

# Microfacet models for reflection and refraction

Steve Marschner

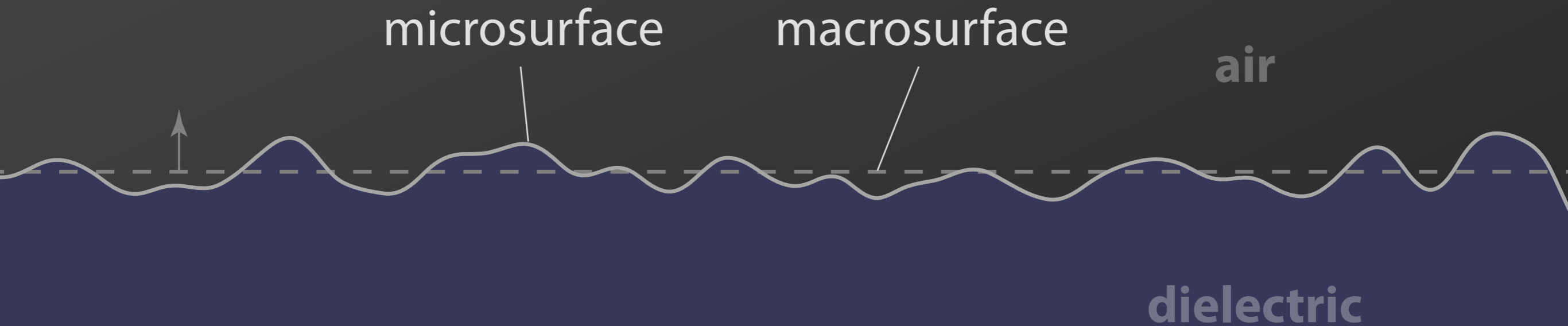
Cornell University CS 5625 Spring 2016

(based on presentation for  
Walter, Marschner, Li, and Torrance EGSR '07)

# Microfacet scattering models

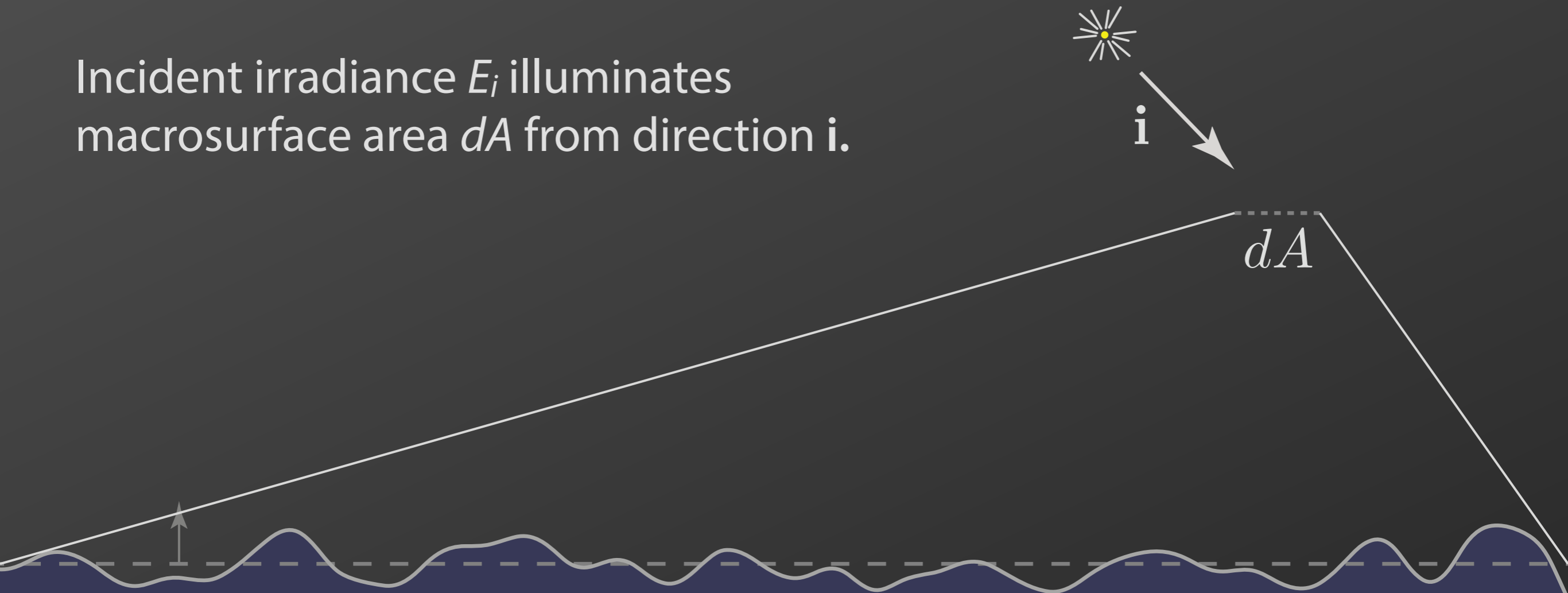
## Rough dielectric surface

- smooth at wavelength scale
- rough at microscale
- flat at macroscale



# Microfacet scattering models

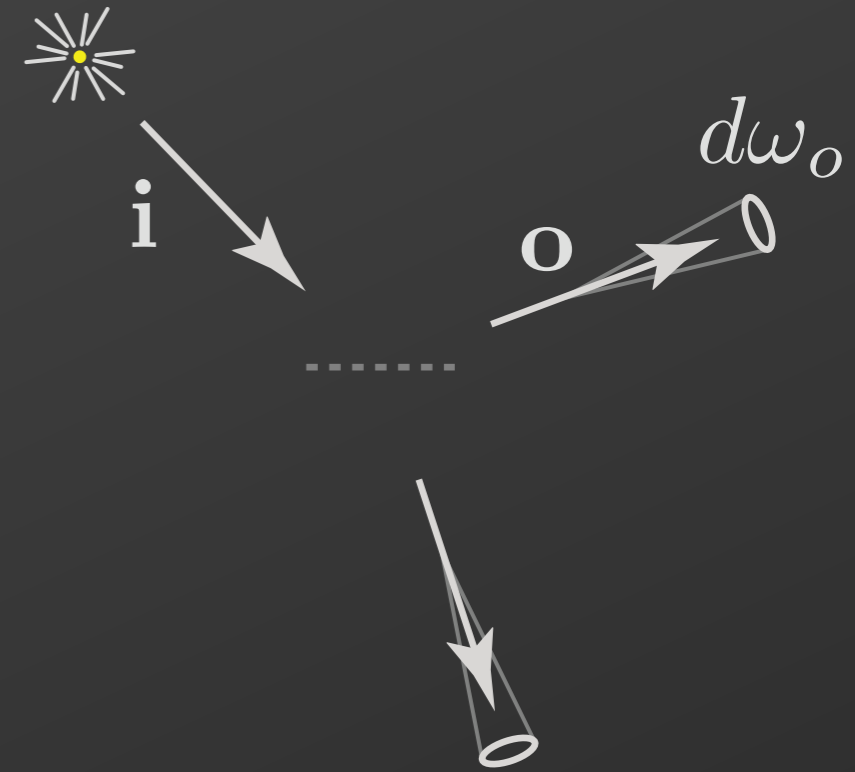
Incident irradiance  $E_i$  illuminates  
macrosurface area  $dA$  from direction  $\mathbf{i}$ .



# Microfacet scattering models

Incident irradiance  $E_i$  illuminates macrosurface area  $dA$  from direction  $\mathbf{i}$ .

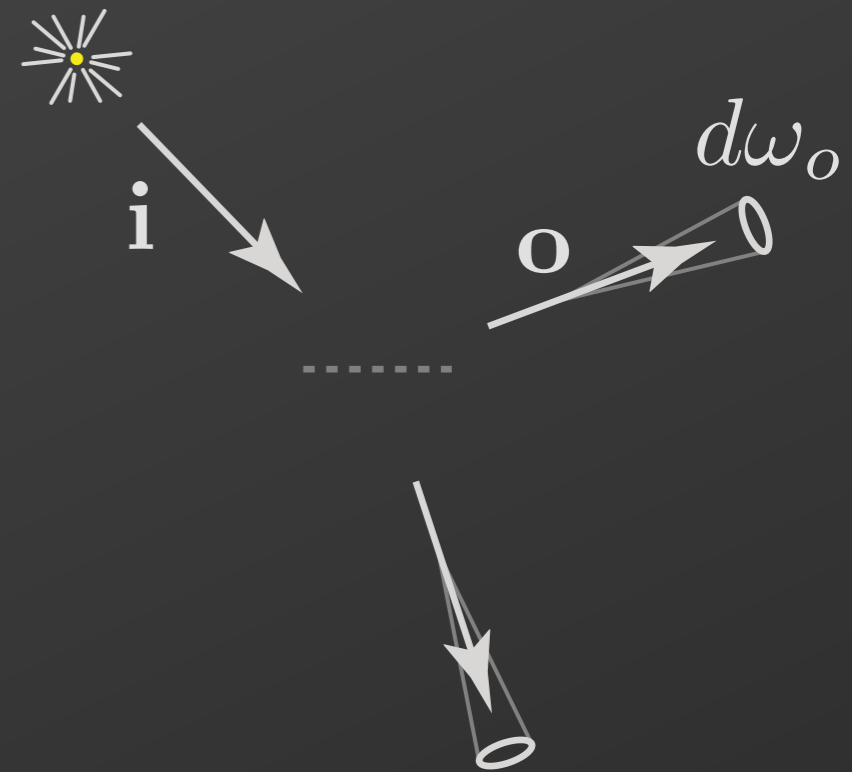
Scattered radiance  $L_r$  or  $L_t$  measured in direction  $\mathbf{o}$  in solid angle  $d\omega_o$ .



# Microfacet scattering models

Incident irradiance  $E_i$  illuminates macrosurface area  $dA$  from direction  $\mathbf{i}$ .

Scattered radiance  $L_r$  or  $L_t$  measured in direction  $\mathbf{o}$  in solid angle  $d\omega_o$ .



$$f_s(\mathbf{i}, \mathbf{o}) = \frac{L_{r,t}}{E_i}$$

**Bidirectional Scattering Distribution Function**

“half-vector” function

$$\mathbf{h}(\mathbf{i}, \mathbf{o})$$

normal distribution

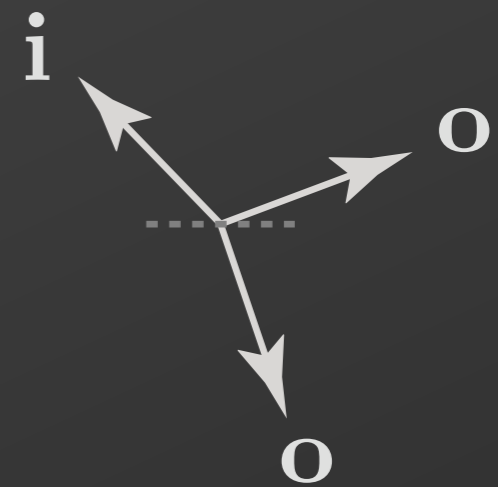
$$D(\mathbf{m})$$

shadowing–masking

$$G(\mathbf{i}, \mathbf{o}, \mathbf{m})$$

attenuation

$$\rho(\mathbf{i}, \mathbf{o})$$



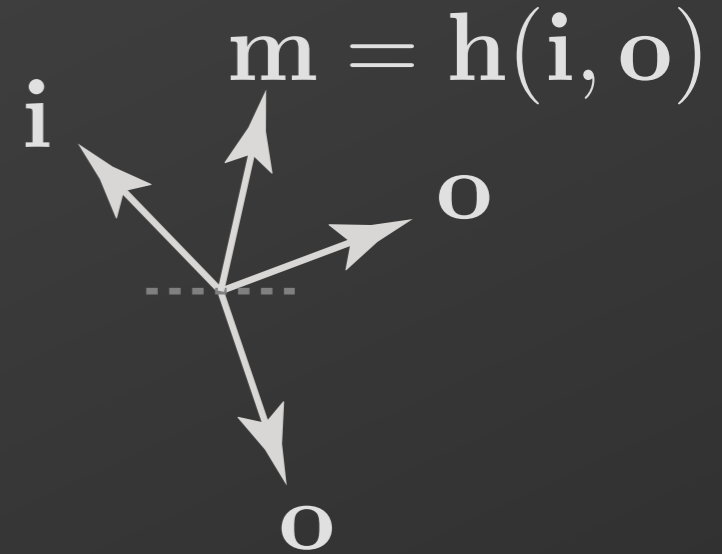
“half-vector” function  
 $\mathbf{h}(\mathbf{i}, \mathbf{o})$

normal distribution  
 $D(\mathbf{m})$

shadowing–masking  
 $G(\mathbf{i}, \mathbf{o}, \mathbf{m})$

attenuation  
 $\rho(\mathbf{i}, \mathbf{o})$

Gives the one microsurface normal  $\mathbf{m}$  that will scatter light from  $\mathbf{i}$  to  $\mathbf{o}$ .



“half-vector” function  
 $\mathbf{h}(\mathbf{i}, \mathbf{o})$

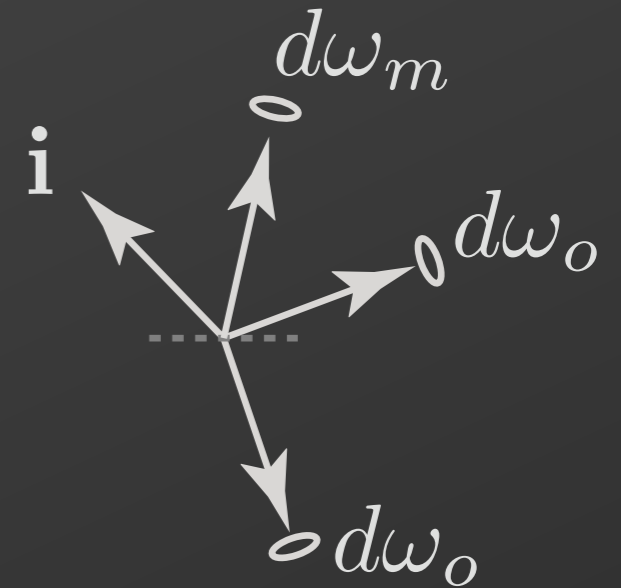
normal distribution  
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attenuation  
 $\rho(\mathbf{i}, \mathbf{o})$

Gives the one microsurface normal  $\mathbf{m}$  that will scatter light from  $\mathbf{i}$  to  $\mathbf{o}$ .

The size of the set of relevant normals  $d\omega_m$  relative to the receiving solid angle  $d\omega_o$  is determined by  $\mathbf{h}$ .





“half-vector” function

$$\mathbf{h}(\mathbf{i}, \mathbf{o})$$

normal distribution

$$D(\mathbf{m})$$

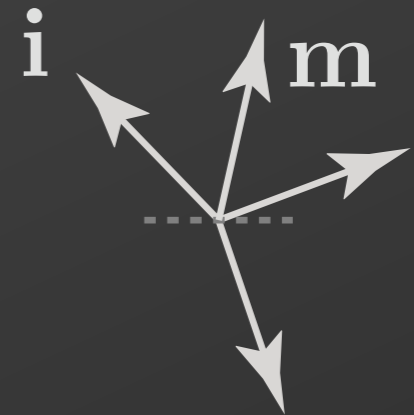
shadowing–masking

$$G(\mathbf{i}, \mathbf{o}, \mathbf{m})$$

attenuation

$$\rho(\mathbf{i}, \mathbf{o})$$

Measures density of microsurface area with respect to microsurface normal.



A diagram of a wavy surface with a differential area element  $dA$ . The surface is represented by a white wavy line on a dark blue background. Below the surface, a horizontal double-headed arrow spans a width, with the label  $dA$  centered below it.

$$dA$$

“half-vector” function

$$\mathbf{h}(\mathbf{i}, \mathbf{o})$$

normal distribution

$$D(\mathbf{m})$$

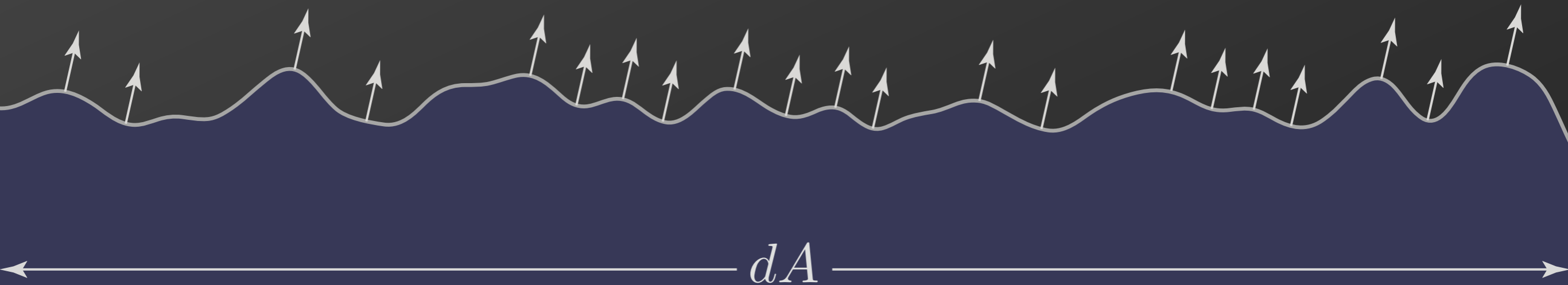
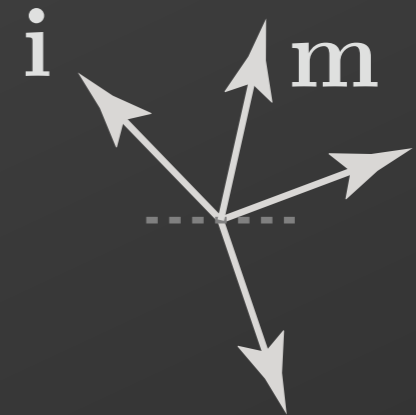
shadowing–masking

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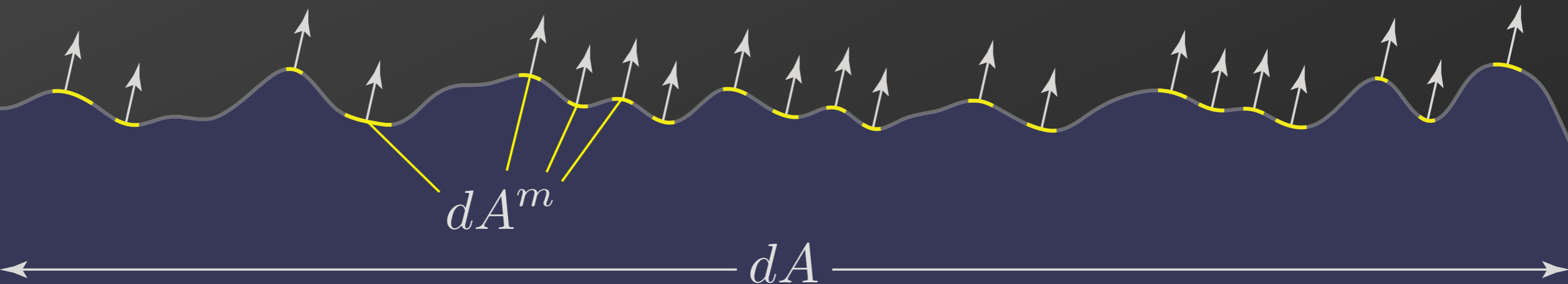
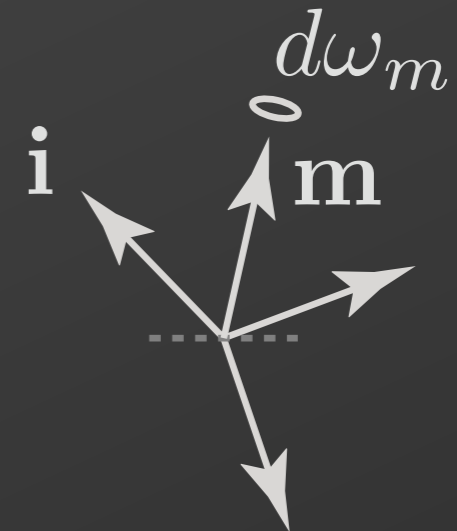
$$G(\mathbf{i}, \mathbf{o}, \mathbf{m})$$

attenuation

$$\rho(\mathbf{i}, \mathbf{o})$$

Measures density of microsurface area with respect to microsurface normal.

The ratio of relevant microsurface area  $dA^m$  to macrosurface area  $dA$  is  $D(\mathbf{m})d\omega_m$ .



$$dA^m = D(\mathbf{m}) d\omega_m dA$$

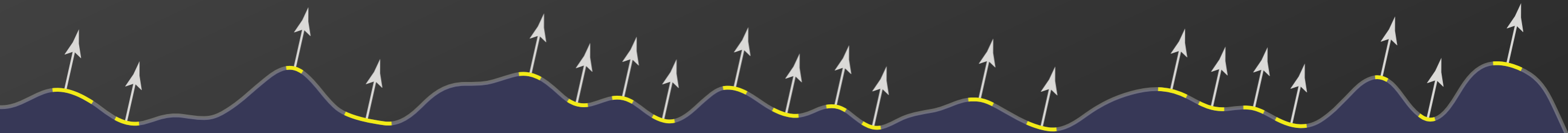
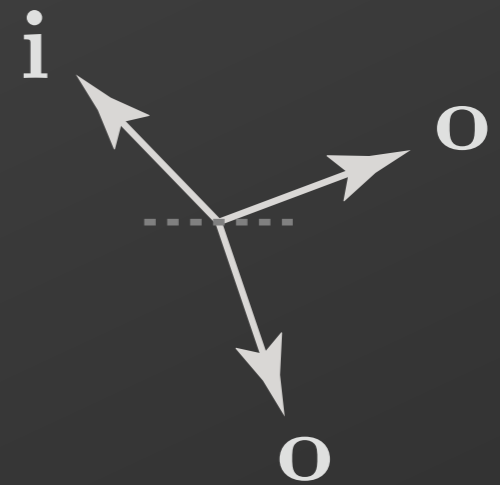
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 $\mathbf{h}(\mathbf{i}, \mathbf{o})$

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shadowing–masking  
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attenuation  
 $\rho(\mathbf{i}, \mathbf{o})$

Measures the fraction of points with microsurface normal  $\mathbf{m}$  that are visible in directions  $\mathbf{i}$  and  $\mathbf{o}$ .



$$dA^m = D(\mathbf{m}) d\omega_m dA$$

“half-vector” function

$$\mathbf{h}(\mathbf{i}, \mathbf{o})$$

normal distribution

$$D(\mathbf{m})$$

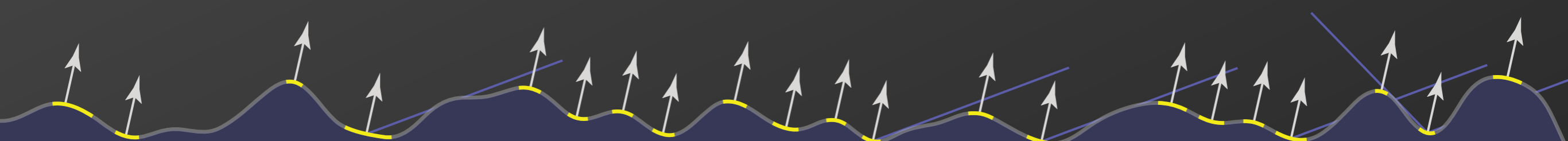
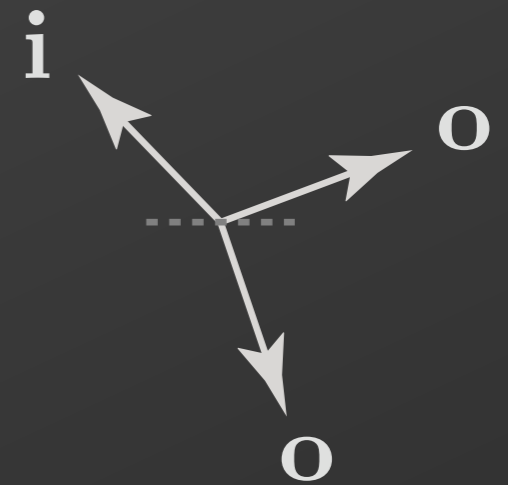
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attenuation

$$\rho(\mathbf{i}, \mathbf{o})$$

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$$dA^m = D(\mathbf{m}) d\omega_m dA$$

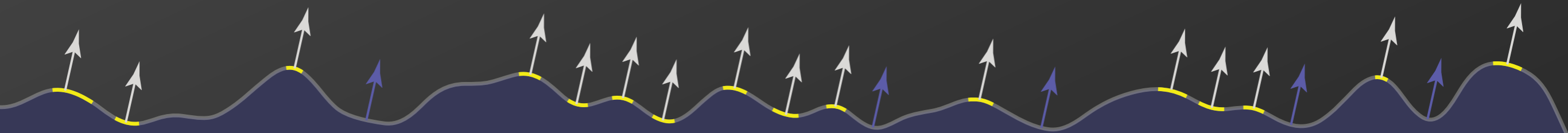
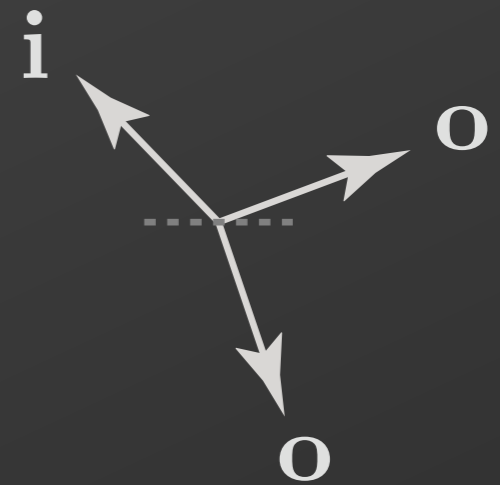
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$$dA^m = D(\mathbf{m}) G(\mathbf{i}, \mathbf{o}, \mathbf{m}) d\omega_m dA$$

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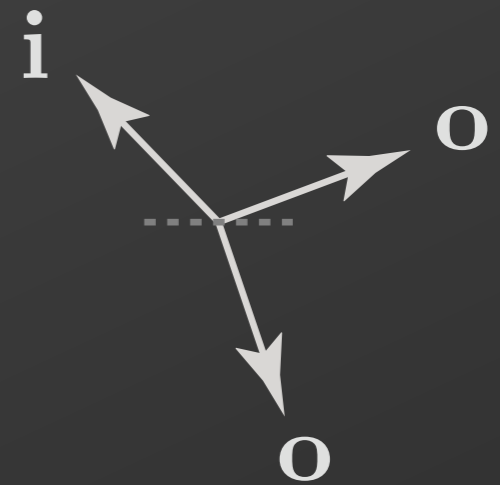
$$G(\mathbf{i}, \mathbf{o}, \mathbf{m})$$

attenuation

$$\rho(\mathbf{i}, \mathbf{o})$$

Measures the fraction of points with microsurface normal  $\mathbf{m}$  that are visible in directions  $\mathbf{i}$  and  $\mathbf{o}$ .

We now know the size of the **scattering area**, which determines how much light reflects.



$$dA^m = D(\mathbf{m}) G(\mathbf{i}, \mathbf{o}, \mathbf{m}) d\omega_m dA$$

“half-vector” function

$$\mathbf{h}(\mathbf{i}, \mathbf{o})$$

normal distribution

$$D(\mathbf{m})$$

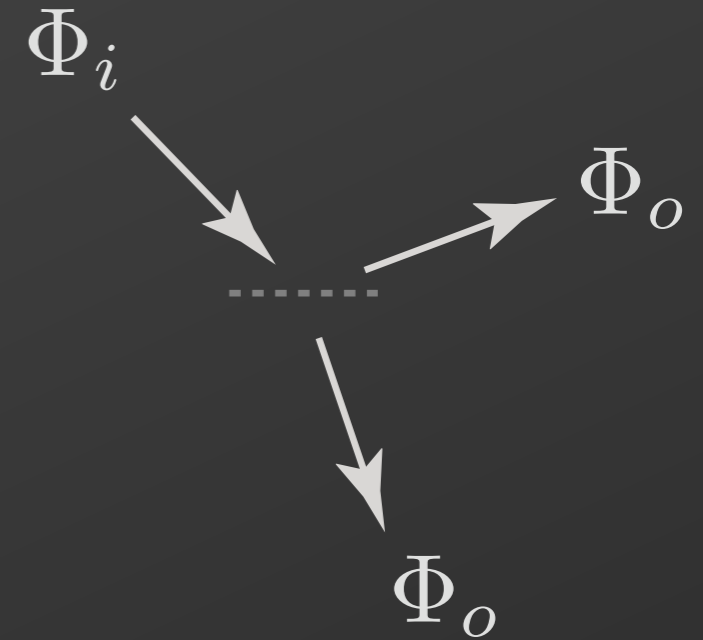
shadowing–masking

$$G(\mathbf{i}, \mathbf{o}, \mathbf{m})$$

attenuation

$$\rho(\mathbf{i}, \mathbf{o})$$

Gives the fraction of the power incident on the **scattering area**  $dA^m$  that is scattered.



$$d\Phi_o^m = \frac{|\mathbf{i} \cdot \mathbf{m}|}{|\mathbf{i} \cdot \mathbf{n}|} \rho(\mathbf{i}, \mathbf{o}) dA^m dE_i$$

$$dA^m = D(\mathbf{m}) G(\mathbf{i}, \mathbf{o}, \mathbf{m}) d\omega_m dA$$



“half-vector” function

$$\mathbf{h}(\mathbf{i}, \mathbf{o})$$

normal distribution

$$D(\mathbf{m})$$

shadowing–masking

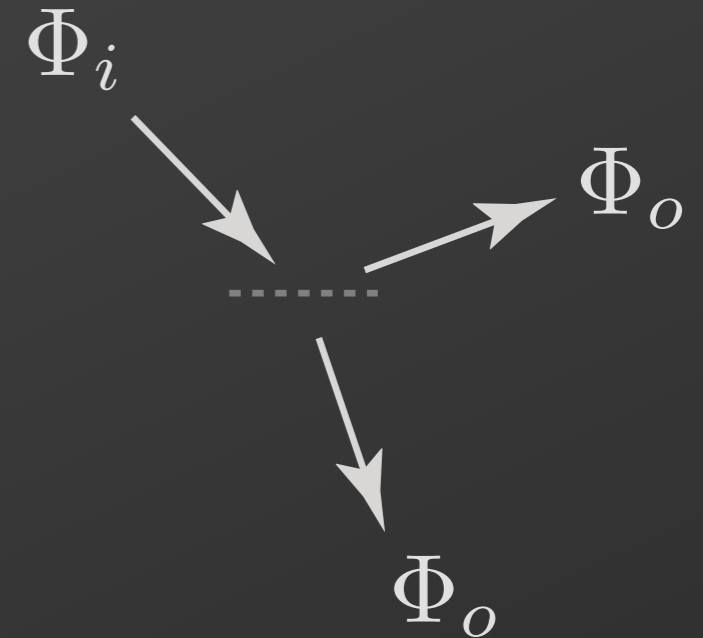
$$G(\mathbf{i}, \mathbf{o}, \mathbf{m})$$

attenuation

$$\rho(\mathbf{i}, \mathbf{o})$$

Gives the fraction of the power incident on the **scattering area**  $dA^m$  that is scattered.

This scattered power is related to the incident irradiance by the attenuation and the **scattering area**, projected in the incident direction.



$$d\Phi_o^m = \frac{|\mathbf{i} \cdot \mathbf{m}|}{|\mathbf{i} \cdot \mathbf{n}|} \rho(\mathbf{i}, \mathbf{o}) dA^m dE_i$$

$$dA^m = D(\mathbf{m}) G(\mathbf{i}, \mathbf{o}, \mathbf{m}) d\omega_m dA$$

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attenuation

$$\rho(\mathbf{i}, \mathbf{o})$$

The BSDF is the ratio of scattered radiance to incident irradiance:

$$f_s(\mathbf{i}, \mathbf{o}) = \frac{dL_o}{dE_i} = \frac{d\Phi_o^m / (dA |\mathbf{o} \cdot \mathbf{n}| d\omega_o)}{dE_i}$$

$$d\Phi_o^m = \frac{|\mathbf{i} \cdot \mathbf{m}|}{|\mathbf{i} \cdot \mathbf{n}|} \rho(\mathbf{i}, \mathbf{o}) dA^m dE_i$$

$$dA^m = D(\mathbf{m}) G(\mathbf{i}, \mathbf{o}, \mathbf{m}) d\omega_m dA$$

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attenuation

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The BSDF is the ratio of scattered radiance to incident irradiance:

$$f_s(\mathbf{i}, \mathbf{o}) = \frac{|\mathbf{i} \cdot \mathbf{m}|}{|\mathbf{i} \cdot \mathbf{n}| |\mathbf{o} \cdot \mathbf{n}|} \rho(\mathbf{i}, \mathbf{o}) D(\mathbf{m}) G(\mathbf{i}, \mathbf{o}, \mathbf{m}) \frac{d\omega_m}{d\omega_o}$$

$$d\Phi_o^m = \frac{|\mathbf{i} \cdot \mathbf{m}|}{|\mathbf{i} \cdot \mathbf{n}|} \rho(\mathbf{i}, \mathbf{o}) dA^m dE_i$$

$$dA^m = D(\mathbf{m}) G(\mathbf{i}, \mathbf{o}, \mathbf{m}) d\omega_m dA$$

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normal distribution

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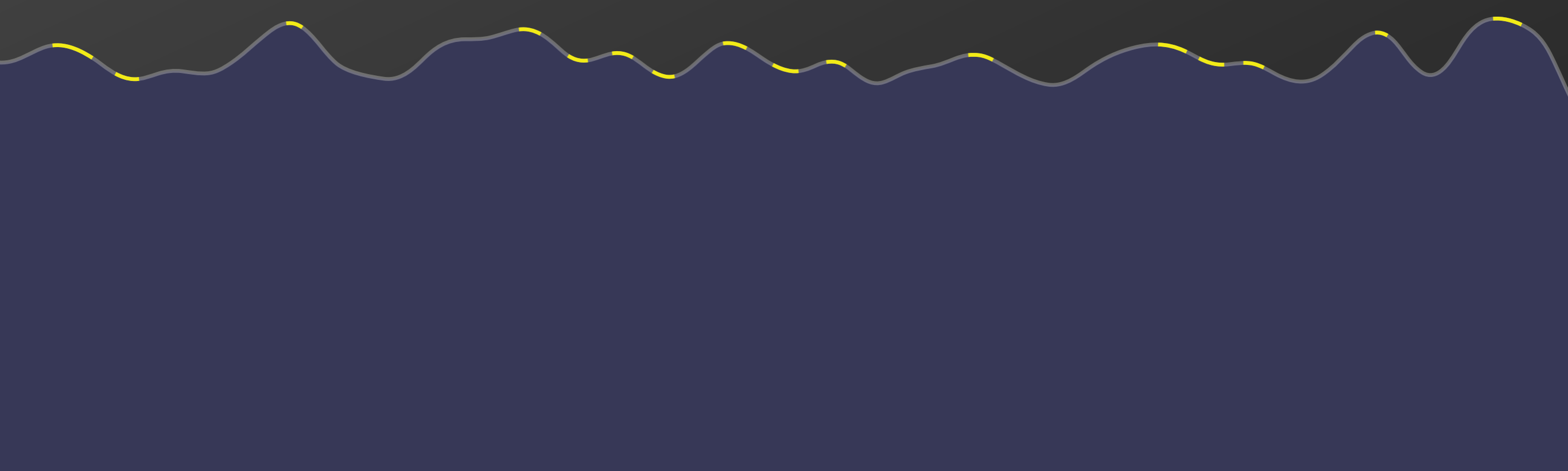
shadowing–masking

$$G(\mathbf{i}, \mathbf{o}, \mathbf{m})$$

attenuation

$$\rho(\mathbf{i}, \mathbf{o})$$

$$f_s(\mathbf{i}, \mathbf{o}) = \frac{|\mathbf{i} \cdot \mathbf{m}|}{|\mathbf{i} \cdot \mathbf{n}| |\mathbf{o} \cdot \mathbf{n}|} \rho(\mathbf{i}, \mathbf{o}) D(\mathbf{m}) G(\mathbf{i}, \mathbf{o}, \mathbf{m}) \frac{d\omega_m}{d\omega_o}$$



“half-vector” function

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normal distribution

$$D(\mathbf{m})$$

shadowing–masking

$$G(\mathbf{i}, \mathbf{o}, \mathbf{m})$$

attenuation

$$\rho(\mathbf{i}, \mathbf{o})$$

Fresnel reflection

$$f_s(\mathbf{i}, \mathbf{o}) = \frac{|\mathbf{i} \cdot \mathbf{m}|}{|\mathbf{i} \cdot \mathbf{n}| |\mathbf{o} \cdot \mathbf{n}|} \rho(\mathbf{i}, \mathbf{o}) D(\mathbf{m}) G(\mathbf{i}, \mathbf{o}, \mathbf{m}) \frac{d\omega_m}{d\omega_o}$$

“half-vector” function

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normal distribution

$$D(\mathbf{m})$$

shadowing–masking

$$G(\mathbf{i}, \mathbf{o}, \mathbf{m})$$

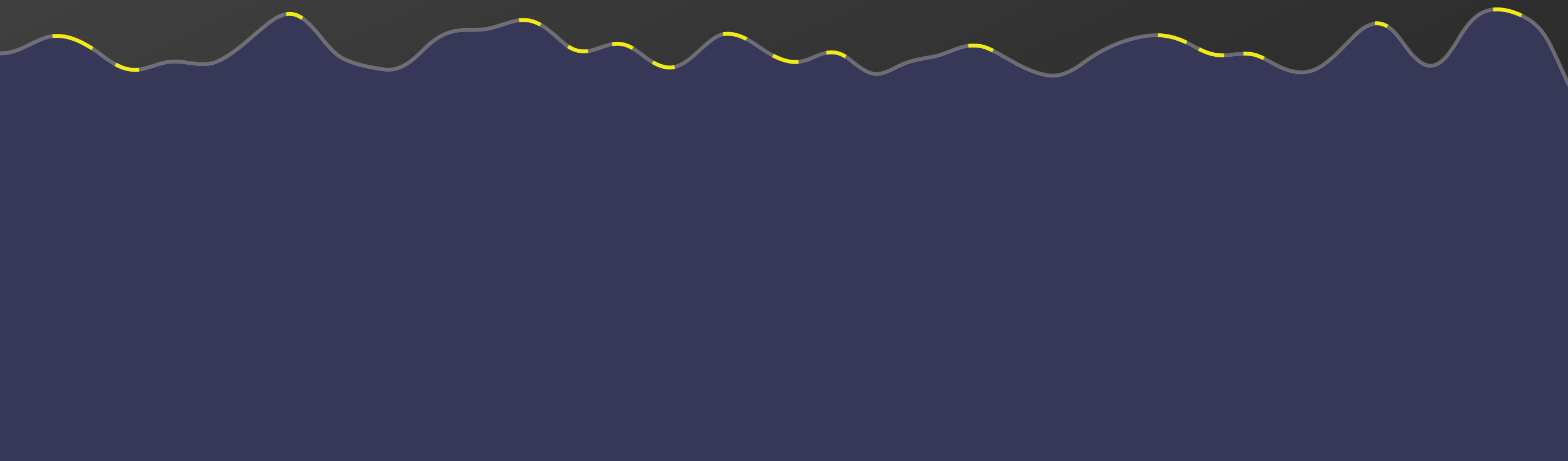
attenuation

$$\rho(\mathbf{i}, \mathbf{o})$$

Fresnel reflection

surface roughness

$$f_s(\mathbf{i}, \mathbf{o}) = \frac{|\mathbf{i} \cdot \mathbf{m}|}{|\mathbf{i} \cdot \mathbf{n}| |\mathbf{o} \cdot \mathbf{n}|} \rho(\mathbf{i}, \mathbf{o}) D(\mathbf{m}) G(\mathbf{i}, \mathbf{o}, \mathbf{m}) \frac{d\omega_m}{d\omega_o}$$



“half-vector” function

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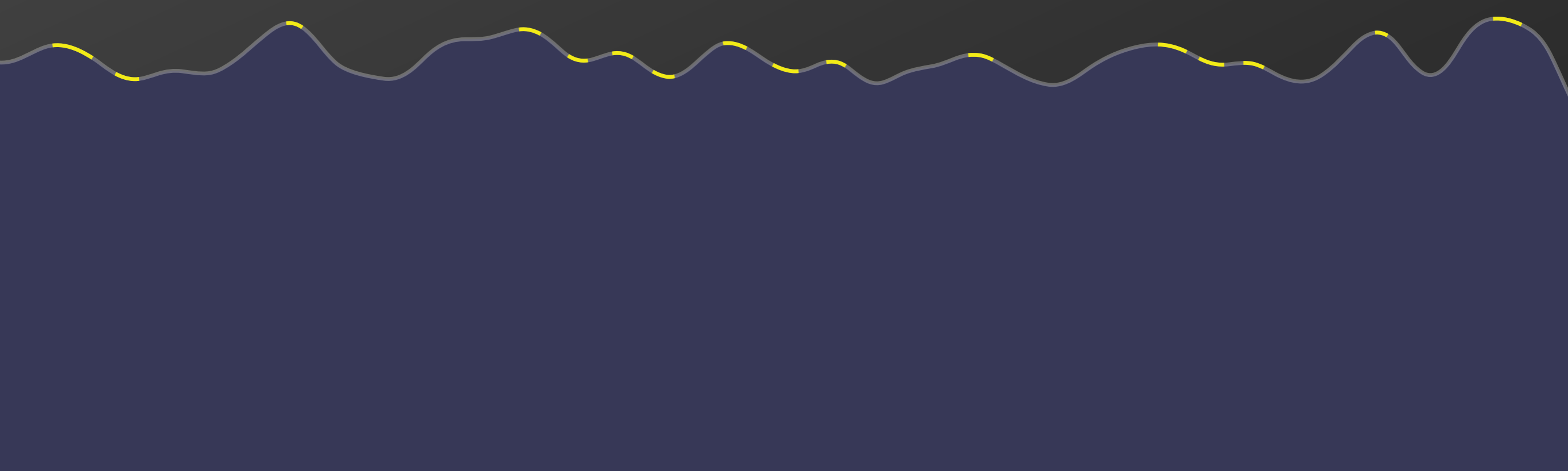
$$\rho(\mathbf{i}, \mathbf{o})$$

Fresnel reflection

surface roughness

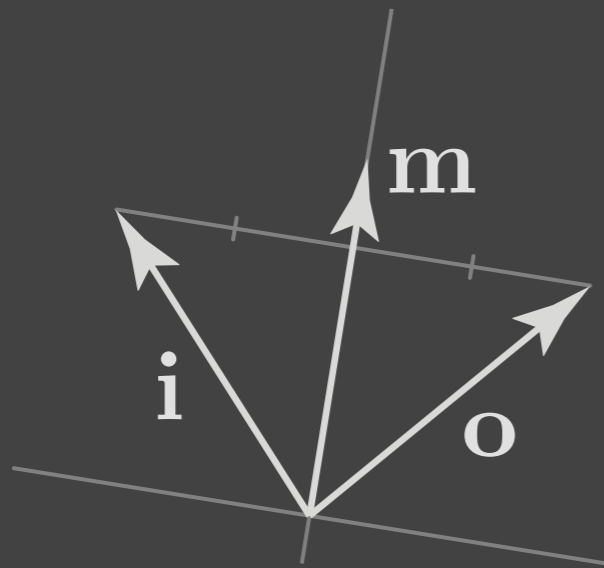
$$f_s(\mathbf{i}, \mathbf{o}) = \frac{|\mathbf{i} \cdot \mathbf{m}|}{|\mathbf{i} \cdot \mathbf{n}| |\mathbf{o} \cdot \mathbf{n}|} \rho(\mathbf{i}, \mathbf{o}) D(\mathbf{m}) G(\mathbf{i}, \mathbf{o}, \mathbf{m}) \frac{d\omega_m}{d\omega_o}$$

determined by geometry



# Construction of half-vector

reflection



$\mathbf{i} + \mathbf{o}$  parallel to  $\mathbf{m}$

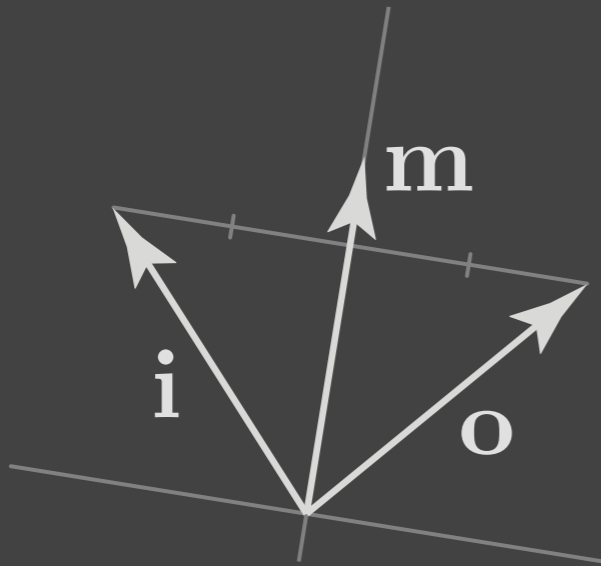
refraction



# Construction of half-vector

reflection

$$\mathbf{h}_r = \text{normalize}(\mathbf{i} + \mathbf{o})$$



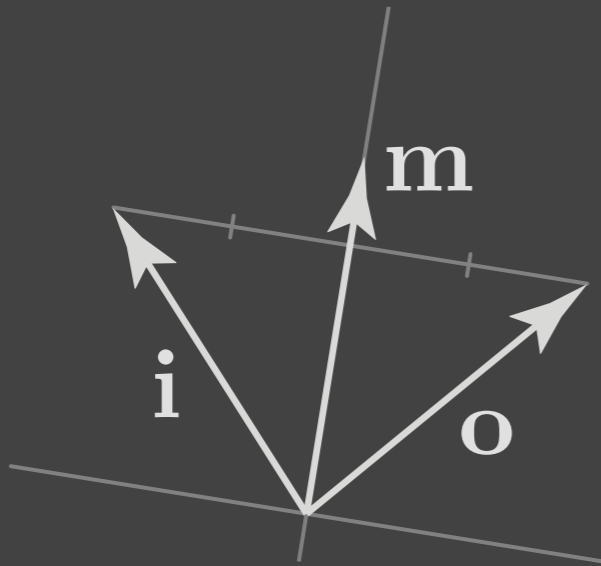
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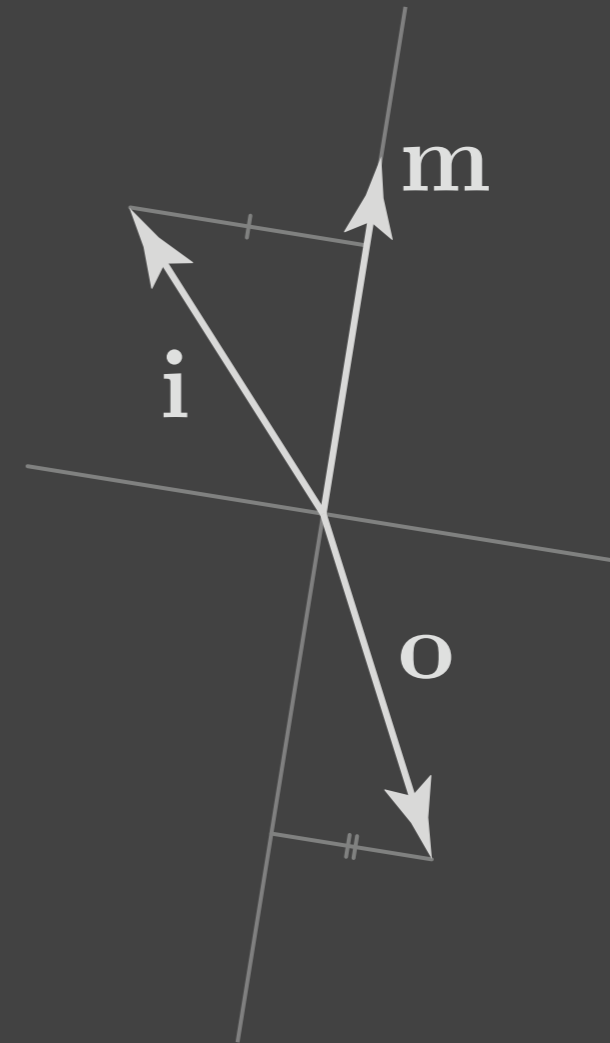
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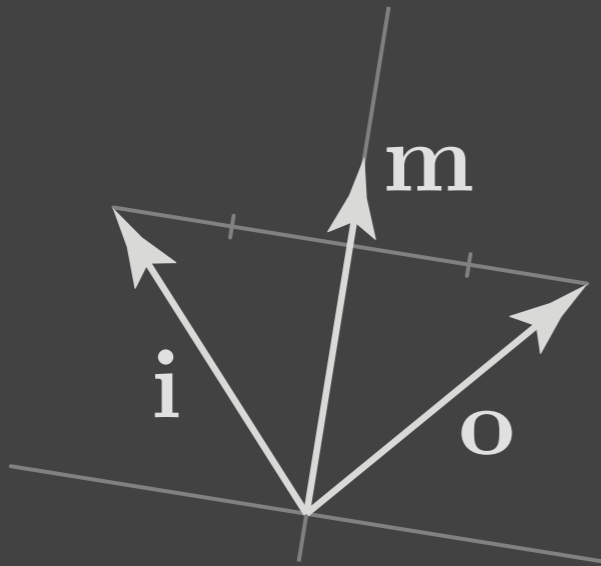
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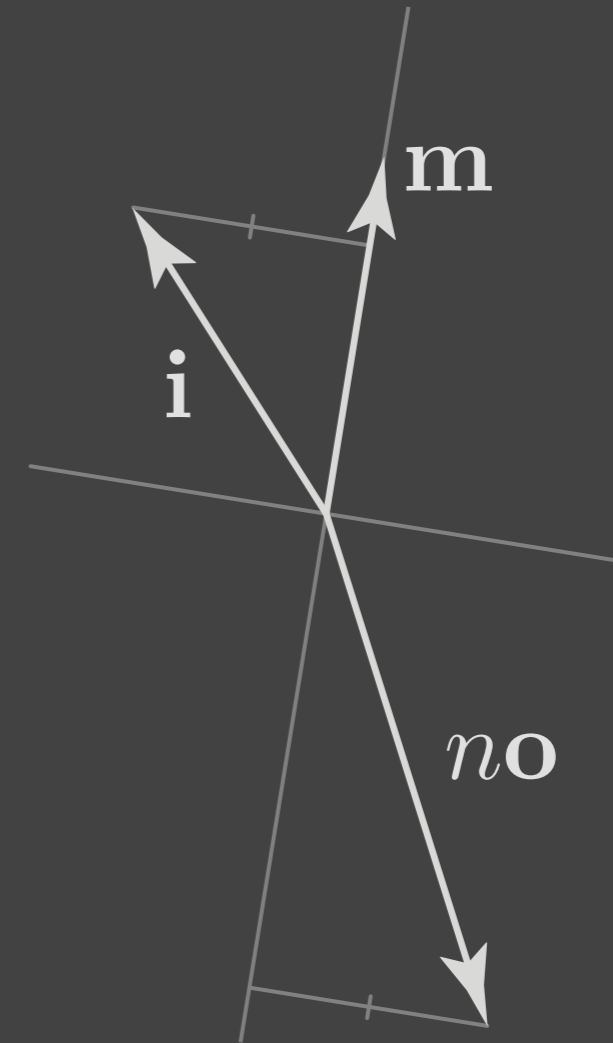
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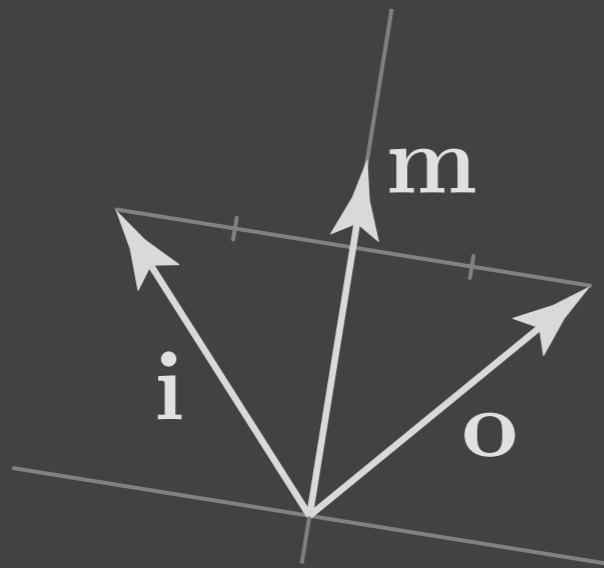


$\mathbf{i} + n\mathbf{o}$  parallel to  $\mathbf{m}$

# Construction of half-vector

reflection

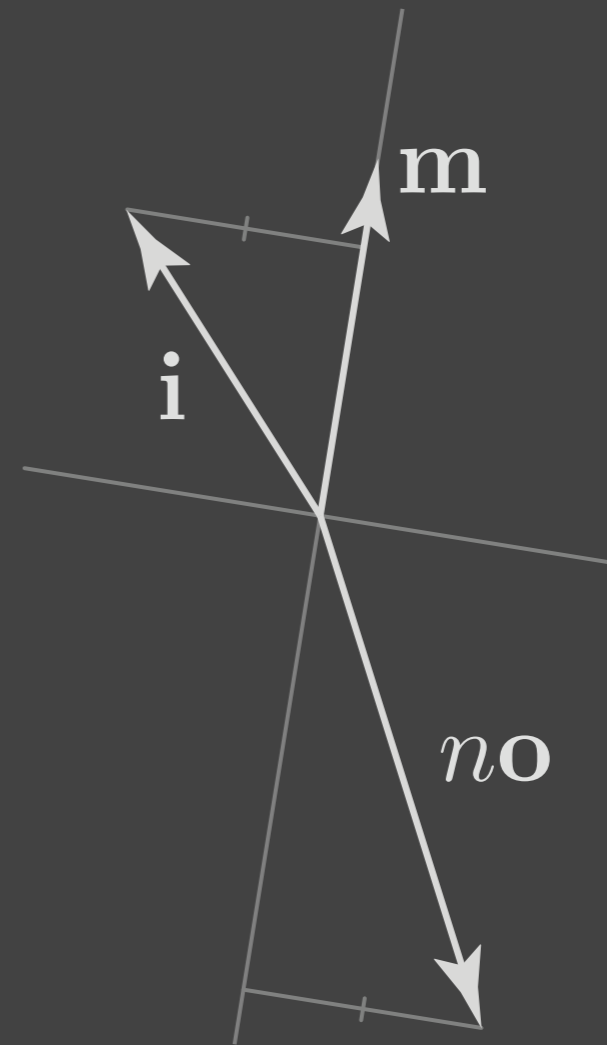
$$\mathbf{h}_r = \text{normalize}(\mathbf{i} + \mathbf{o})$$



$\mathbf{i} + \mathbf{o}$  parallel to  $\mathbf{m}$

refraction

$$\mathbf{h}_t = -\text{normalize}(\mathbf{i} + n\mathbf{o})$$



$\mathbf{i} + n\mathbf{o}$  parallel to  $\mathbf{m}$

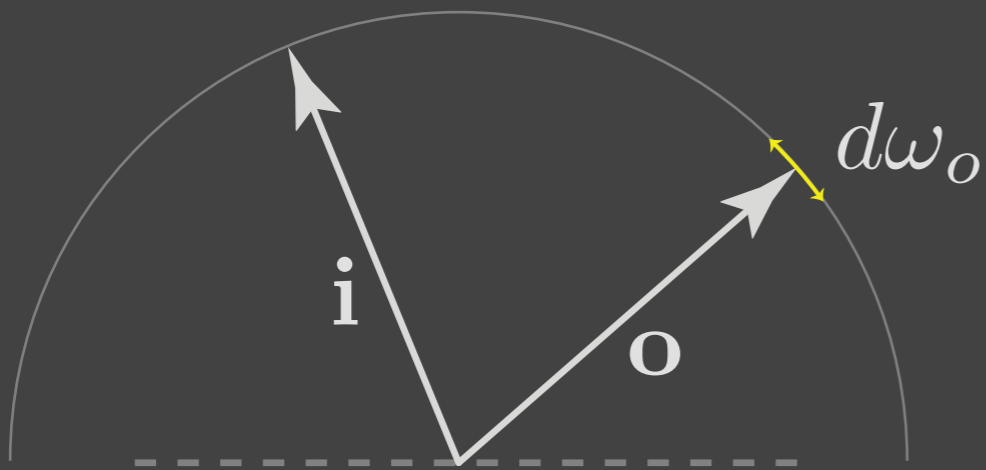
# Construction of half-vector solid angle

reflection

$$\mathbf{h}_r = \text{normalize}(\mathbf{i} + \mathbf{o})$$

refraction

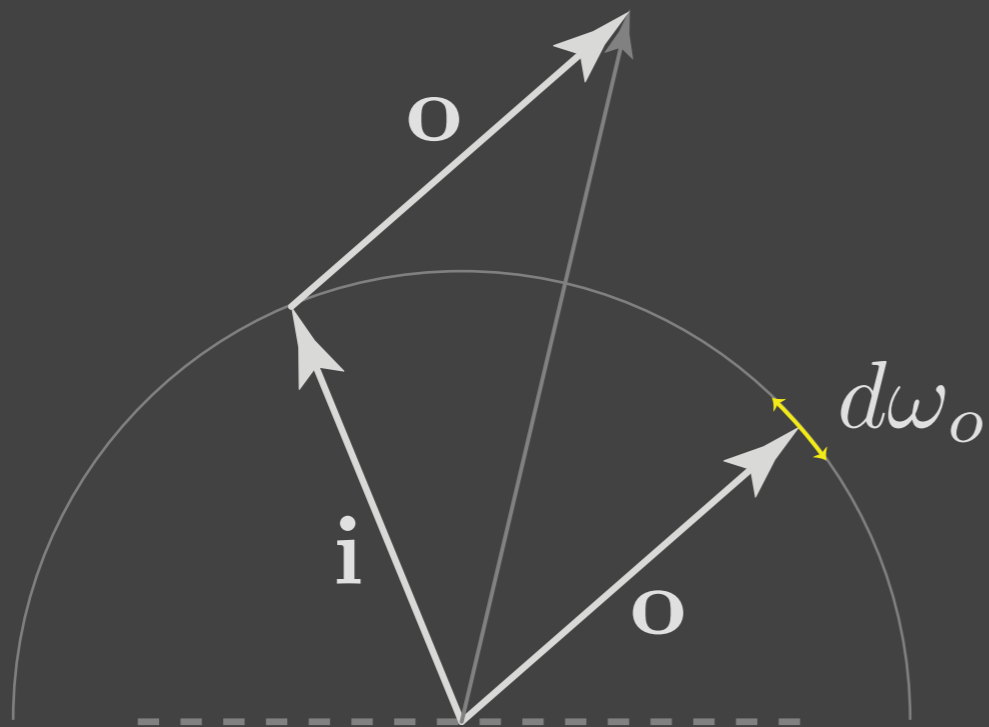
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# Construction of half-vector solid angle

reflection

$$\mathbf{h}_r = \text{normalize}(\mathbf{i} + \mathbf{o})$$



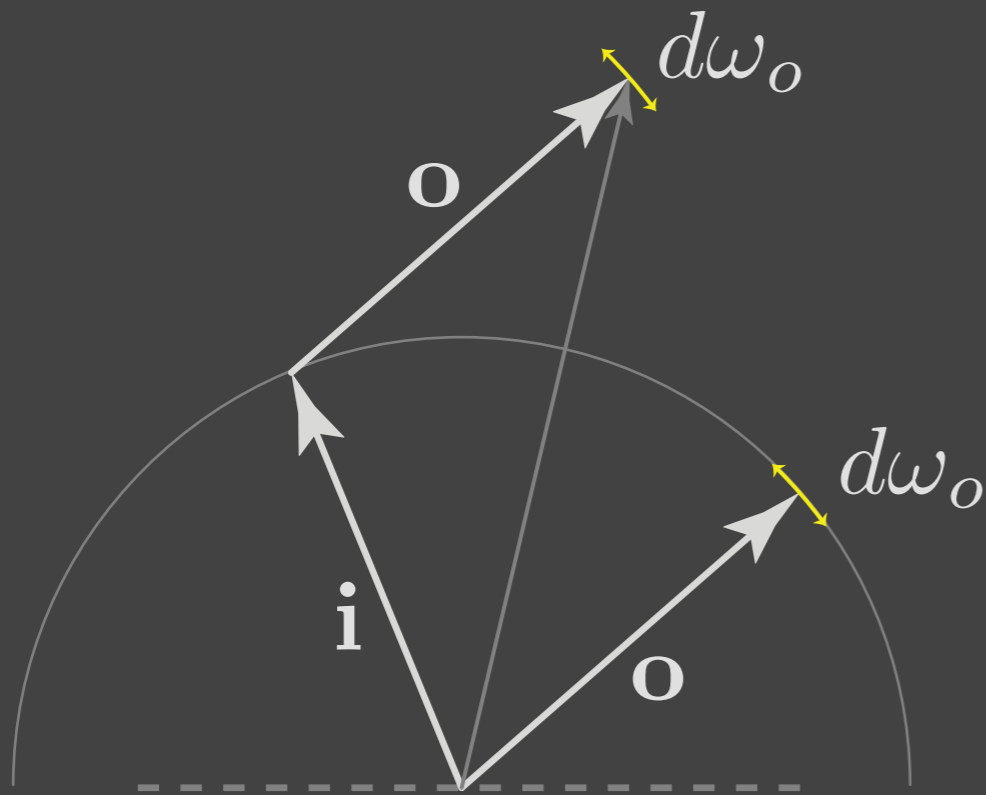
refraction

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# Construction of half-vector solid angle

reflection

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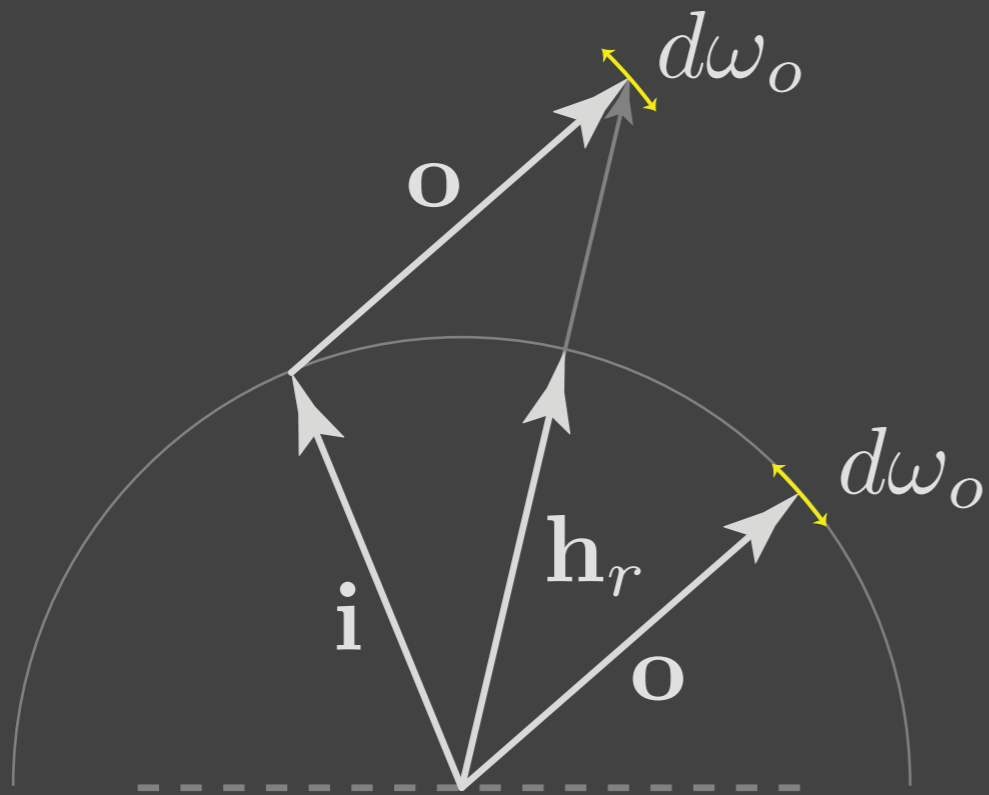
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# Construction of half-vector solid angle

reflection

$$\mathbf{h}_r = \text{normalize}(\mathbf{i} + \mathbf{o})$$



refraction

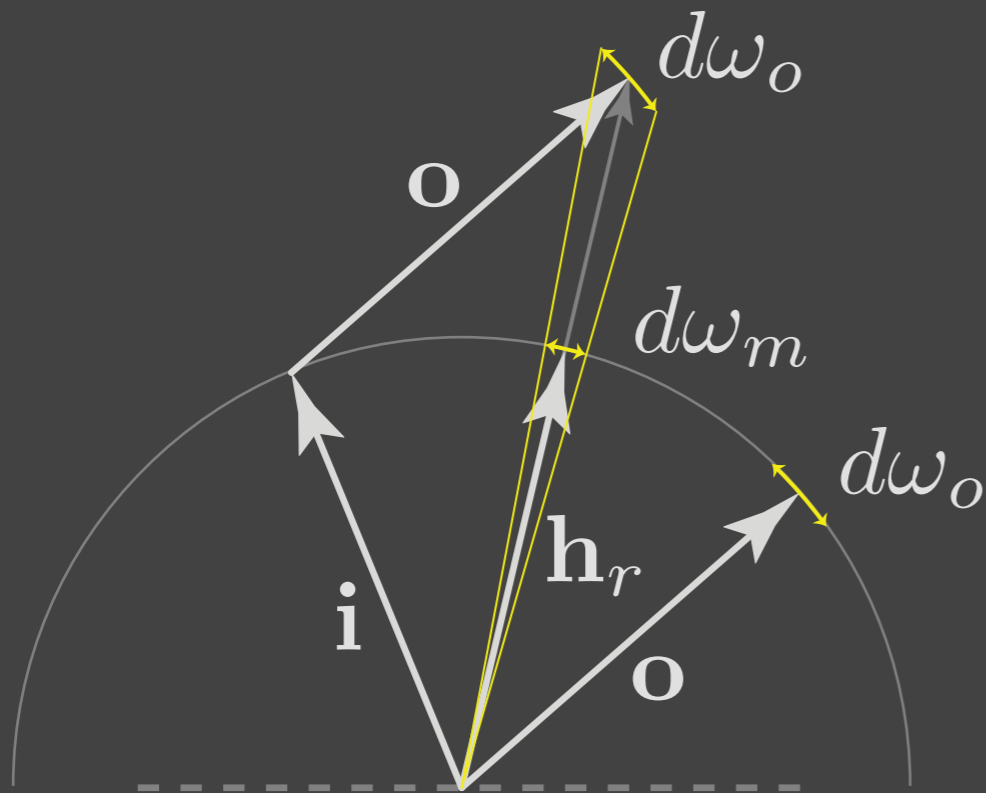
$$\mathbf{h}_t = -\text{normalize}(\mathbf{i} + n\mathbf{o})$$



# Construction of half-vector solid angle

reflection

$$\mathbf{h}_r = \text{normalize}(\mathbf{i} + \mathbf{o})$$



$$d\omega_m = \frac{|\mathbf{o} \cdot \mathbf{h}_r|}{\|\mathbf{i} + \mathbf{o}\|^2} d\omega_o$$

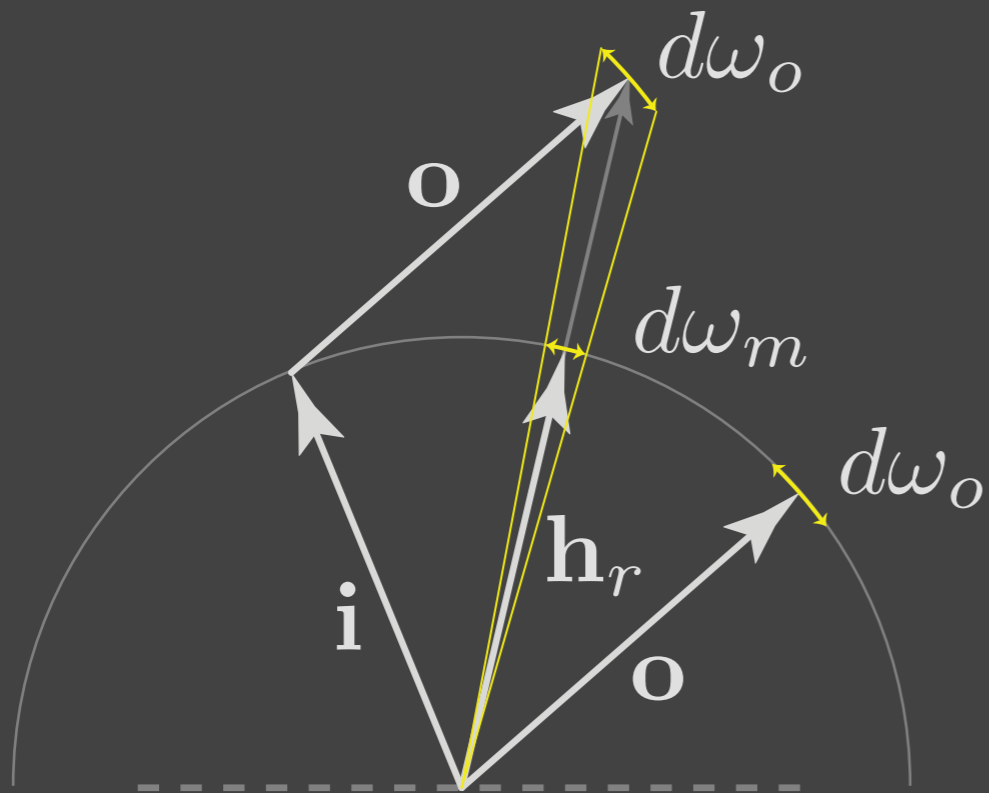
refraction

$$\mathbf{h}_t = -\text{normalize}(\mathbf{i} + n\mathbf{o})$$

# Construction of half-vector solid angle

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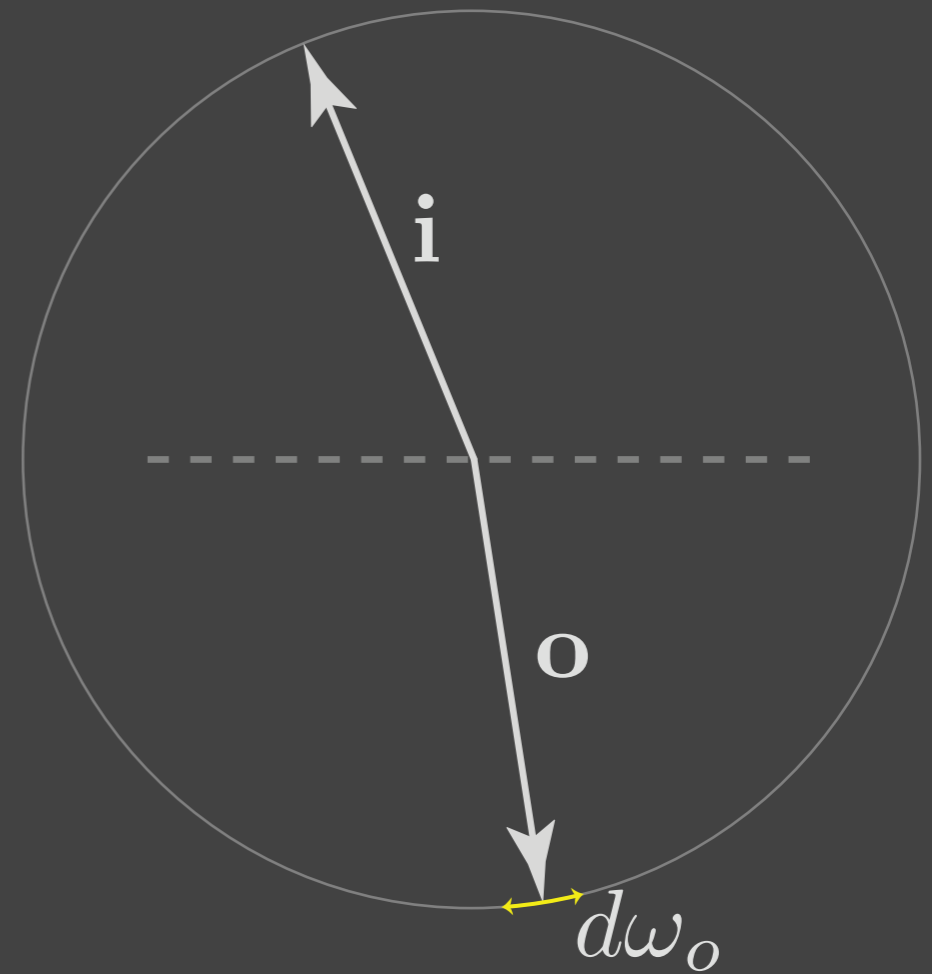
$$\mathbf{h}_r = \text{normalize}(\mathbf{i} + \mathbf{o})$$



$$d\omega_m = \frac{|\mathbf{o} \cdot \mathbf{h}_r|}{\|\mathbf{i} + \mathbf{o}\|^2} d\omega_o$$

refraction

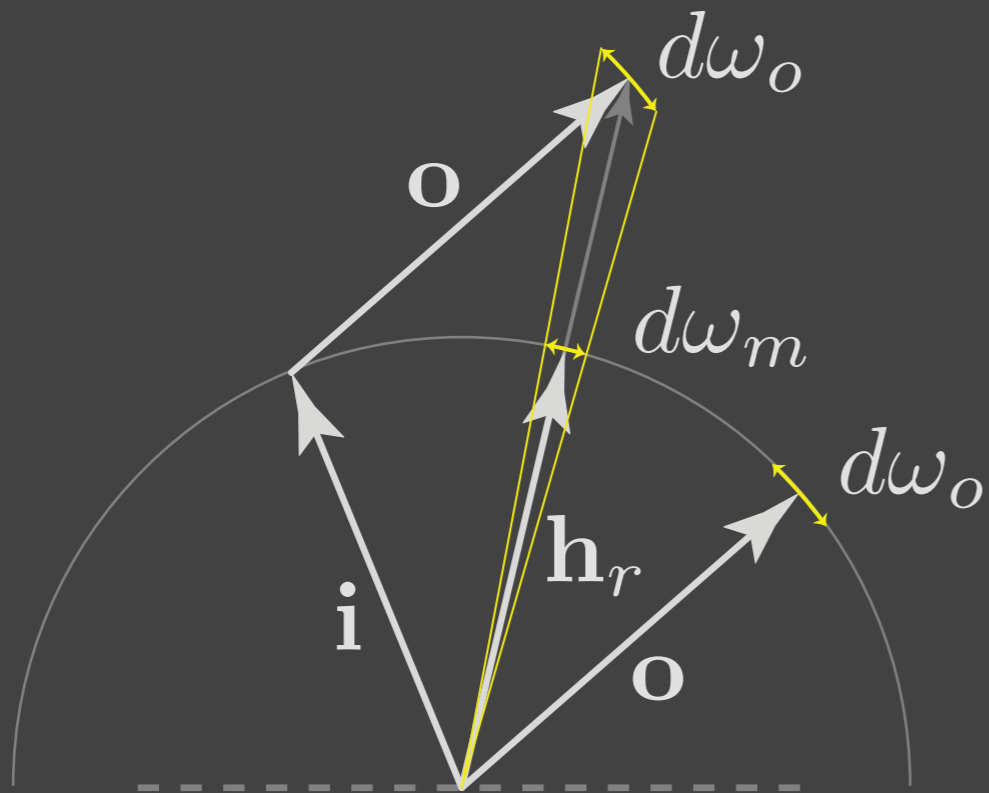
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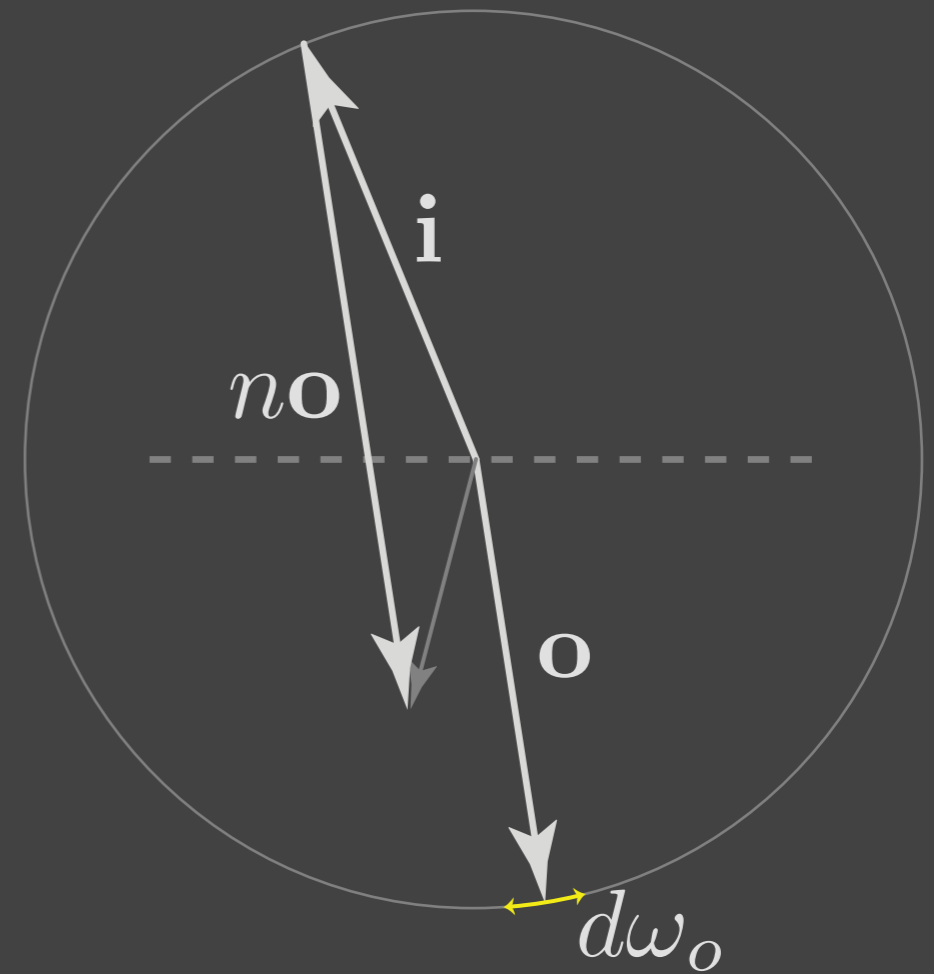
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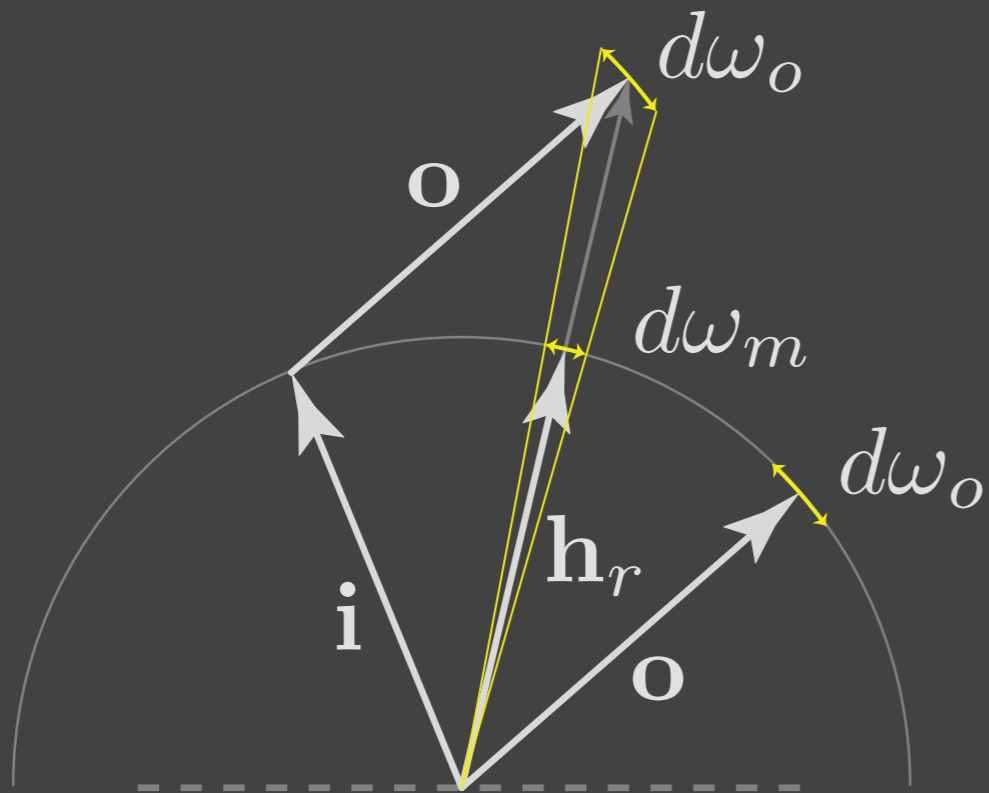
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# Construction of half-vector solid angle

reflection

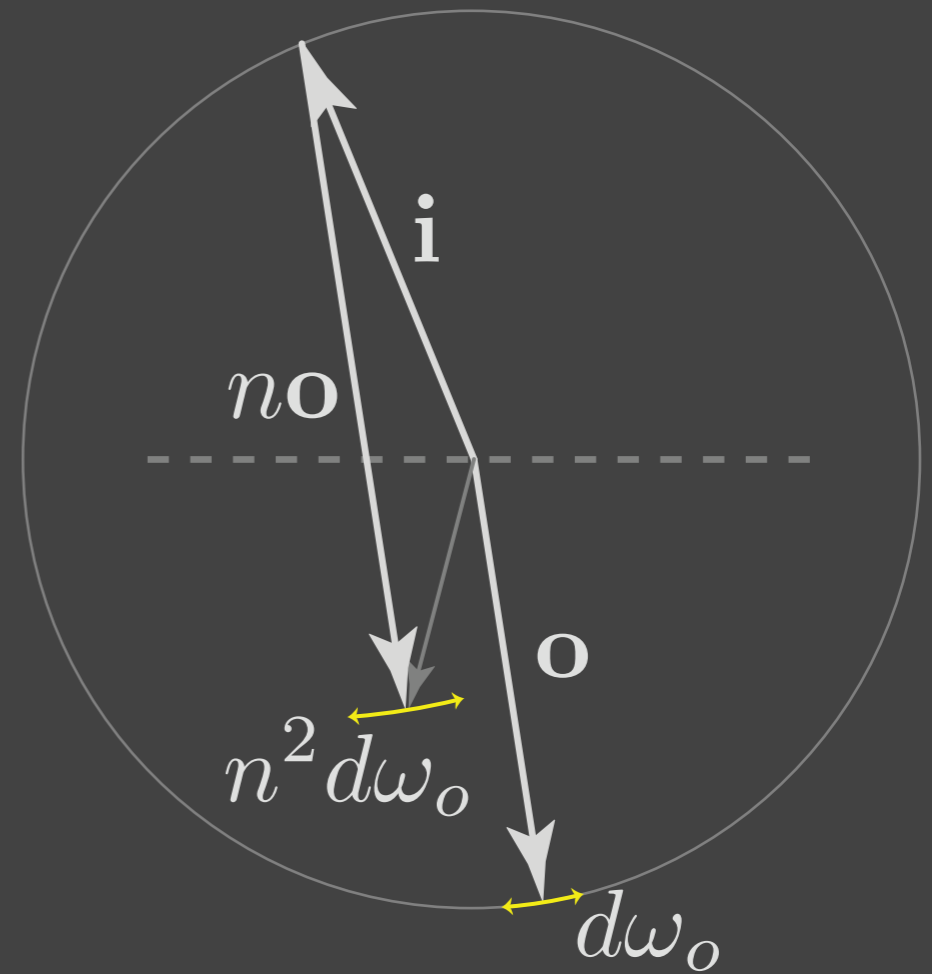
$$\mathbf{h}_r = \text{normalize}(\mathbf{i} + \mathbf{o})$$



$$d\omega_m = \frac{|\mathbf{o} \cdot \mathbf{h}_r|}{\|\mathbf{i} + \mathbf{o}\|^2} d\omega_o$$

refraction

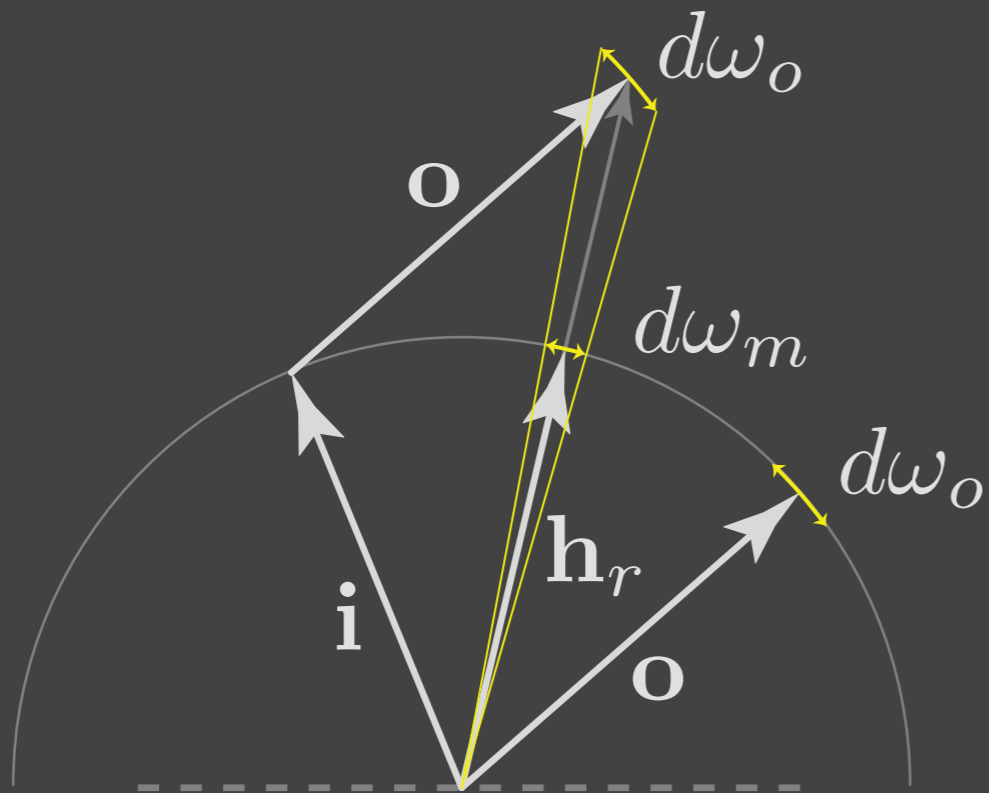
$$\mathbf{h}_t = -\text{normalize}(\mathbf{i} + n\mathbf{o})$$



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reflection

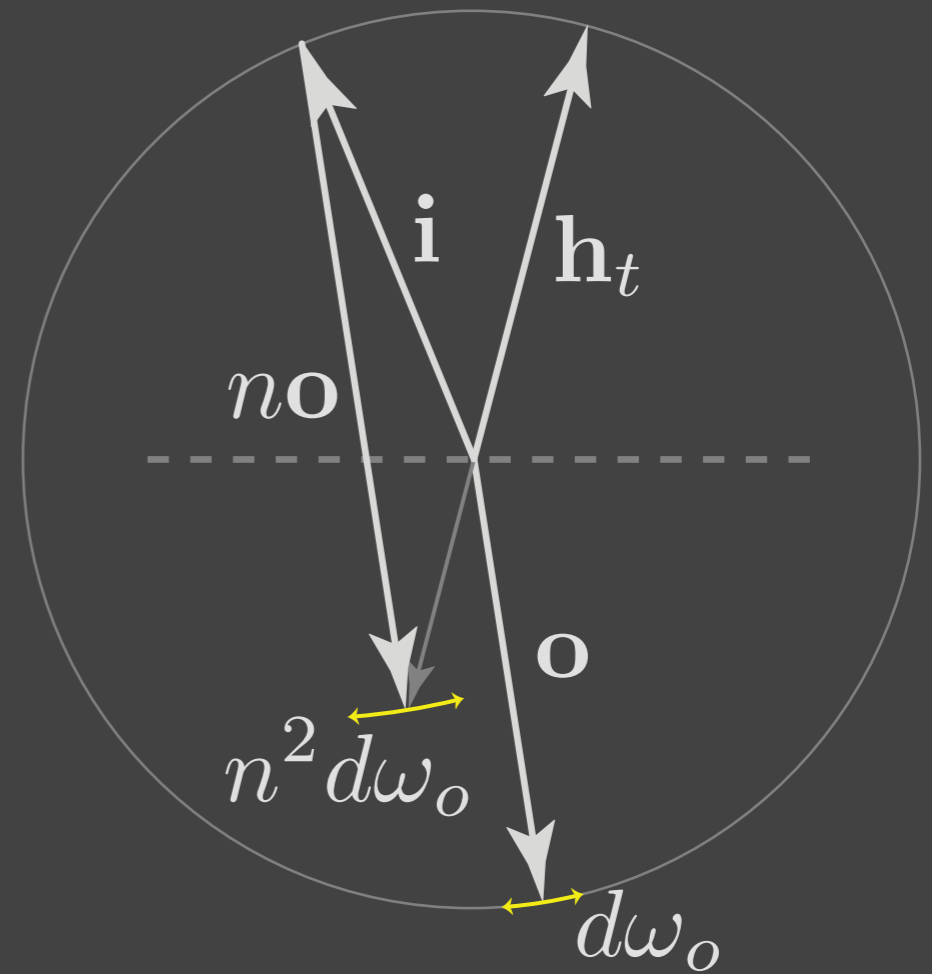
$$\mathbf{h}_r = \text{normalize}(\mathbf{i} + \mathbf{o})$$



$$d\omega_m = \frac{|\mathbf{o} \cdot \mathbf{h}_r|}{\|\mathbf{i} + \mathbf{o}\|^2} d\omega_o$$

refraction

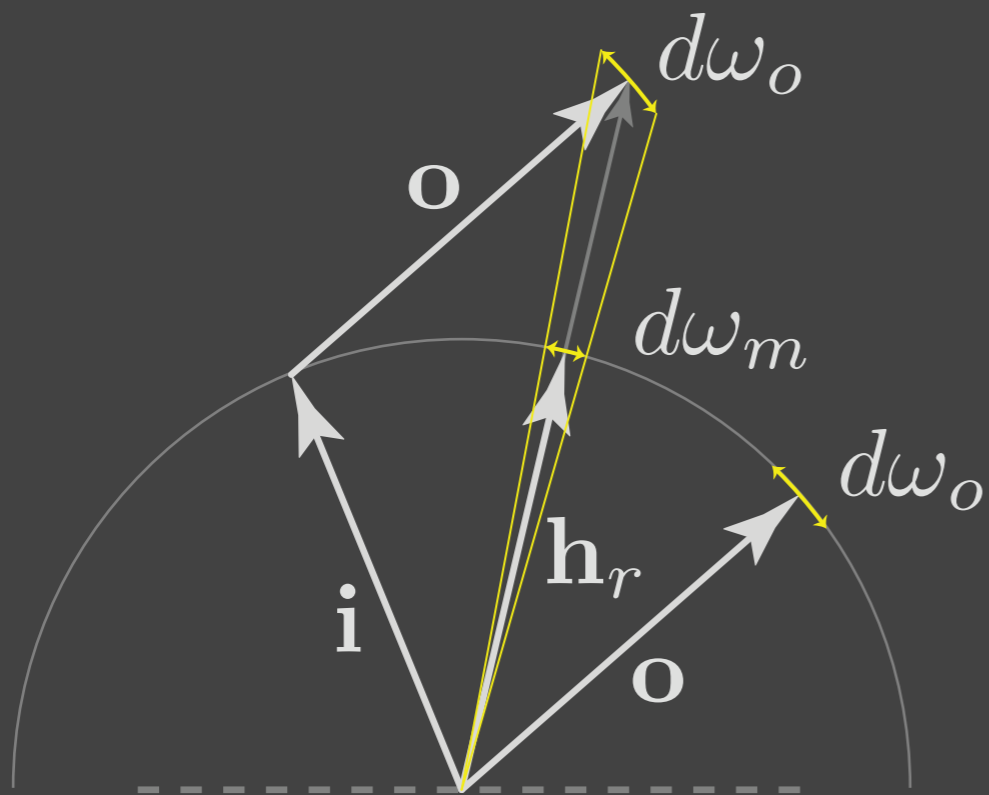
$$\mathbf{h}_t = -\text{normalize}(\mathbf{i} + n\mathbf{o})$$



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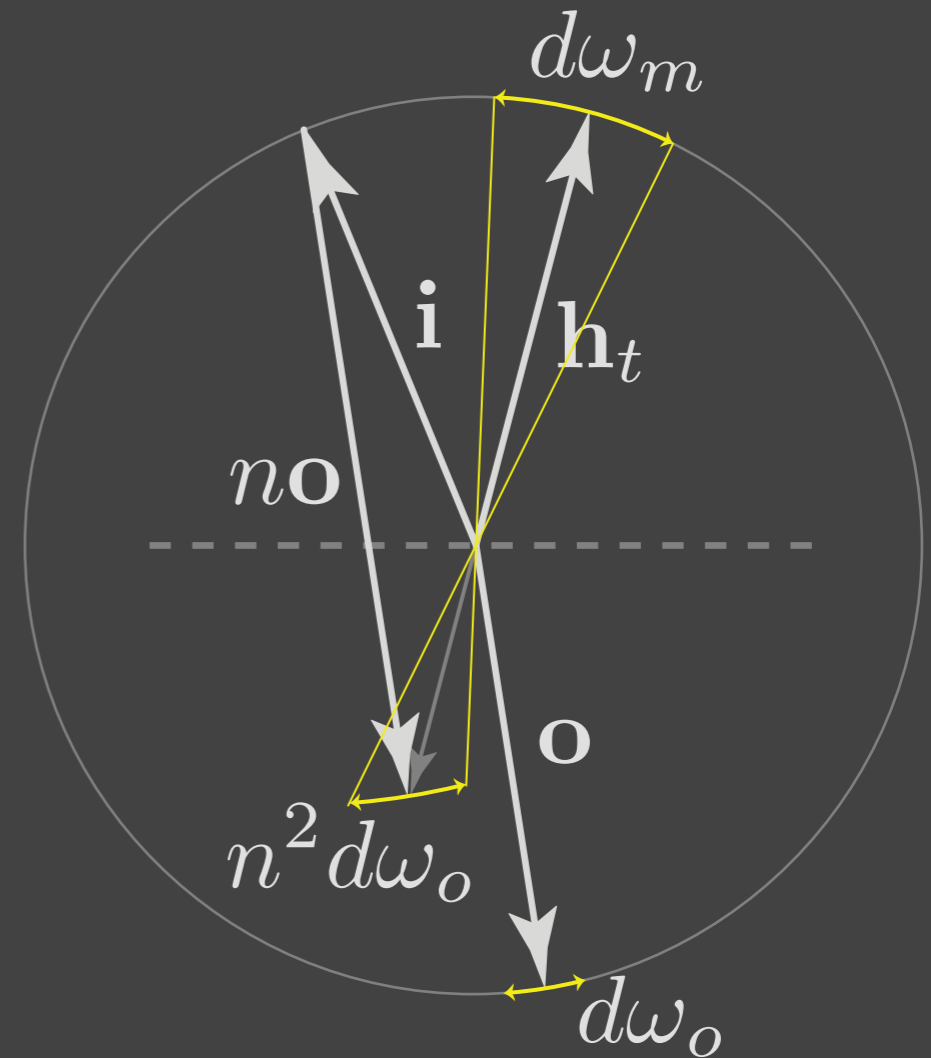
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refraction

$$\mathbf{h}_t = -\text{normalize}(\mathbf{i} + n\mathbf{o})$$



$$d\omega_m = \frac{|\mathbf{o} \cdot \mathbf{h}_t|}{\|\mathbf{i} + n\mathbf{o}\|^2} n^2 d\omega_o$$

# Result: scattering functions

reflection

$$f_s(\mathbf{i}, \mathbf{o}) = \frac{|\mathbf{i} \cdot \mathbf{m}|}{|\mathbf{i} \cdot \mathbf{n}| |\mathbf{o} \cdot \mathbf{n}|} \rho(\mathbf{i}, \mathbf{o}) D(\mathbf{m}) G(\mathbf{i}, \mathbf{o}, \mathbf{m}) \frac{d\omega_m}{d\omega_o}$$

transmission

$$f_s(\mathbf{i}, \mathbf{o}) = \frac{|\mathbf{i} \cdot \mathbf{m}|}{|\mathbf{i} \cdot \mathbf{n}| |\mathbf{o} \cdot \mathbf{n}|} \rho(\mathbf{i}, \mathbf{o}) D(\mathbf{m}) G(\mathbf{i}, \mathbf{o}, \mathbf{m}) \frac{d\omega_m}{d\omega_o}$$

# Result: scattering functions

reflection

$$f_r(\mathbf{i}, \mathbf{o}) = \frac{|\mathbf{i} \cdot \mathbf{m}|}{|\mathbf{i} \cdot \mathbf{n}| |\mathbf{o} \cdot \mathbf{n}|} F(\mathbf{i}, \mathbf{m}) D(\mathbf{m}) G(\mathbf{i}, \mathbf{o}, \mathbf{m}) \frac{|\mathbf{o} \cdot \mathbf{m}|}{\|\mathbf{i} + \mathbf{o}\|^2}$$

transmission

$$f_s(\mathbf{i}, \mathbf{o}) = \frac{|\mathbf{i} \cdot \mathbf{m}|}{|\mathbf{i} \cdot \mathbf{n}| |\mathbf{o} \cdot \mathbf{n}|} \rho(\mathbf{i}, \mathbf{o}) D(\mathbf{m}) G(\mathbf{i}, \mathbf{o}, \mathbf{m}) \frac{d\omega_m}{d\omega_o}$$



# Result: scattering functions

reflection

$$f_r(\mathbf{i}, \mathbf{o}) = \frac{|\mathbf{i} \cdot \mathbf{m}|}{|\mathbf{i} \cdot \mathbf{n}| |\mathbf{o} \cdot \mathbf{n}|} F(\mathbf{i}, \mathbf{m}) D(\mathbf{m}) G(\mathbf{i}, \mathbf{o}, \mathbf{m}) \frac{|\mathbf{o} \cdot \mathbf{m}|}{\|\mathbf{i} + \mathbf{o}\|^2}$$

transmission

$$f_t(\mathbf{i}, \mathbf{o}) = \frac{|\mathbf{i} \cdot \mathbf{m}|}{|\mathbf{i} \cdot \mathbf{n}| |\mathbf{o} \cdot \mathbf{n}|} (1 - F(\mathbf{i}, \mathbf{m})) D(\mathbf{m}) G(\mathbf{i}, \mathbf{o}, \mathbf{m}) \frac{n^2 |\mathbf{o} \cdot \mathbf{m}|}{\|\mathbf{i} + n\mathbf{o}\|^2}$$

# Result: scattering functions

reflection

$$f_r(\mathbf{i}, \mathbf{o}) = \frac{|\mathbf{i} \cdot \mathbf{m}| |\mathbf{o} \cdot \mathbf{m}|}{|\mathbf{i} \cdot \mathbf{n}| |\mathbf{o} \cdot \mathbf{n}|} \frac{F(\mathbf{i}, \mathbf{m}) D(\mathbf{m}) G(\mathbf{i}, \mathbf{o}, \mathbf{m})}{\|\mathbf{i} + \mathbf{o}\|^2}$$

transmission

$$f_t(\mathbf{i}, \mathbf{o}) = \frac{|\mathbf{i} \cdot \mathbf{m}|}{|\mathbf{i} \cdot \mathbf{n}| |\mathbf{o} \cdot \mathbf{n}|} (1 - F(\mathbf{i}, \mathbf{m})) D(\mathbf{m}) G(\mathbf{i}, \mathbf{o}, \mathbf{m}) \frac{n^2 |\mathbf{o} \cdot \mathbf{m}|}{\|\mathbf{i} + n\mathbf{o}\|^2}$$

# Result: scattering functions

reflection

$$f_r(\mathbf{i}, \mathbf{o}) = \frac{|\mathbf{i} \cdot \mathbf{m}| |\mathbf{o} \cdot \mathbf{m}|}{|\mathbf{i} \cdot \mathbf{n}| |\mathbf{o} \cdot \mathbf{n}|} \frac{F(\mathbf{i}, \mathbf{m}) D(\mathbf{m}) G(\mathbf{i}, \mathbf{o}, \mathbf{m})}{\|\mathbf{i} + \mathbf{o}\|^2}$$

transmission

$$f_t(\mathbf{i}, \mathbf{o}) = \frac{|\mathbf{i} \cdot \mathbf{m}| |\mathbf{o} \cdot \mathbf{m}|}{|\mathbf{i} \cdot \mathbf{n}| |\mathbf{o} \cdot \mathbf{n}|} \frac{n^2 (1 - F(\mathbf{i}, \mathbf{m})) D(\mathbf{m}) G(\mathbf{i}, \mathbf{o}, \mathbf{m})}{\|\mathbf{i} + n\mathbf{o}\|^2}$$

# Result: scattering functions

reflection

$$f_r(\mathbf{i}, \mathbf{o}) = \frac{1}{|\mathbf{i} \cdot \mathbf{n}| |\mathbf{o} \cdot \mathbf{n}|} \frac{F(\mathbf{i}, \mathbf{m}) D(\mathbf{m}) G(\mathbf{i}, \mathbf{o}, \mathbf{m})}{4}$$

transmission

$$f_t(\mathbf{i}, \mathbf{o}) = \frac{|\mathbf{i} \cdot \mathbf{m}| |\mathbf{o} \cdot \mathbf{m}|}{|\mathbf{i} \cdot \mathbf{n}| |\mathbf{o} \cdot \mathbf{n}|} \frac{n^2 (1 - F(\mathbf{i}, \mathbf{m})) D(\mathbf{m}) G(\mathbf{i}, \mathbf{o}, \mathbf{m})}{\|\mathbf{i} + n\mathbf{o}\|^2}$$

# Result: scattering functions

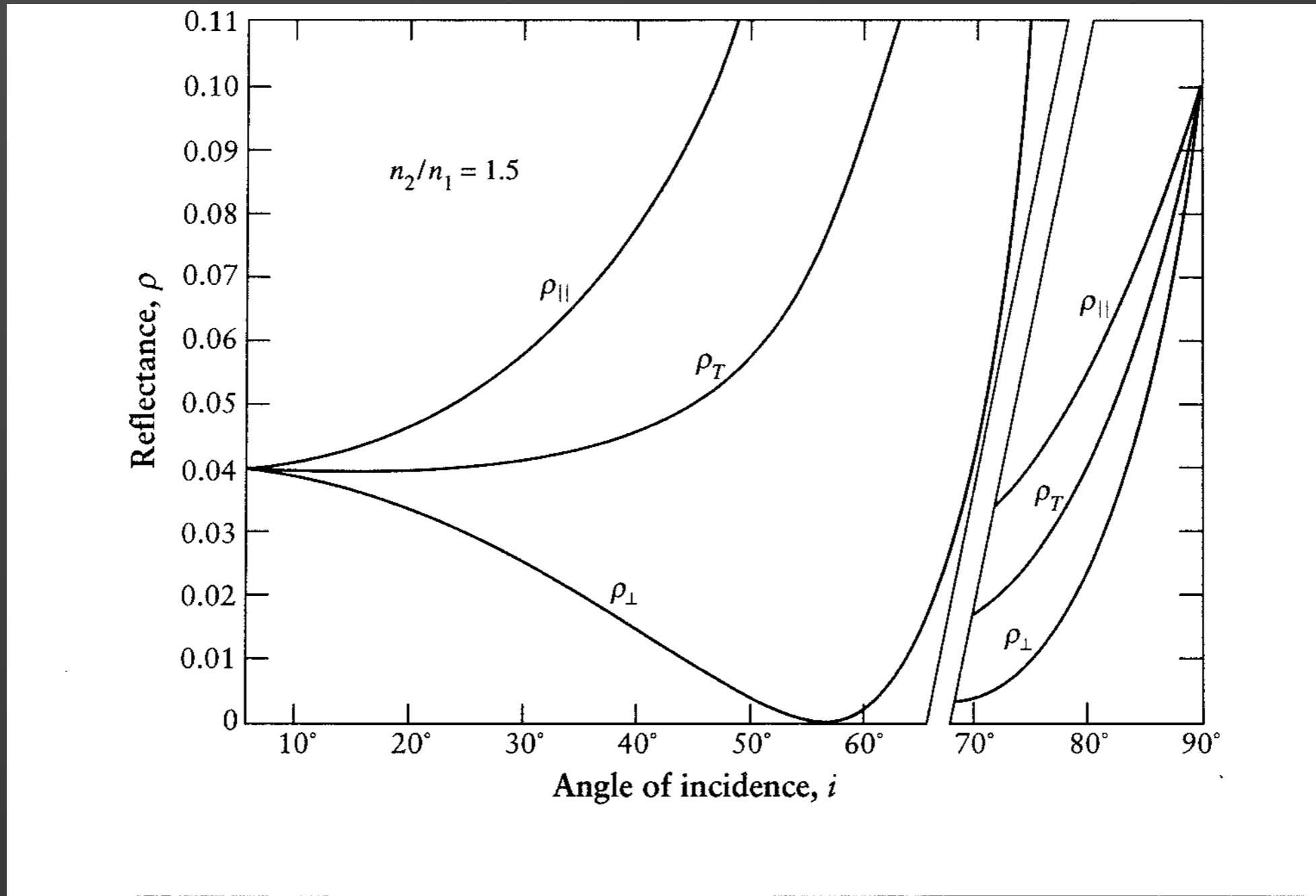
reflection

$$f_r(\mathbf{i}, \mathbf{o}) = \frac{F(\mathbf{i}, \mathbf{m}) D(\mathbf{m}) G(\mathbf{i}, \mathbf{o}, \mathbf{m})}{4|\mathbf{i} \cdot \mathbf{n}| |\mathbf{o} \cdot \mathbf{n}|}$$

transmission

$$f_t(\mathbf{i}, \mathbf{o}) = \frac{|\mathbf{i} \cdot \mathbf{m}| |\mathbf{o} \cdot \mathbf{m}|}{|\mathbf{i} \cdot \mathbf{n}| |\mathbf{o} \cdot \mathbf{n}|} \frac{n^2 (1 - F(\mathbf{i}, \mathbf{m})) D(\mathbf{m}) G(\mathbf{i}, \mathbf{o}, \mathbf{m})}{\|\mathbf{i} + n\mathbf{o}\|^2}$$

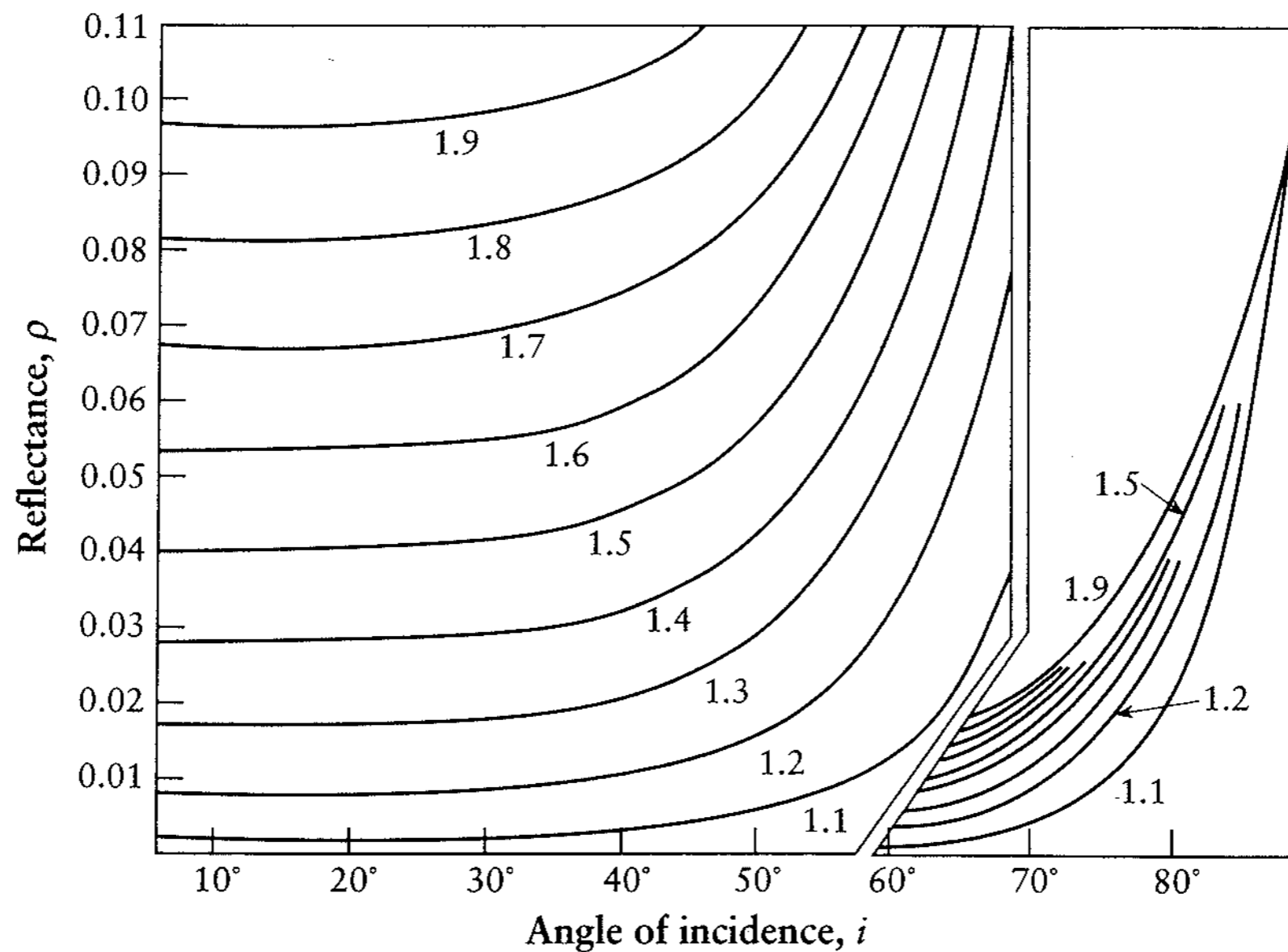
# Fresnel reflectance



**FIGURE 15.8**

The Fresnel reflectance for an air-glass boundary with index of refraction 1.5. We show the two polarized components and the term for unpolarized light. Redrawn from Judd and Wyszecki, *Color in Business, Science and Industry*, fig. 3.2, p. 400.

# Fresnel reflectance



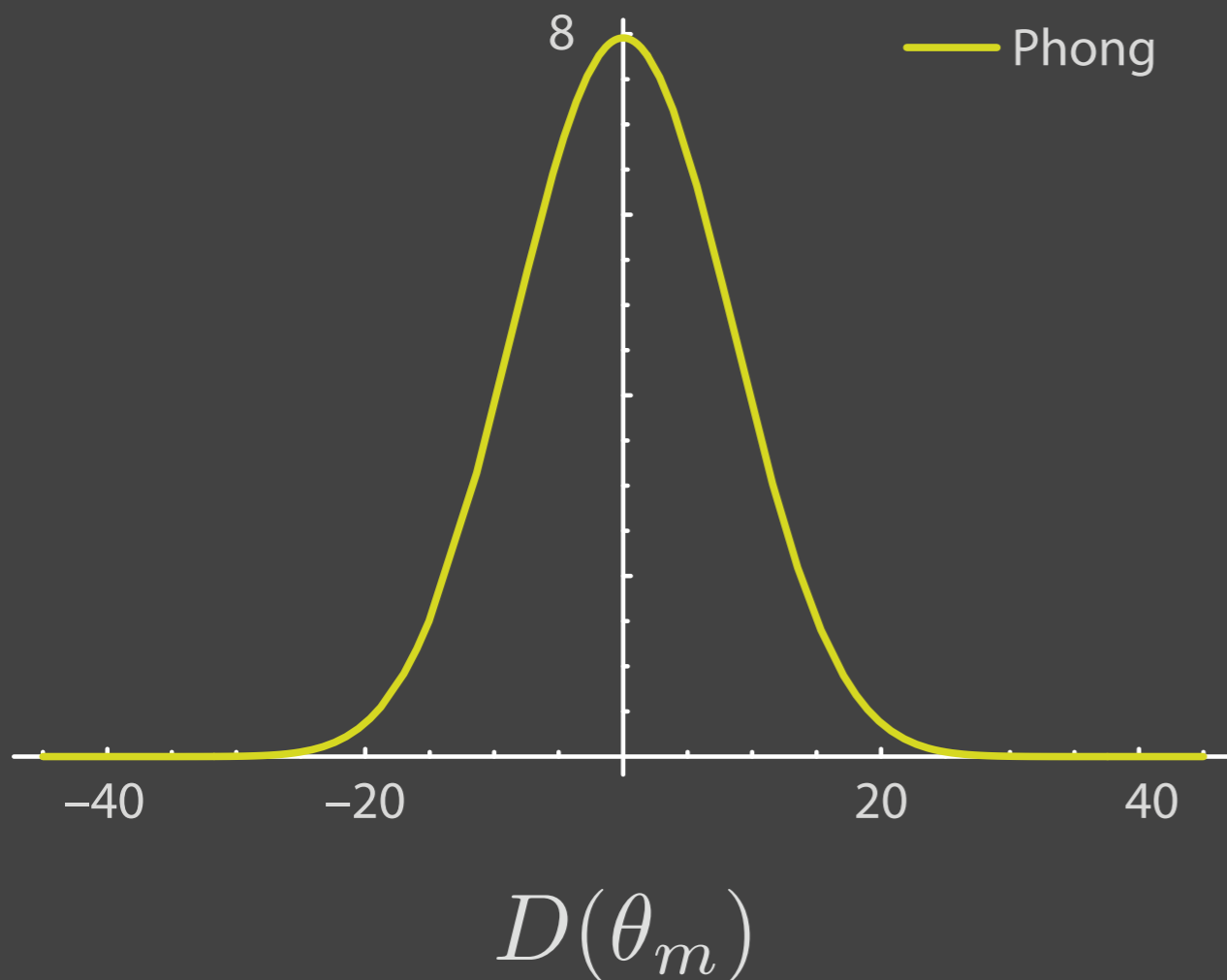
**FIGURE 15.9**

The Fresnel reflection for unpolarized light for different indices of refraction. Redrawn from Judd and Wyszecki, *Color in Business, Science and Industry*, fig. 3.3, p. 401.

# Normal distributions

Choice of distribution is determined by surface

- Phong, Beckman are popular choices
- “GGX” distribution is another option
- [Smith 67] gives a way to produce smooth Gs

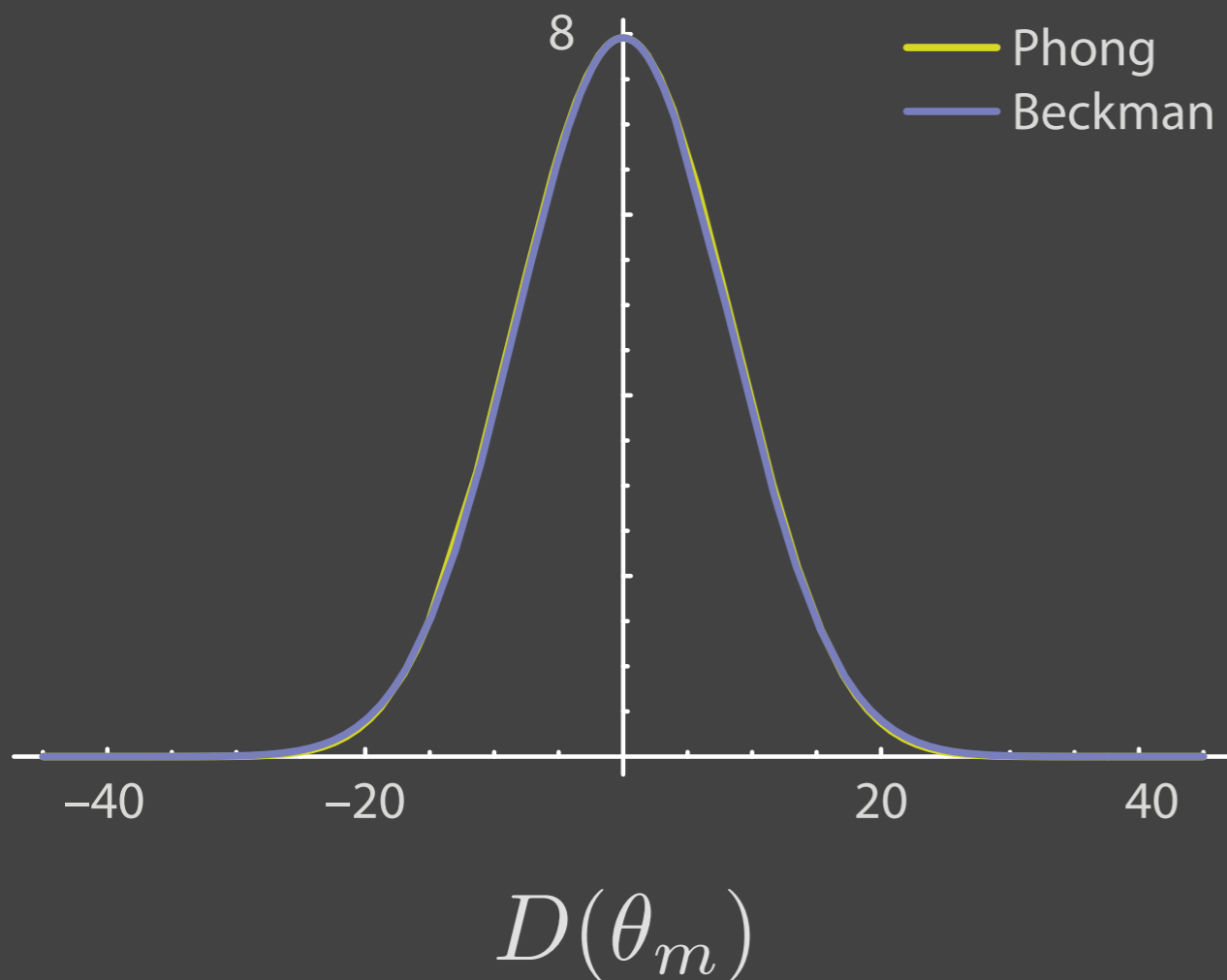




# Normal distributions

Choice of distribution is determined by surface

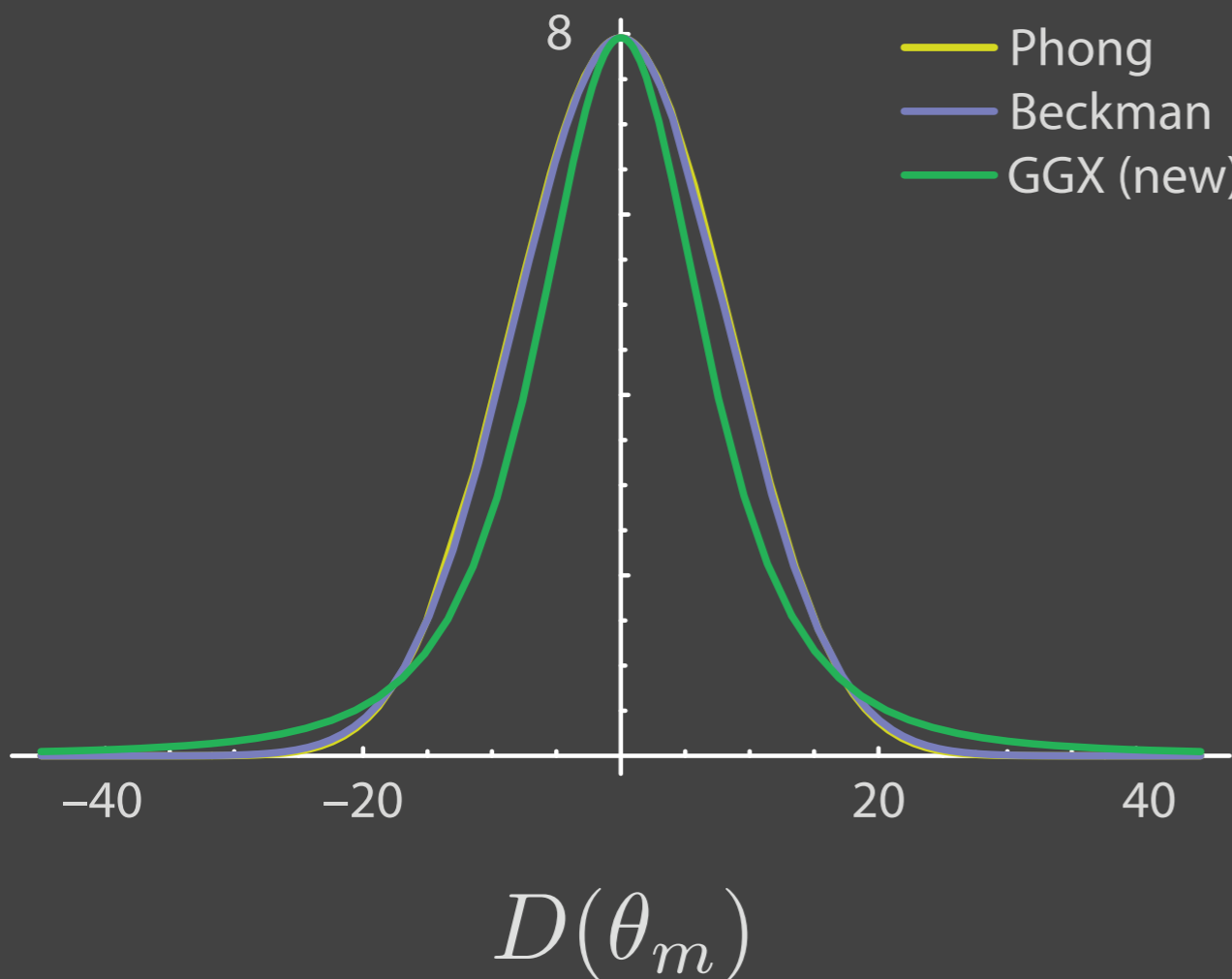
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