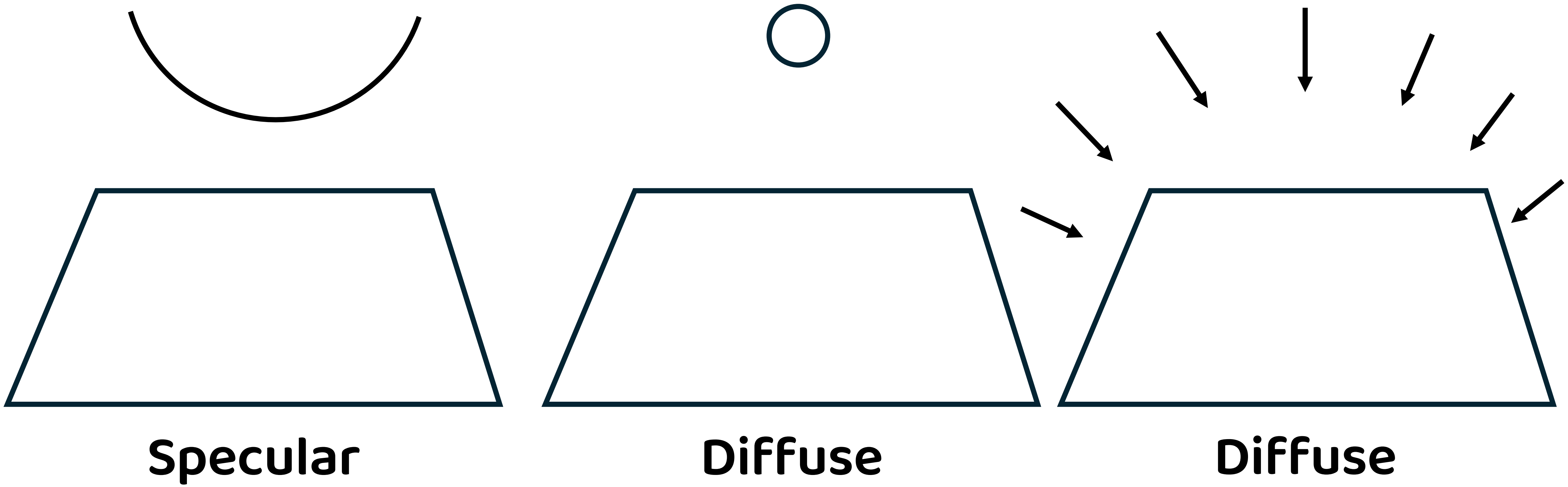
$$\omega = \frac{(x - p)}{\|x - p\|}$$

$$p(\omega) = p_{\Delta} \frac{1}{A_{\Delta}} \frac{\|x - p\|^2}{(n_{\Delta}, \omega)}$$



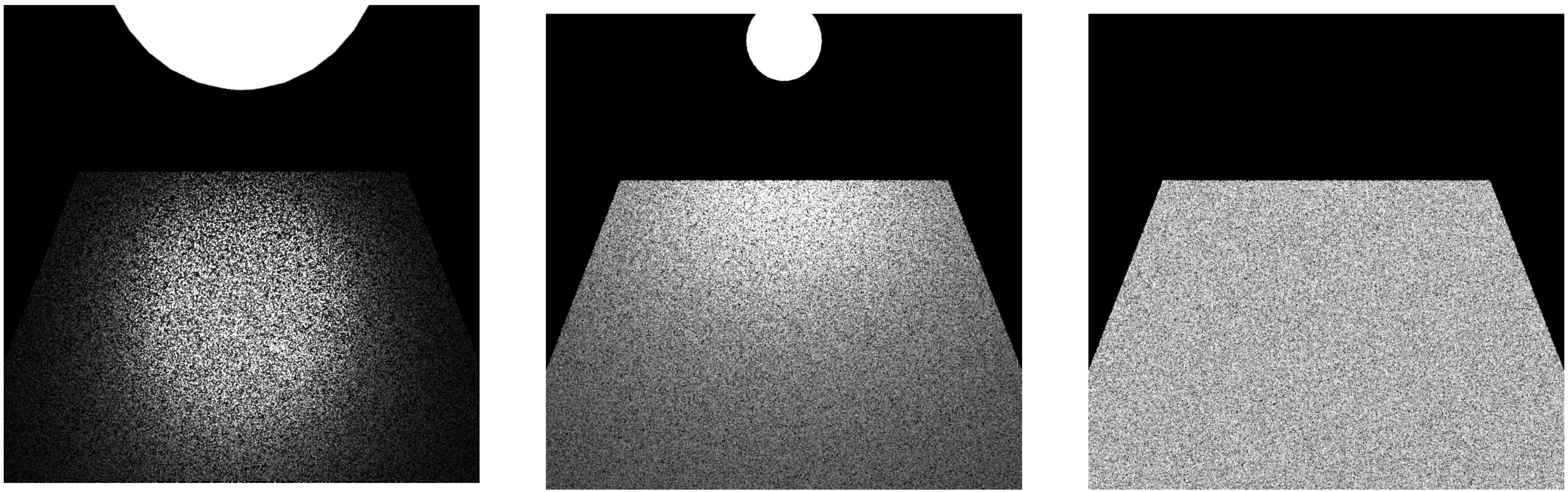
Emitter Sampling: Area Lights

$$\overline{L_o^{em}}(\omega_o) \approx \frac{1}{N}\sum_{k=1}^{N} f_r(\omega_k, \omega_o) L_i(\omega_k)(\omega_k \cdot n)\left(\frac{A_\Delta(n_\Delta, \omega_k)}{p_\Delta \|x - p\|^2}\right)$$



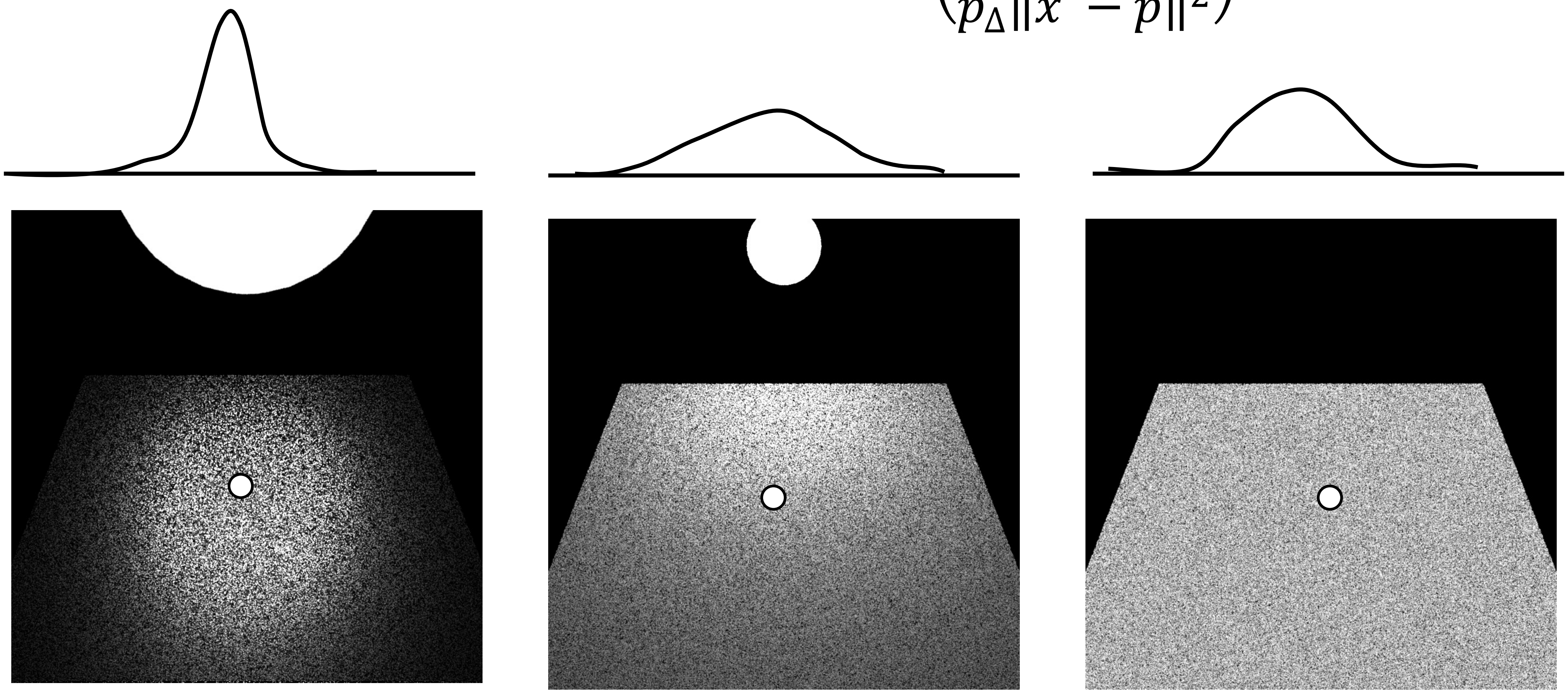
Emitter Sampling: Area Lights

$$\overline{L_o^{em}}(\omega_o) \approx \frac{1}{N}\sum_{k=1}^{N} f_r(\omega_k, \omega_o) L_i(\omega_k)(\omega_k \cdot n)\left(\frac{A_\Delta(n_\Delta, \omega_k)}{p_\Delta \|x - p\|^2}\right)$$

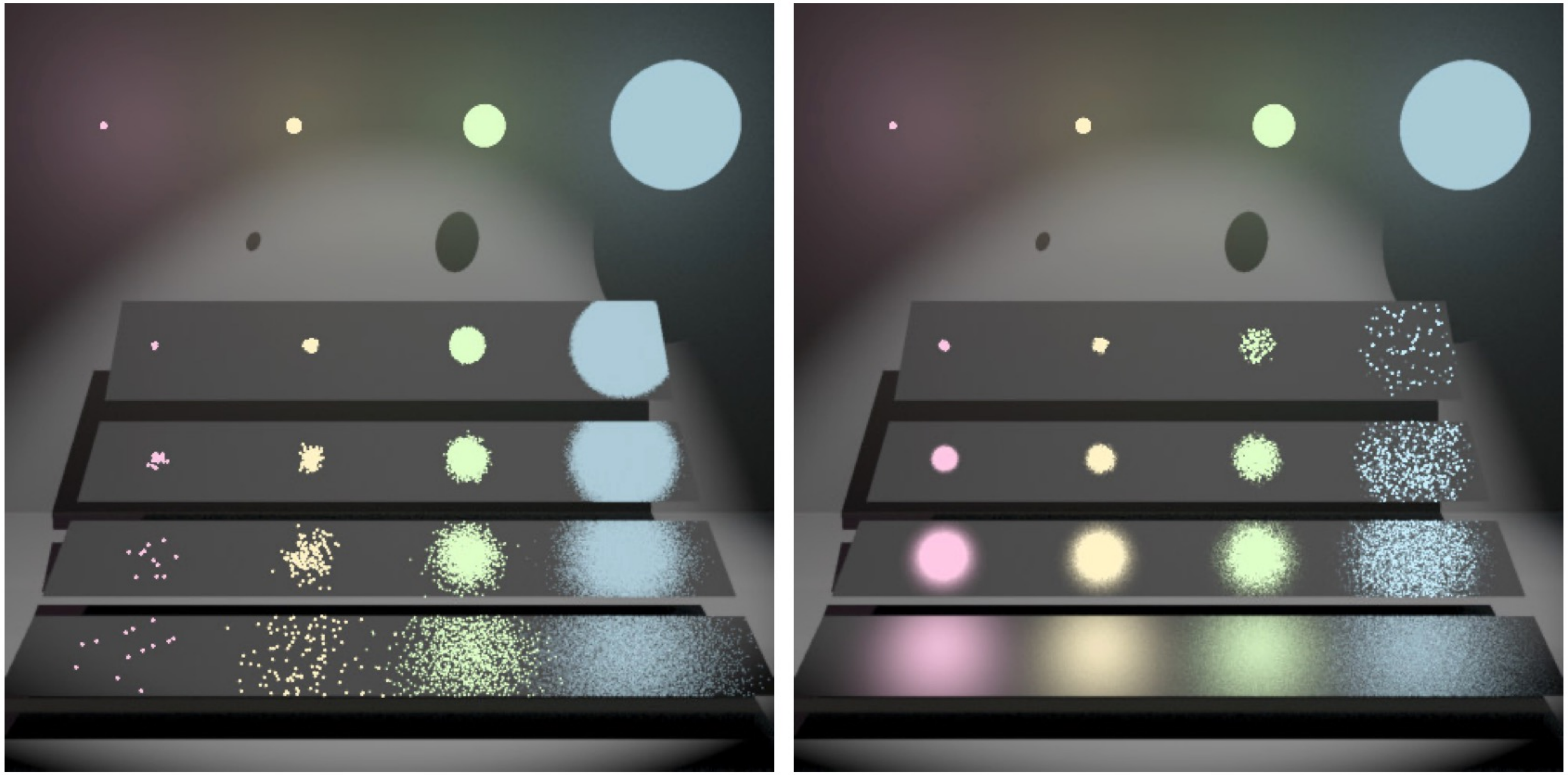


Emitter Sampling: Area Lights

$$f_r(\omega_k, \omega_o) L_i(\omega_k)(\omega_k \cdot n) \left(\frac{A_\Delta(n_\Delta, \omega_k)}{p_\Delta \|x - p\|^2} \right)$$



Emitter Sampling: Area Lights



(a) Sampling the BSDF (b) Sampling the light sources

Source: Eric Veach

Emitter Sampling: Area Lights

What if there are multiple area lights?

Emitter Sampling: Area Lights

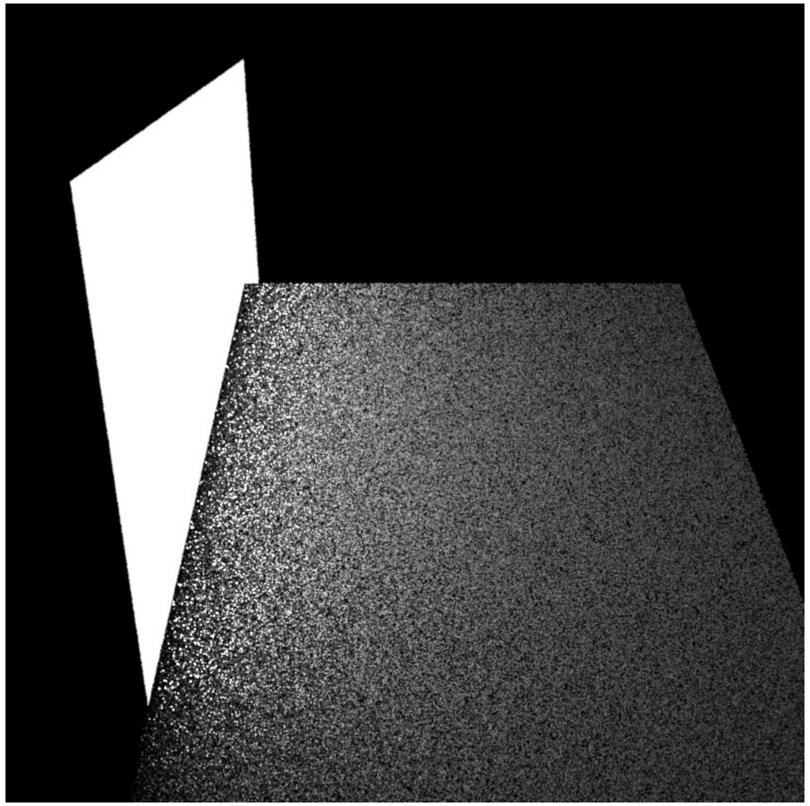
$$f_r(\omega_k, \omega_o) L_i(\omega_k)(\omega_k \cdot n) \left(\frac{A_\Delta(n_\Delta, \omega_k)}{p_\Delta \|x \; - p\|^2} \right)$$

Is there an issue with such sampling density?

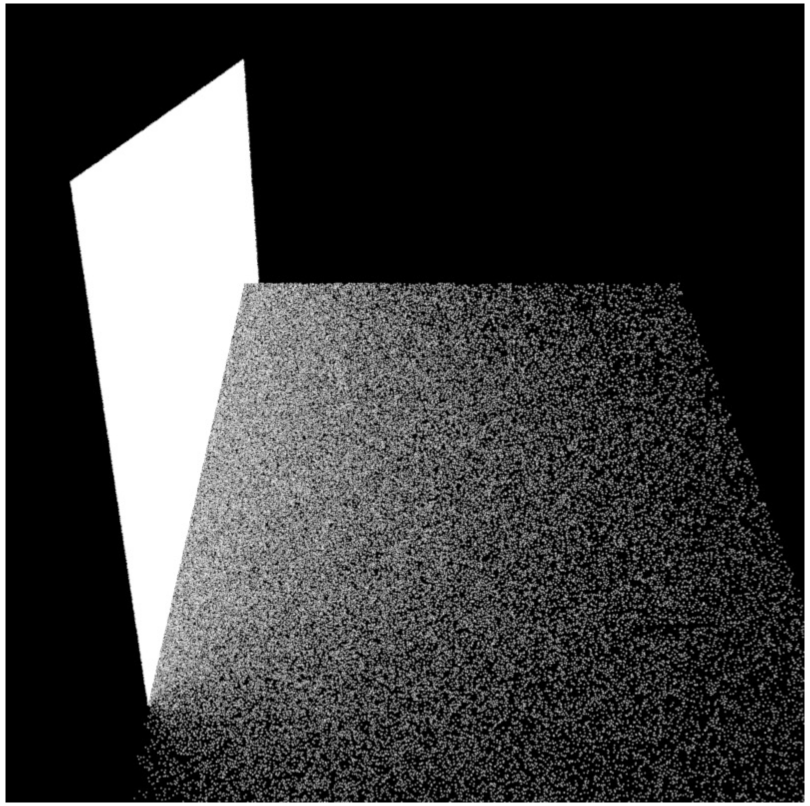
Emitter Sampling: Area Lights

$$f_r(\omega_k, \omega_o) L_i(\omega_k)(\omega_k \cdot n)\left(\frac{A_\Delta(n_\Delta, \omega_k)}{p_\Delta \|x - p\|^2}\right)$$

Emitter Sampling



Cosine Sampling



Other importance sampling strategies?

- Multiple importance sampling (next lecture)
- Cosine weighted solid angle sampling
- BRDF weighted polygon sampling
- Path guiding methods