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IMAGING DATA EVALUATION AND ANALYTICS LAB (IDEAL)

CS5540: Computational Techniques for Analyzing Clinical Data

Lecture 17:

Dynamic MRI Image Reconstruction

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Parallel Imaging For Dynamic Images

– Maximum a posteriori reconstruction for dynamic images, using Gaussian prior on the dynamic part: MAP-SENSE

– MAP reconstruction under smoothness priors time: k-t SESNE

– MAP recon under sparsity priors in x-f space

Dynamic images:

What is the right imaging model?

SENSE

$$y(t) = H\,x(t) + n(t), \quad n \text{ is Gaussian} \qquad (1)$$

MAP Sense

$$y(t) = H\,x(t) + n(t), \quad n \text{ Gaussian around } 0,$$
$$x \text{ Gaussian around mean image} \qquad (2)$$

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$$x \text{ is smooth in time} \qquad (3)$$

Generalized series, RIGR, TRICKS

K-t SENSE

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$$x \text{ is smooth in x-t, space, sparse in k-f space}$$
$$x \text{ is sparse in x-f space} \qquad (4)$$

compressed sensing

"MAP-SENSE" – Maximum A Posteriori Parallel Reconstruction Under Sensitivity Errors

- MAP-Sense : optimal for Gaussian distributed images
- Like a spatially variant Wiener filter
- But much faster, due to our stochastic MR image model

Recall: Bayesian Estimation

- Bayesian methods maximize the posterior probability:

$$Pr(x|y) \square Pr(y|x) \,.\, Pr(x)$$

- $Pr(y|x)$ (likelihood function) = $\exp(- ||y-Hx||^2)$
- $Pr(x)$ (prior PDF) = Gaussian prior:

$$\mathrm{Pr}(\mathrm{x}) = \exp\{- \tfrac{1}{2}\ \mathrm{x}^T \mathrm{R}_x^{-1}\ \mathrm{x}\}$$

- MAP estimate:

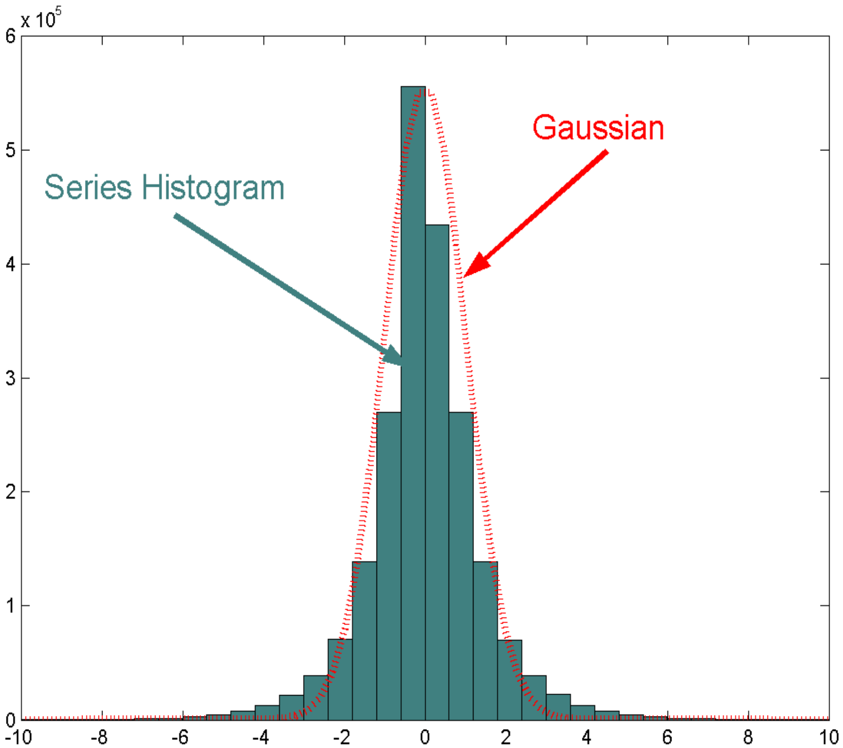
$$x_{est} = arg\ min\ ||y-Hx||^2 + G(x)$$

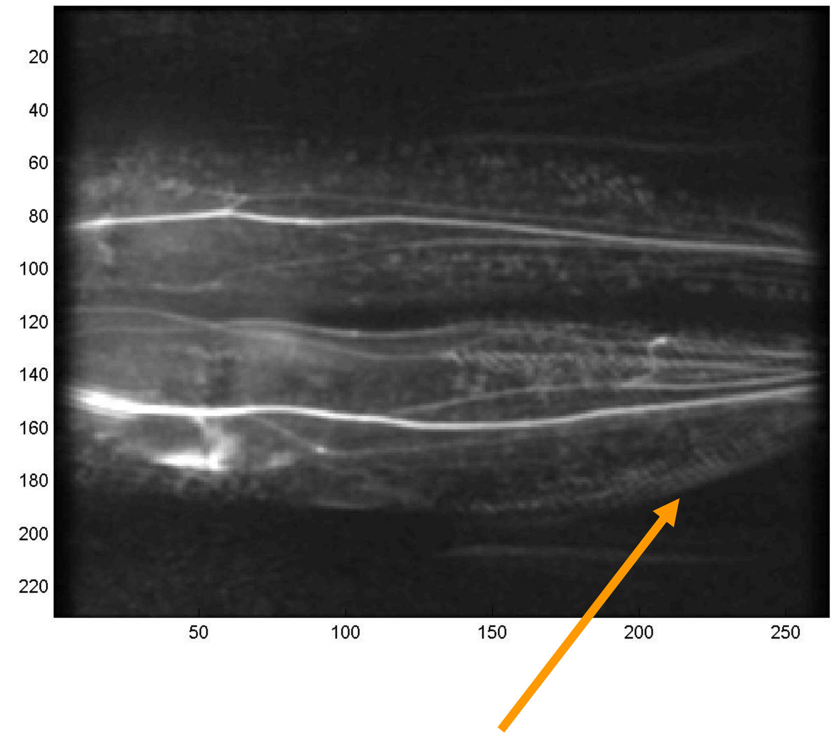
- MAP estimate for Gaussian everything is known as Wiener estimate

Spatial Priors For Images - Example

Frames are tightly distributed around mean

After subtracting mean, images are close to Gaussian





envelope *a(i,j)*

Prior: -mean is μ_x

-local std.dev. varies as *a(i,j)*

Spatial Priors for MR images

stationary process

- Stochastic MR image model:

$$x(i,j) = \boldsymbol{\mu}_{\boldsymbol{x}}(i,j) + a(i,j)\,.\,(h ** p)(i,j) \qquad (1)$$

** denotes 2D convolution

$\boldsymbol{\mu}_{\boldsymbol{x}}(i,j)$ is mean image for class
$p(i,j)$ is a unit variance i.i.d. stochastic process
$a(i,j)$ is an envelope function
$h(i,j)$ simulates correlation properties of image x

$$x = ACp + \boldsymbol{\mu} \qquad (2)$$

where $A = diag(a)$, and C is the Toeplitz matrix generated by h

- Can model many important stationary and non-stationary cases

MAP estimate for Imaging Model (3)

- The Wiener estimate

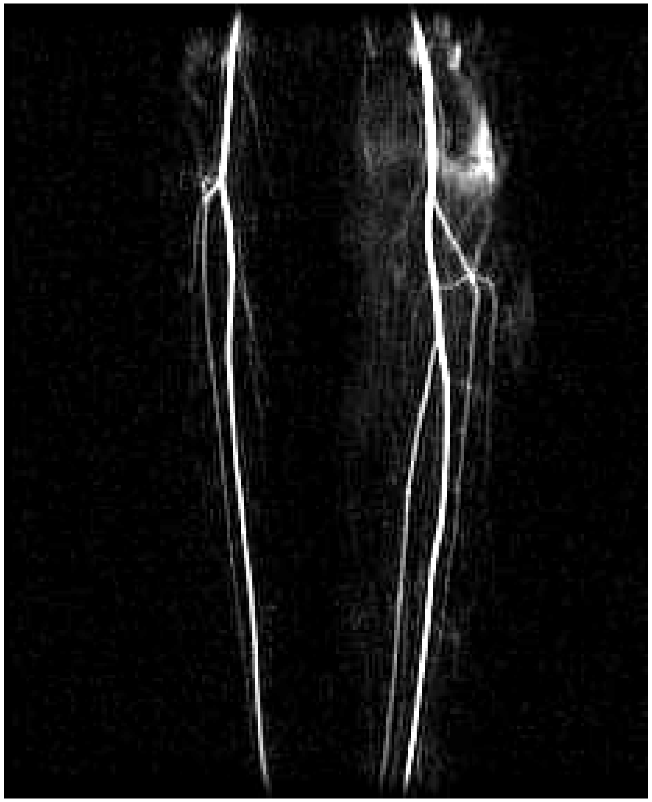
$$x_{MAP} - \boldsymbol{\mu}_x = HR_x (HR_x H^H + R_n)^{-1} (y - \boldsymbol{\mu}_y) \quad (3)$$

R_x, R_n = covariance matrices of x and n

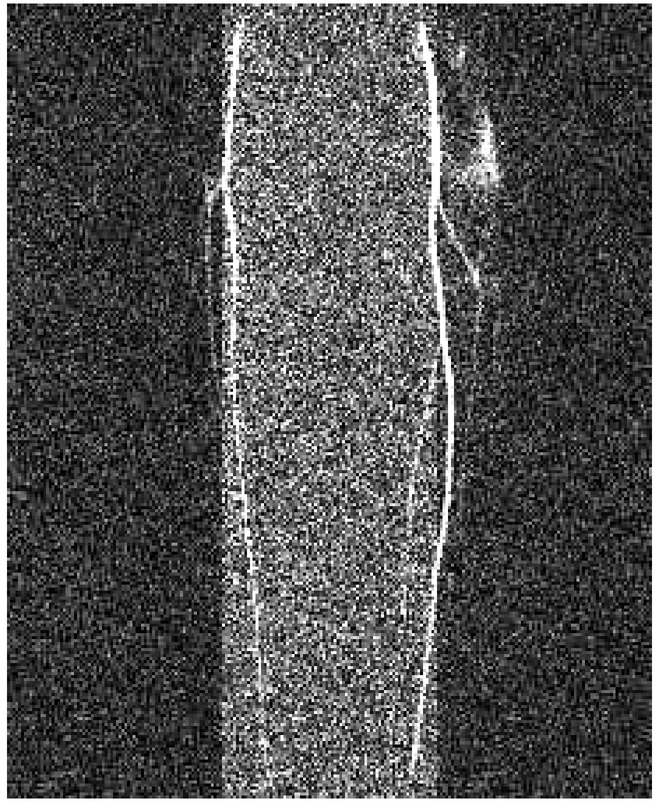
MAP-SENSE Preliminary Results

- Scans acceleraty 5x
- The angiogram was computed by:

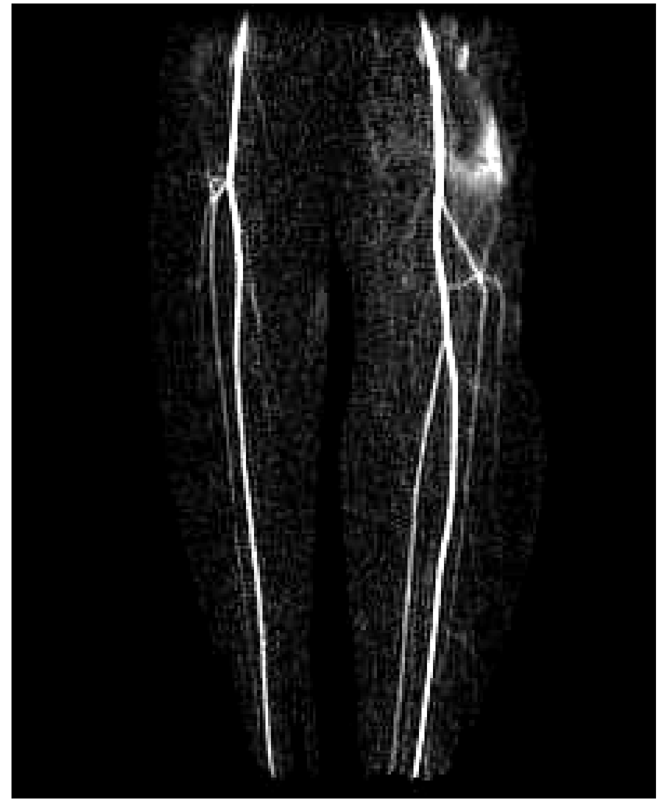
  avg(post-contrast) – avg(pre-contrast)



Unaccelerated



5x faster: SENSE



5x faster: MAP-SENSE

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K-t SENSE

compressed sensing

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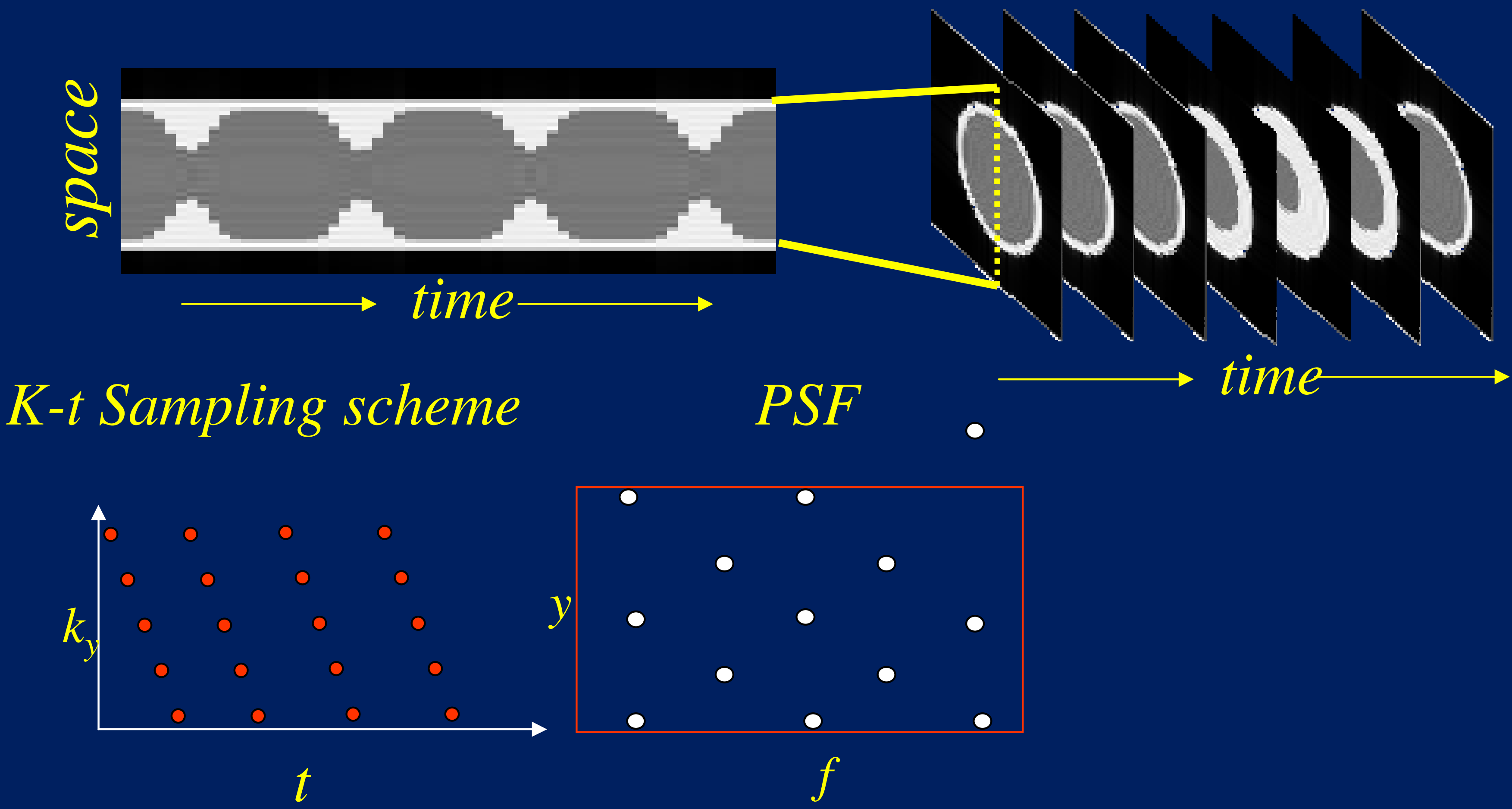
"k-t SENSE" – Maximum A Posteriori Parallel Reconstruction Under smoothness of images in x-t space (ie sparsity in k-f space)

- Exploits the smoothness of spatio-temporal signals
- ➔ sparseness (finite support) in k-f space
- The k-f properties deduced from low spatial frequency training data
- Then the model is applied to undersampled acquisition

Jeffrey Tsao, Peter Boesiger, and Klaas P. Pruessmann. ***k-t BLAST and k-t SENSE: Dynamic MRI With High Frame Rate Exploiting Spatiotemporal Correlations.*** *Magnetic Resonance in Medicine 50:1031–1042 (2003)*

K-t SENSE: Sparsity in k-F space



- By formulating the x-f sparsity model as a prior distribution, we can think of k-t SENSE as a Bayesian method!

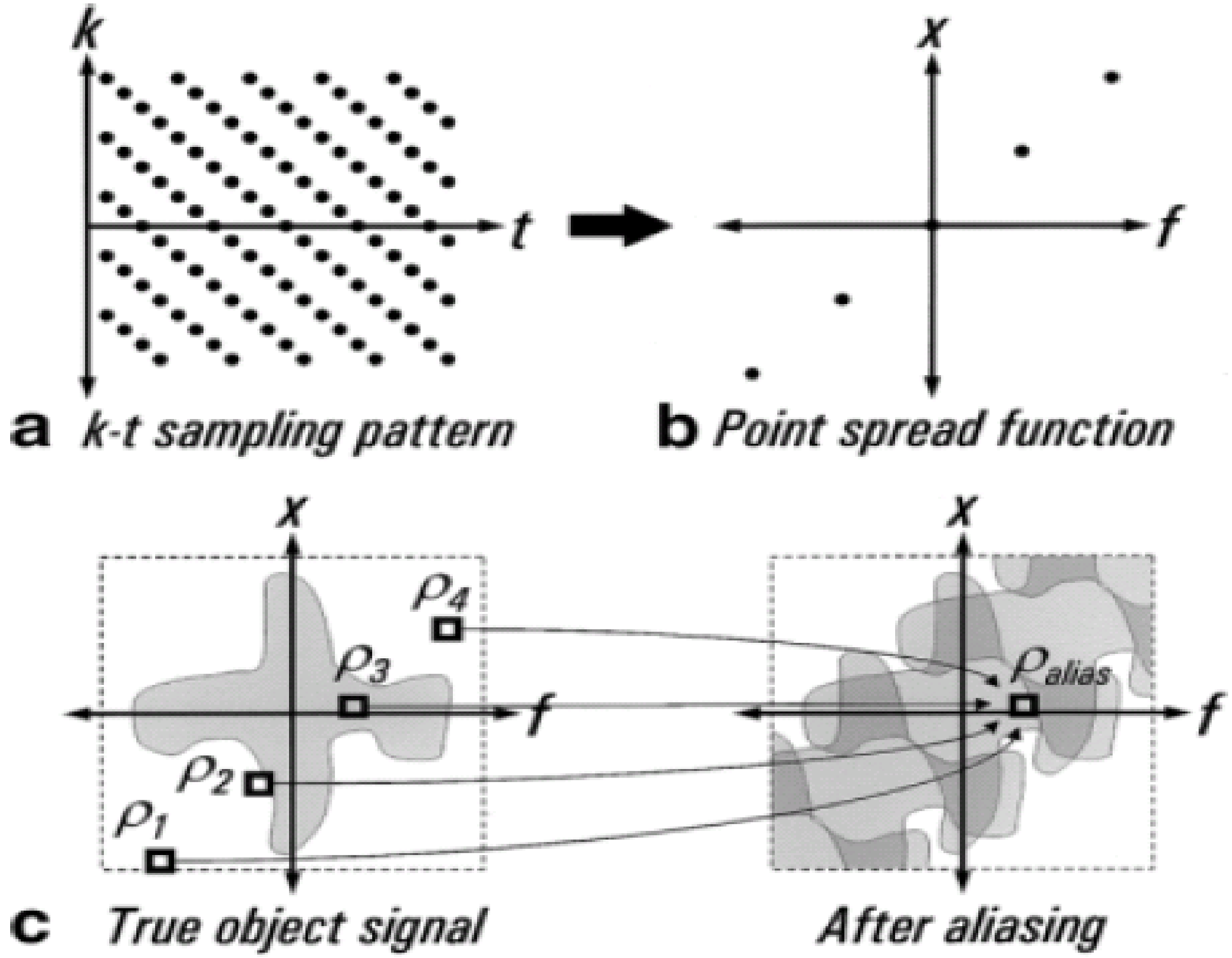
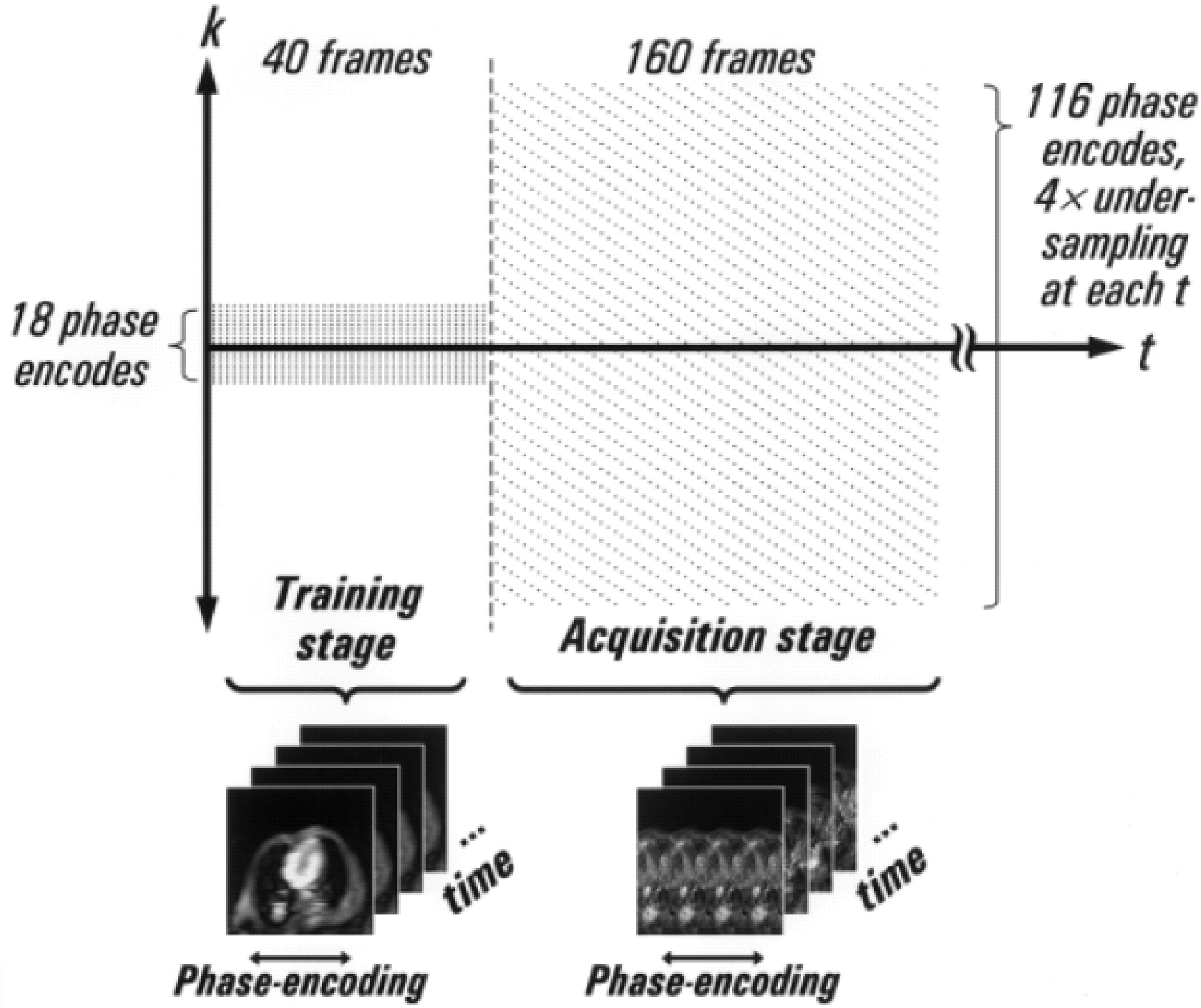


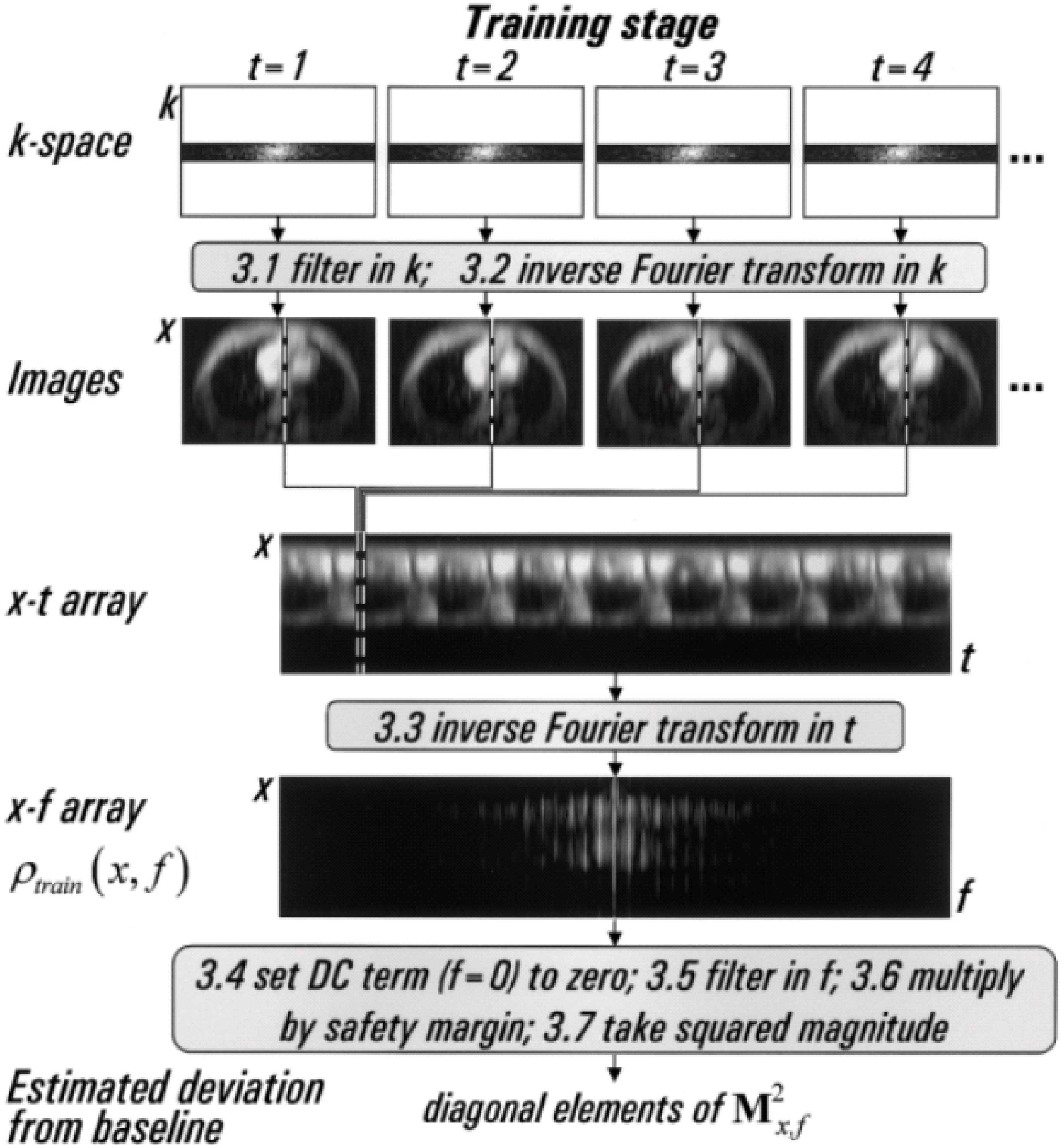
FIG. 1. **a:** *k-t* space sampling pattern with a 4-fold acceleration. "*k*" and "*t*" refer to the phase-encode index and time, respectively. The sampling pattern is equivalent to sampling on a sheared grid. **b:** Resulting point spread function in *x-f* space. "*x*" and "*f*" refer to the spatial position along the phase-encoding direction and temporal frequency, respectively. **c:** Convolution of object signals in *x-f* space with point spread function, resulting in aliasing which maps ρ_1, ρ_2, ρ_3, and ρ_4 onto a single aliased voxel ρ_{alias}. To avoid clutter, not all the signal replicates are shown.

•*Tsao et al, MRM03*

"kT-SENSE" – 2 stages of scanning: training and acquisition



Processing steps of Training stage



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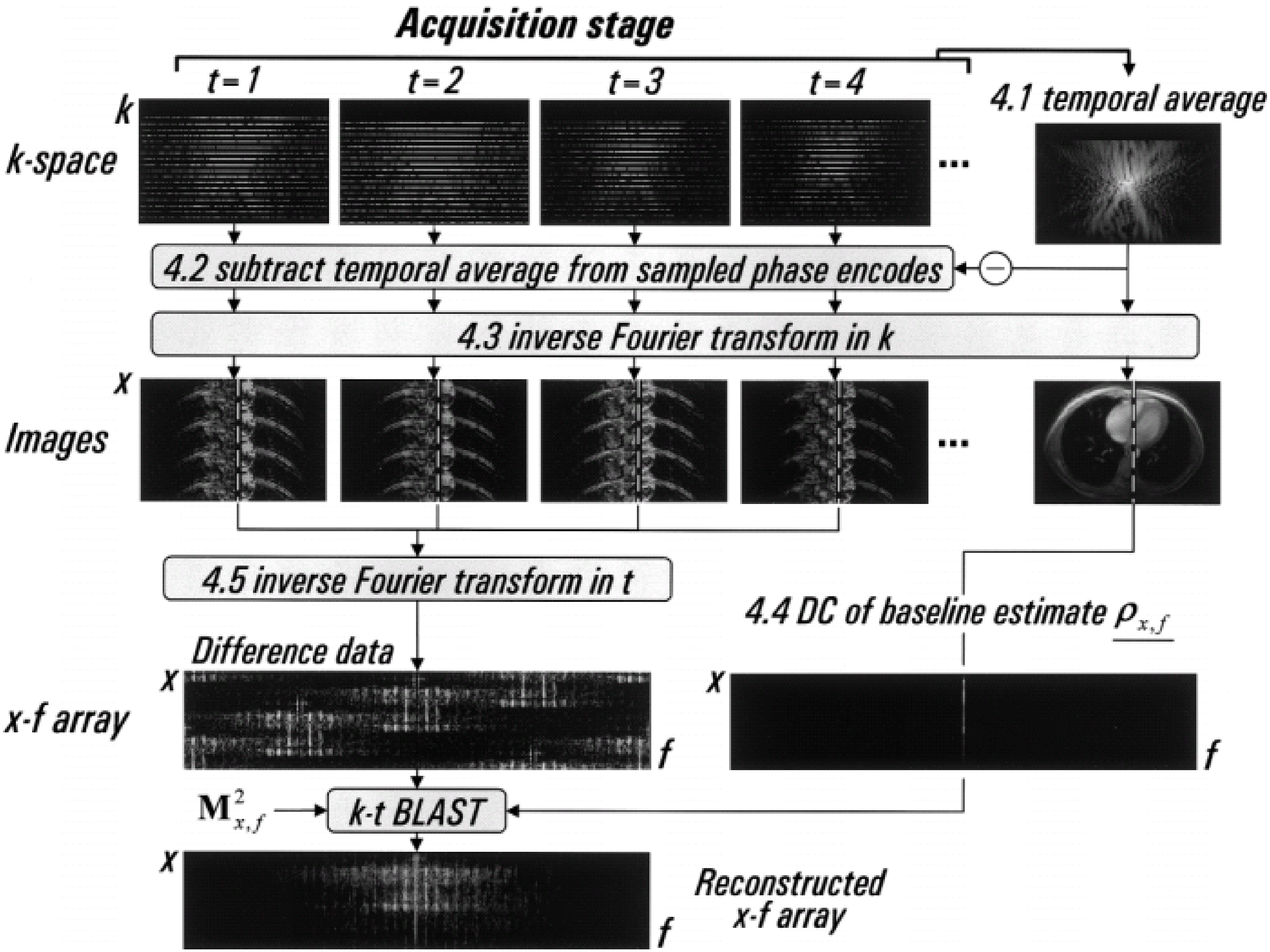
Processing steps of Acquisition stage



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Possible neuroimaging applications

All modalities are applicable, but esp. ones that are time-sensitive
Perfusion, flow, DTI, etc

Accelerating Cine Phase-Contrast Flow Measurements Using k-t BLAST and k-t SENSE. *Christof Baltes, Sebastian Kozerke, Michael S. Hansen, Klaas P. Pruessmann, Jeffrey Tsao, and Peter Boesiger. Magnetic Resonance in Medicine 54:1430–1438 (2005)*

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D Xu, L Ying, ZP Liang, ***Parallel generalized series MRI: algorithm and application to cancer imaging.*** *Engineering in Medicine and Biology Society, 2004. IEMBS'04*

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References

Overview

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- Website: http://www.cs.cornell.edu/~rdz/SENSE.htm

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Regularized SENSE

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Compressed Sensing in MR

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