Preliminary Definitions

Given a message m sent from m.source to m.dest,

$Depend(m)$ $\left\{ p \in P \;\middle|\; \begin{array}{l} \vee (p = m.dest) \text{ and } p \text{ delivered } m \\ \vee (\exists e_p : (deliver_{m.dest}(m) \rightarrow e_p)) \end{array} \right\}$

$Log(m)$ set of processes with a copy of the determinant of m in their volatile memory

p orphan of a set C of crashed processes:

$$(p \notin C) \wedge \exists m : (Log(m) \subseteq C \wedge p \in Depend(m))$$

The "No-Orphans" Consistency Condition

No orphans after crash C if:

$$\forall m : (Log(m) \subseteq C) \Rightarrow (Depend(m) \subseteq C)$$

No orphans after any C if:

$$\forall m : (Depend(m) \subseteq Log(m))$$

The Consistency Condition

$$\forall m : (\neg stable(m) \Rightarrow (Depend(m) \subseteq Log(m)))$$

Optimistic and Pessi

No orphans after crash C if:

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Optimistic weakens it to:

$$\forall m : (Log(m) \subseteq C) \Rightarrow \Diamond(Depend(m) \subseteq C)$$

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No orphans after any crash if:

$$\forall m : (\neg stable(m) \Rightarrow (Depend(m) \subseteq Log(m)))$$

Pessimistic strengthens it to:

$$\forall m : (\neg stable(m) \Rightarrow |Depend(m)| \leq 1)$$

Causal Message Logging

No orphans after any crash of size at most f if:

$$\forall m : (\neg stable(m) \Rightarrow (Depend(m) \subseteq Log(m)))$$

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$$\forall m : (\neg stable(m) \Rightarrow (Depend(m) \subseteq Log(m)))$$

Causal strengthens it to:

$$\forall m : \left(\neg stable(m) \Rightarrow \begin{pmatrix} \wedge(Depend(m) \subseteq Log(m)) \\ \wedge\Diamond(Depend(m) = Log(m)) \end{pmatrix}\right)$$

(so that $Depend$ is only as large as necessary)

An Example

Causal Logging:

$$\forall m : (\neg stable(m) \Rightarrow (Depend(m) \subseteq Log(m)))$$

If $f = 1$, $\quad stable(m) \equiv |Log(m)| \geq 2$

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$$\texttt{If } f = 1, \quad stable(m) \equiv |Log(m)| \geq 2$$

