Validity

Initially $V = \{v_i\}$

To execute propose(v_i)

round k, $1 \leq k \leq f+1$

1: send $\{v \in V : p_i$ has not already sent $v\}$ to all
2: for all j, $0 \leq j \leq n-1$, $j \neq i$ do
3: receive S_j from p_j
4: $V := V \cup S_j$

decide(x) occurs as follows:

5: if $k = f+1$ then
6: decide min(V)

Validity

Initially $V = \{v_i\}$

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round k, $1 \leq k \leq f+1$

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3: receive S_j from p_j
4: $V := V \cup S_j$

decide(x) occurs as follows:
5: if $k = f+1$ then
6: decide min(V)

- Suppose every process proposes v^*
- Since only crash model, only v^* can be sent
- By Validity of send and receive, v^* sent by correct processes will be received by correct processes
- By Uniform Integrity of send and receive, only v^* can be received
- By protocol, $V = \{v^*\}$
- min(V) = v^*
- decide(v^*)

Agreement

Initially $V = \{v_i\}$

To execute propose(v_i)

round k, $1 \leq k \leq f+1$

1: send $\{v \in V : p_i$ has not already sent $v\}$ to all
2: for all j, $0 \leq j \leq n-1$, $j \neq i$ do
3: receive S_j from p_j
4: $V := V \cup S_j$

decide(x) occurs as follows:

5: if $k = f+1$ then
6: decide min(V)

Lemma 1

For any $r \geq 1$, if a process p receives a value v in round r, then there exists a sequence of processes $p_0, p_1, \ldots, p_r$ such that $p_r = p$, p_0 is v's proponent, and in each round p_{k-1} sends v and p_k receives it. Furthermore, all processes in the sequence are distinct.

Proof

By induction on the length of the sequence

Agreement

Initially $V = \{v_i\}$

To execute propose(v_i)

round k, $1 \leq k \leq f+1$

1: send $\{v \in V : p_i$ has not already sent $v\}$ to all
2: for all j, $0 \leq j \leq n-1$, $j \neq i$ do
3: receive S_j from p_j
4: $V := V \cup S_j$

decide(x) occurs as follows:

5: if $k = f+1$ then
6: decide min(V)

Lemma 2:

In every execution, at the end of round $f+1$, $V_i = V_j$ for every correct processes p_i and p_j

Agreement follows from Lemma 2, since min is a deterministic function

Agreement

Initially $V = \{v_i\}$

To execute propose(v_i)

round k, $1 \leq k \leq f+1$

1: send $\{v \in V : p_i$ has not already sent $v\}$ to all
2: for all j, $0 \leq j \leq n-1$, $j \neq i$ do
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decide(x) occurs as follows:

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6: decide min(V)

Lemma 2:

In every execution, at the end of round $f+1$, $V_i = V_j$ for every correct processes p_i and p_j

Agreement follows from Lemma 2, since min is a deterministic function

Proof:

- Show that if a correct p has x in its V at the end of round $f+1$, then every correct p has x in its V at the end of round $f+1$

Agreement

Initially $V = \{v_i\}$

To execute propose(v_i)

round k, $1 \le k \le f+1$

1: send $\{v \in V : p_i$ has not already sent $v\}$ to all
2: for all j, $0 \le j \le n-1$, $j \ne i$ do
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In every execution, at the end of round $f+1$, $V_i = V_j$ for every correct processes p_i and p_j

Agreement follows from Lemma 2, since min is a deterministic function

Proof:

- Show that if a correct p has x in its V at the end of round $f+1$, then every correct p has x in its V at the end of round $f+1$
- Let r be earliest round x is added to the V of a correct p. Let that process be p^*
- If $r \le f$, then p^* sends x in round $r+1 \le f+1$; every correct process receives x and adds x to its V in round $r+1$

Agreement

Initially $V = \{v_i\}$

To execute propose(v_i)

round k, $1 \le k \le f+1$

1: send $\{v \in V : p_i$ has not already sent $v\}$ to all
2: for all j, $0 \le j \le n-1$, $j \ne i$ do
3: receive S_j from p_j
4: $V := V \cup S_j$

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Proof:

- Show that if a correct p has x in its V at the end of round $f+1$, then every correct p has x in its V at the end of round $f+1$
- Let r be earliest round x is added to the V of a correct p. Let that process be p^*
- If $r \le f$, then p^* sends x in round $r+1 \le f+1$; every correct process receives x and adds x to its V in round $r+1$
- What if $r = f+1$?

Agreement

Initially $V = \{v_i\}$

To execute propose(v_i)

round k, $1 \leq k \leq f+1$

1: send $\{v \in V : p_i$ has not already sent $v\}$ to all
2: for all j, $0 \leq j \leq n-1$, $j \neq i$ do
3: receive S_j from p_j
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decide(x) occurs as follows:

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Lemma 2:

In every execution, at the end of round $f+1$, $V_i = V_j$ for every correct processes p_i and p_j

Agreement follows from Lemma 2, since min is a deterministic function

Proof:

- Show that if a correct p has x in its V at the end of round $f+1$, then every correct p has x in its V at the end of round $f+1$
- Let r be earliest round x is added to the V of a correct p. Let that process be p^*
- If $r \leq f$, then p^* sends x in round $r+1 \leq f+1$; every correct process receives x and adds x to its V in round $r+1$
- What if $r = f+1$?
- By Lemma 1, there exists a sequence of distinct processes $p_0, \ldots, p_{f+1} = p^*$
- Consider processes $p_0, \ldots, p_f$
- $f+1$ processes; only f faulty
- one of $p_0, \ldots, p_f$ is correct, and adds x to its V before p^* does it in round r

CONTRADICTION!

Terminating Reliable Broadcast

Validity	If the sender is correct and broadcasts a message m, then all correct processes eventually deliver m
Agreement	If a correct process delivers a message m, then all correct processes eventually deliver m
Integrity	Every correct process delivers at most one message, and if it delivers $m \neq$ SF, then some process must have broadcast m
Termination	Every correct process eventually delivers some message

TRB for benign failures

Sender in round 1:

```
1:  send m to all
```

Process p in round k, $1 \le k \le f+1$

```
1:  if delivered m in round k-1 then
2:    if p ≠ sender then
3:      send m to all
4:    halt
5:  receive round k messages
6:  if received m then
7:    deliver(m)
8:    if  k = f+1 then halt
9:  else if k = f+1
10:   deliver(SF)
11:   halt
```

TRB for benign failures

```
Sender in round 1:
1:   send m to all

Process p in round    k, 1 ≤ k ≤ f+1
1:   if delivered m in round k-1 then
2:     if p ≠ sender then
3:       send m to all
4:     halt
5:   receive round k messages
6:   if received m then
7:     deliver(m)
8:     if  k = f+1 then halt
9:   else if k = f+1
10:    deliver(SF)
11:    halt
```

Terminates in $f+1$ rounds

How can we do better?

Find a protocol whose round complexity is proportional to t -the number of failures that actually occurred- rather than to f -the max number of failures that may occur

Early stopping: the idea

- Suppose processes can detect the set of processes that have failed by the end of round i
- Call that set $faulty(p, i)$
- If $|faulty(p, i)| < i$ there can be no active dangerous chains, and p can safely deliver SF

Early Stopping: The Protocol

$faulty(p,k) \equiv$ set of processes that failed to send a message to p in some round

1: if p = sender then value := m else value:= ?

Process p in round $k, 1 \leq k \leq f+1$

2: send value to all
3: if delivered in round $k-1$ then halt
4: receive round k values from all
5: $faulty(p,k) := faulty(p,k-1) \cup \{q \mid p$ received no value from q in round $k\}$
6: if received value $v \neq ?$ then
7: value := v
8: deliver value
9: if p = sender then value := ?
10: else if $k = f+1$ or $|faulty(p,k)| < k$ then
11: value := SF
12: deliver value
13: if $k = f+1$ then halt

Termination

Let $faulty(p,k)$ be the set of processes that have failed to send a message to p in any round $1,\ldots,k$

```
1:   if p = sender then value := m else value:= ?

     Process p in round k, 1≤k≤f+1

2:   send value to all
3:   if delivered in round k−1 then halt
4:   receive round k values from all
5:   faulty(p,k) := faulty(p,k−1) ∪ {q | p
     received no value from q in round k}
6:   if received value v ≠ ? then
7:       value := v
8:       deliver value
9:       if p = sender then value := ?
10:  else if k=f+1 or |faulty(p,k)|<k then
11:      value := SF
12:      deliver value
13:      if k=f+1 then halt
```

Termination

Let $faulty(p,k)$ be the set of processes that have failed to send a message to p in any round $1,\ldots,k$

```
1:   if p = sender then value := m else value:= ?

     Process p in round k, 1≤k≤f+1

2:   send value to all
3:   if delivered in round k−1 then halt
4:   receive round k values from all
5:   faulty(p,k) := faulty(p,k−1) ∪ {q | p
     received no value from q in round k}
6:   if received value v ≠ ? then
7:       value := v
8:       deliver value
9:       if p = sender then value := ?
10:  else if k=f+1 or |faulty(p,k)|<k then
11:      value := SF
12:      deliver value
13:      if k=f+1 then halt
```

- If in any round a process receives a value, then it delivers the value in that round
- If a process has received only "?" for $f+1$ rounds, then it delivers SF in round $f+1$

Validity

Let $faulty(p,k)$ be the set of processes that have failed to send a message to p in any round $1,\ldots,k$

```
1:   if p = sender then value := m else value:= ?

     Process p in round k, 1≤k≤f+1

2:   send value to all
3:   if delivered in round k−1 then halt
4:   receive round k values from all
5:   faulty(p,k) := faulty(p,k−1) ∪ {q | p
     received no value from q in round k}
6:   if received value v ≠ ? then
7:       value := v
8:       deliver value
9:       if p = sender then value := ?
10:  else if k=f+1 or |faulty(p,k)|<k then
11:      value := SF
12:      deliver value
13:      if k=f+1 then halt
```

Validity

Let $faulty(p,k)$ be the set of processes that have failed to send a message to p in any round $1,\ldots,k$

```
1:   if p = sender then value := m else value:= ?

     Process p in round k, 1≤k≤f+1

2:   send value to all
3:   if delivered in round k−1 then halt
4:   receive round k values from all
5:   faulty(p,k) := faulty(p,k−1) ∪ {q | p
     received no value from q in round k}
6:   if received value v ≠ ? then
7:       value := v
8:       deliver value
9:       if p = sender then value := ?
10:  else if k=f+1 or |faulty(p,k)|<k then
11:      value := SF
12:      deliver value
13:      if k=f+1 then halt
```

- If the sender is correct then it sends m to all in round 1
- By Validity of the underlying send and receive, every correct process will receive m by the end of round 1
- By the protocol, every correct process will deliver m by the end of round 1

Agreement - 1

Let $faulty(p,k)$ be the set of processes that have failed to send a message to p in any round $1,\ldots,k$

```
1:   if p = sender then value := m else value:= ?

Process p in round k, 1≤k≤f+1

2:   send value to all
3:   if delivered in round k−1 then halt
4:   receive round k values from all
5:   faulty(p,k) := faulty(p,k−1) ∪ {q | p
     received no value from q in round k}
6:   if received value v ≠ ? then
7:       value := v
8:       deliver value
9:       if p = sender then value := ?
10:  else if k=f+1 or |faulty(p,k)|<k then
11:      value := SF
12:      deliver value
13:      if k=f+1 then halt
```

Lemma 1:

For any $r \geq 1$, if a process p delivers $m \neq$ SF in round r, then there exists a sequence of processes $p_0, p_1, \ldots, p_r$ such that p_0 = sender, $p_r = p$, and in each round $k, 1 \leq k \leq r$, p_{k-1} sent m and p_k received it. Furthermore, all processes in the sequence are distinct, unless $r = 1$ and $p_0 = p_1 =$ sender

Lemma 2:

For any $r \geq 1$, if a process p sets value to SF in round r, then there exist some $j \leq r$ and a sequence of distinct processes $q_j, q_{j+1}, \ldots, q_r = p$ such that q_j only receives "?" in rounds 1 to j, $|faulty(q_j, j)| < j$, and in each round $k, j+1 \leq k \leq r$, q_{k-1} sends SF to q_k and q_k receives SF

Agreement - 2

Let $faulty(p,k)$ be the set of processes that have failed to send a message to p in any round $1,\ldots,k$

1: if p = sender then value := m else value:= ?

Process p in round $k, 1 \leq k \leq f+1$

2: send value to all
3: if delivered in round $k-1$ then halt
4: receive round k values from all
5: $faulty(p,k) := faulty(p,k-1) \cup \{q \mid p$ received no value from q in round $k\}$
6: if received value $v \neq ?$ then
7: value := v
8: deliver value
9: if p = sender then value := ?
10: else if $k = f+1$ or $|faulty(p,k)| < k$ then
11: value := SF
12: deliver value
13: if $k = f+1$ then halt

Lemma 3:

It is impossible for p and q, not necessarily correct or distinct, to set value in the same round r to m and SF, respectively

Agreement - 2

Let $faulty(p,k)$ be the set of processes that have failed to send a message to p in any round $1,\ldots,k$

1: if p = sender then value := m else value:= ?

Process p in round $k, 1 \leq k \leq f+1$

2: send value to all
3: if delivered in round $k-1$ then halt
4: receive round k values from all
5: $faulty(p,k) := faulty(p,k-1) \cup \{q \mid p$ received no value from q in round $k\}$
6: if received value $v \neq ?$ then
7: value := v
8: deliver value
9: if p = sender then value := ?
10: else if $k = f+1$ or $|faulty(p,k)| < k$ then
11: value := SF
12: deliver value
13: if $k = f+1$ then halt

Lemma 3:

It is impossible for p and q, not necessarily correct or distinct, to set value in the same round r to m and SF, respectively

Proof

By contradiction

Suppose p sets value = m and q sets value = SF

By Lemmas 1 and 2 there exist

$$p_0,\ldots,p_r$$

$$q_j,\ldots,q_r$$

with the appropriate characteristics

Since q_j did not receive m from process p_{k-1} $1 \leq k \leq j$ in round k

q_j must conclude that $p_0,\ldots,p_{j-1}$ are all faulty processes

But then, $|faulty(q_j,j)| \geq j$

CONTRADICTION

Agreement - 3

Let $faulty(p,k)$ be the set of processes that have failed to send a message to p in any round $1,\ldots,k$

1: if p = sender then value := m else value:= ?

Process p in round $k, 1 \leq k \leq f+1$

2: send value to all
3: if delivered in round $k-1$ then halt
4: receive round k values from all
5: $faulty(p,k) := faulty(p,k-1) \cup \{q \mid p$ received no value from q in round $k\}$
6: if received value $v \neq ?$ then
7: value := v
8: deliver value
9: if p = sender then value := ?
10: else if $k = f+1$ or $|faulty(p,k)| < k$ then
11: value := SF
12: deliver value
13: if $k = f+1$ then halt

Agreement - 3

Let $faulty(p,k)$ be the set of processes that have failed to send a message to p in any round $1,\ldots,k$

1: if p = sender then value := m else value:= ?

Process p in round $k, 1 \leq k \leq f+1$

2: send value to all
3: if delivered in round $k-1$ then halt
4: receive round k values from all
5: $faulty(p,k) := faulty(p,k-1) \cup \{q \mid p$ received no value from q in round $k\}$
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7: value := v
8: deliver value
9: if p = sender then value := ?
10: else if $k = f+1$ or $|faulty(p,k)| < k$ then
11: value := SF
12: deliver value
13: if $k = f+1$ then halt

Proof

If no correct process ever receives m, then every correct process delivers SF in round $f+1$

Agreement - 3

Let $faulty(p,k)$ be the set of processes that have failed to send a message to p in any round $1,\ldots,k$

1: if p = sender then value := m else value:= ?

Process p in round $k, 1 \leq k \leq f+1$

2: send value to all
3: if delivered in round $k-1$ then halt
4: receive round k values from all
5: $faulty(p,k) := faulty(p,k-1) \cup \{q \mid p$ received no value from q in round $k\}$
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10: else if $k = f+1$ or $|faulty(p,k)| < k$ then
11: value := SF
12: deliver value
13: if $k = f+1$ then halt

Proof

If no correct process ever receives m, then every correct process delivers SF in round $f+1$

Otherwise, let r be the earliest round in which a correct process delivers value ≠ SF

$r \leq f$

- By Lemma 3, no (correct) process can set value differently in round r
- In round $r+1 \leq f+1$, that correct process sends its value to all
- Every correct process receives and delivers the value in round $r+1 \leq f+1$

$r = f+1$

- By Lemma 1, there exists a sequence $p_0, \cdots, p_{f+1}$ $= p_r$ of distinct processes
- Consider processes $p_0, \cdots, p_f$
  - $f+1$ processes; only f faulty
  - one of $p_0, \cdots, p_f$ is correct-- let it be p_c
  - To send v in round $c+1$, p_c must have set its value to v and delivered v in round $c < r$

CONTRADICTION

Integrity

Let $faulty(p,k)$ be the set of processes that have failed to send a message to p in any round $1,\ldots,k$

1: if p = sender then value := m else value:= ?

Process p in round $k, 1 \leq k \leq f+1$

2: send value to all
3: if delivered in round $k-1$ then halt
4: receive round k values from all
5: $faulty(p,k) := faulty(p,k-1) \cup \{q \mid p$ received no value from q in round $k\}$
6: if received value $v \neq ?$ then
7: value := v
8: deliver value
9: if p = sender then value := ?
10: else if $k = f+1$ or $|faulty(p,k)| < k$ then
11: value := SF
12: deliver value
13: if $k = f+1$ then halt

Integrity

Let $faulty(p,k)$ be the set of processes that have failed to send a message to p in any round $1,\ldots,k$

```
1:   if p = sender then value := m else value:= ?

     Process p in round k, 1≤k≤f+1

2:   send value to all
3:   if delivered in round k−1 then halt
4:   receive round k values from all
5:   faulty(p,k) := faulty(p,k−1) ∪ {q | p
     received no value from q in round k}
6:   if received value v ≠ ? then
7:       value := v
8:       deliver value
9:       if p = sender then value := ?
10:  else if k=f+1 or |faulty(p,k)|<k then
11:      value := SF
12:      deliver value
13:      if k=f+1 then halt
```

- At most one m
  - Failures are benign, and a process executes at most one deliver event before halting
- If $m \neq$ SF, only if m was broadcast
  - From Lemma 1 in the proof of Agreement

FLP

What about the asynchronous model?

Theorem

> There is no deterministic protocol that solves Consensus in a message-passing asynchronous system in which at most one process may fail by crashing

(Fisher, Lynch, and Paterson. Impossibility of distributed consensus with one faulty process. JACM, Vol. 32, no. 2, April 1985, pp. 374-382)

Around FLP in 80 Slides

How can one get around FLP?

Weaken the problem

- Weaken termination
  - use randomization to terminate with arbitrarily high probability
  - guarantee termination only during periods of synchrony
- Weaken agreement
  - ε - agreement
    - real-valued inputs and outputs
    - agreement within real-valued small positive tolerance ε
  - k-set agreement
    - Agreement: In any execution, there is a subset W of the set of input values, |W| =k, s.t. all decision values are in W
    - Validity: In any execution, any decision value for any process is the input value of some process

How can one get around FLP?

Constrain input values

- Characterize the set of input values for which agreement is possible

Strengthen the system model

- Introduce failure detectors to distinguish between crashed processes and very slow processes

Paxos

It's (Almost) Everywhere!

Production use of Paxos [edit]

- The Petal project from DEC SRC was likely the first system to use Paxos, in this case for widely replicated global information (e.g., which machines are in the system).[20]
- Google uses the Paxos algorithm in their Chubby distributed lock service in order to keep replicas consistent in case of failure. Chubby is used by BigTable which is now in production in Google Analytics and other products.
- The Infinit peer-to-peer file system relies on Paxos to maintain consistency among replicas while allowing for quorums to evolve in size.
- Google Spanner and Megastore use the Paxos algorithm internally.
- The OpenReplica replication service uses Paxos to maintain replicas for an open access system that enables users to create fault-tolerant objects. It provides high performance through concurrent rounds and flexibility through dynamic membership changes.
- IBM supposedly uses the Paxos algorithm in their IBM SAN Volume Controller product to implement a general purpose fault-tolerant virtual machine used to run the configuration and control components of the storage virtualization services offered by the cluster.[*citation needed*]
- Microsoft uses Paxos in the Autopilot cluster management service from Bing.
- WANdisco have implemented Paxos within their DConE active-active replication technology.[21]
- XtreemFS uses a Paxos-based lease negotiation algorithm for fault-tolerant and consistent replication of file data and metadata.[22]
- Heroku uses Doozerd which implements Paxos for its consistent distributed data store.
- Ceph uses Paxos as part of the monitor processes to agree which OSDs are up and in the cluster.
- The Clustrix distributed SQL database uses Paxos for distributed transaction resolution.
- Neo4j HA graph database implements Paxos, replacing Apache ZooKeeper from v1.9
- VMware NSX Controller uses Paxos-based algorithm within NSX Controller cluster.
- Amazon Web Services uses the Paxos algorithm extensively to power its platform.[23]
- Nutanix implements the Paxos algorithm in Cassandra for metadata.
- Apache Mesos uses Paxos algorithm for its replicated log coordination.
- Windows Fabric used by many of the Azure services make use of the paxos algorithm for replication between nodes in a cluster
- Oracle NoSQL Database leverages Paxos-based automated fail-over election process in the event of a master replica node failure to minimize downtime.[24]

The Part-Time Parliament

- Parliament determines laws by passing sequence of numbered decrees
- Direct democracy: Citizens/ Legislators leave and enter the chamber at arbitrary times
- No centralized records: each legislator carries a ledger recording the approved decrees

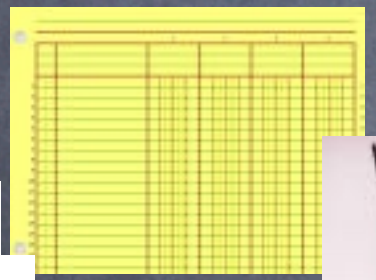
Government 101

- No two ledgers contain contradictory information
- If a majority of legislators are in the Chamber and no one enters or leaves the Chamber for a sufficiently long time, then
  - any decree proposed by a legislator is eventually passed
  - any passed decree appears on the ledger of every legislator

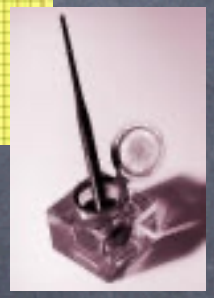
"In a world..."

Political intrigue!

Funky equipment!



ledger



pen with indelible ink

hourglass

messengers!

Λαμπσων

Δἴκστρα

Λισκωφ

Mysterious characters

Σθνειδερ

Σκεεν

Πνυελι

Δωλεφ

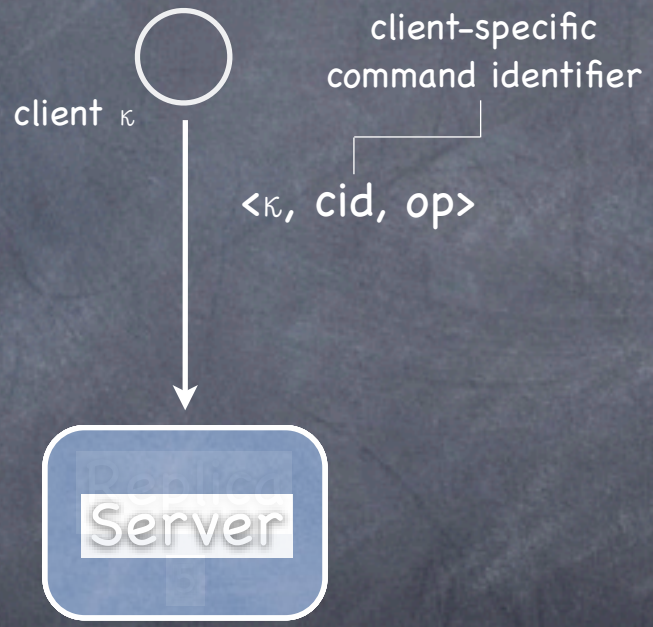
Δφωρκ

and more!

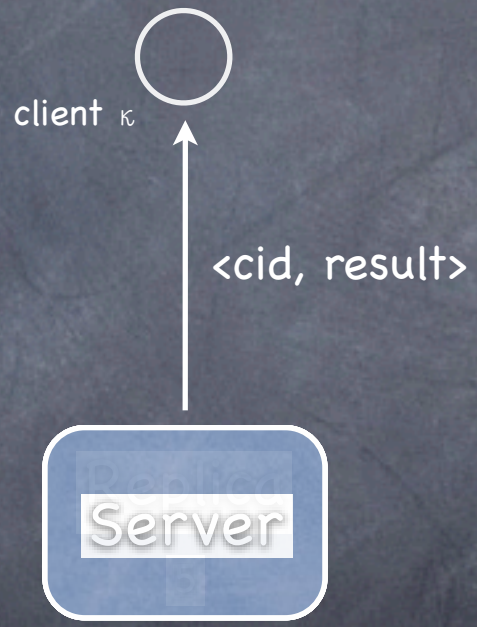
Back to the future

- A protocol for state machine replication in an asynchronous environment that admits crash failures (key ideas already present in earlier work on Viewstamped Replication by Oki and Liskov)
- Messages:
  - between correct endpoints are eventually received
  - can be lost and duplicated, but not corrupted

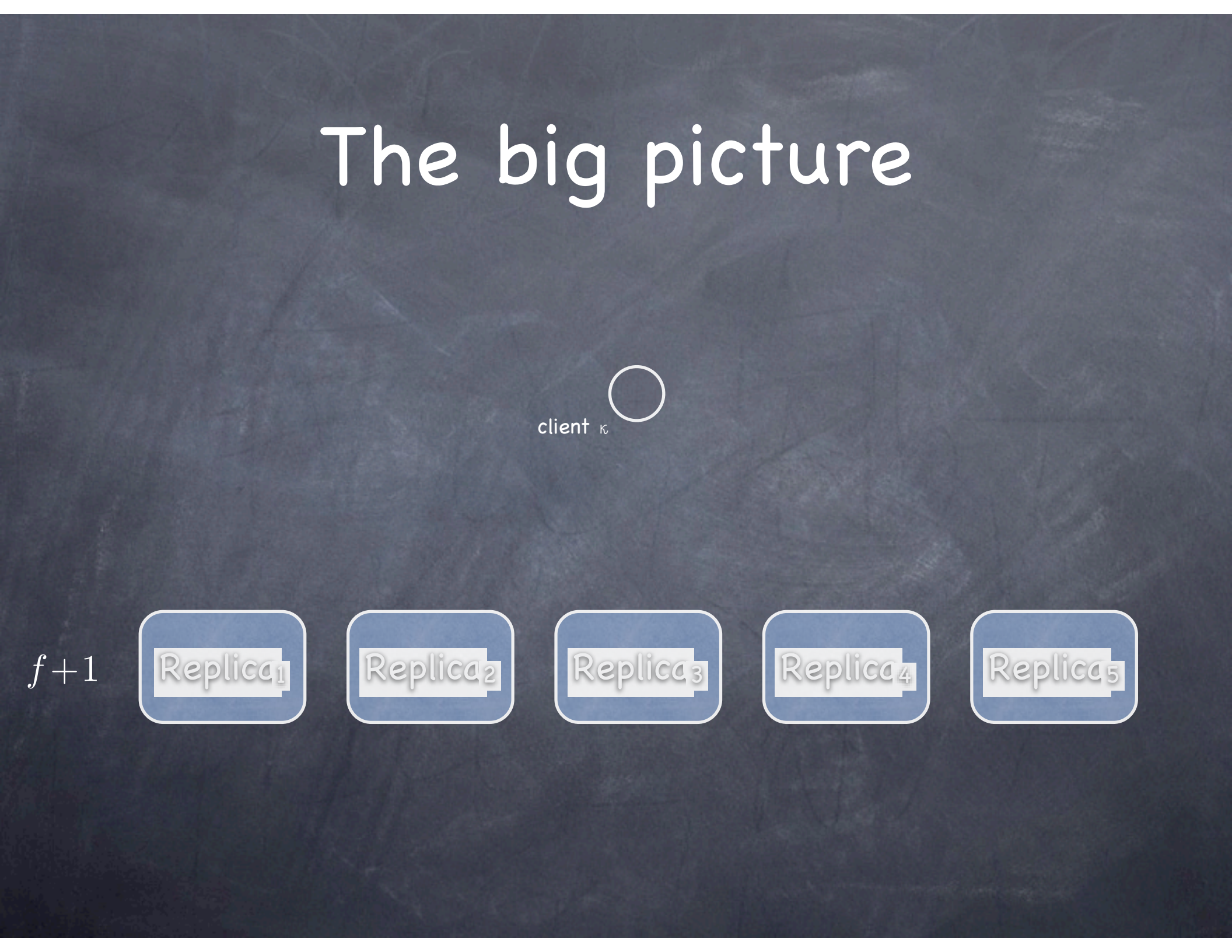
The big picture



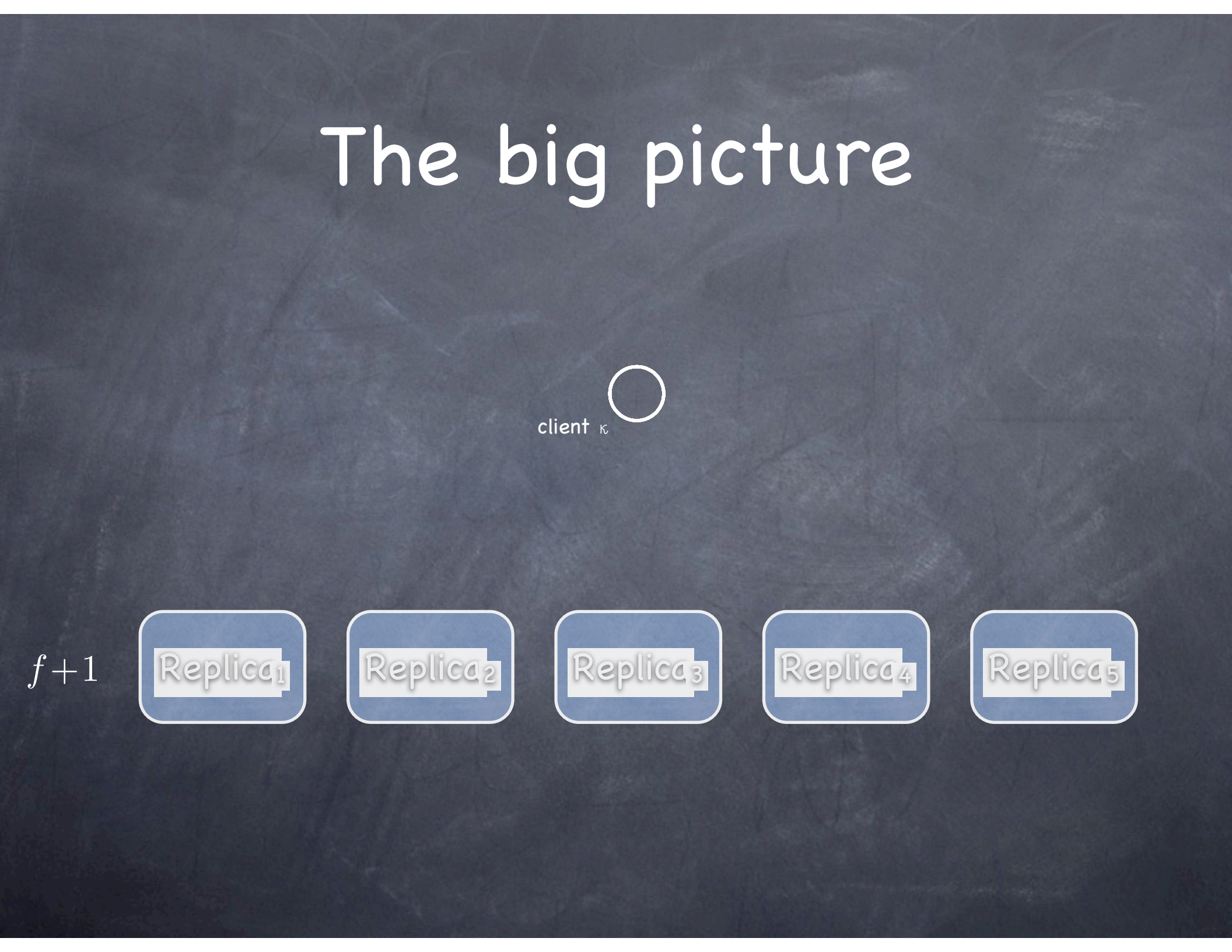
The big picture



The big picture



The big picture



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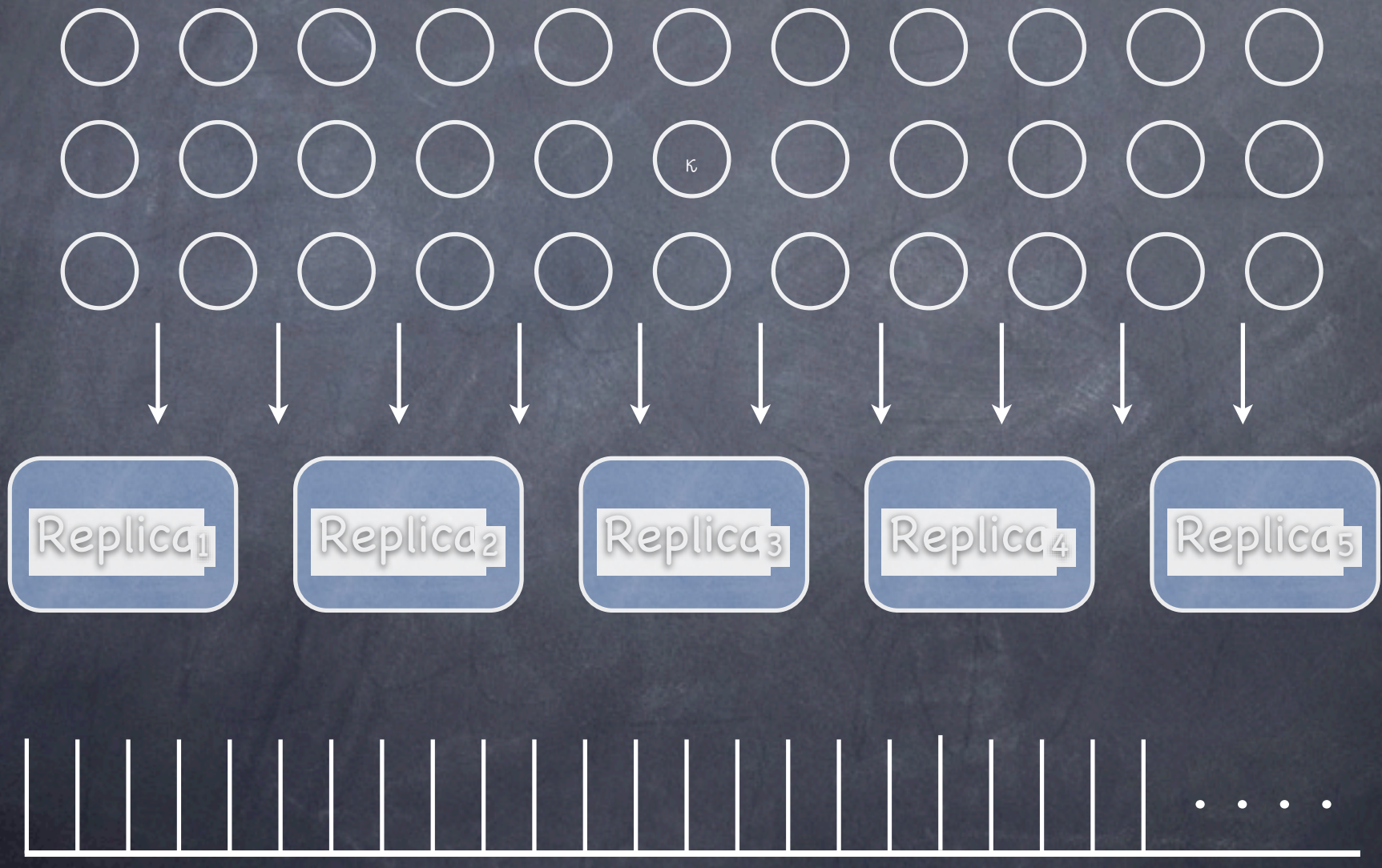
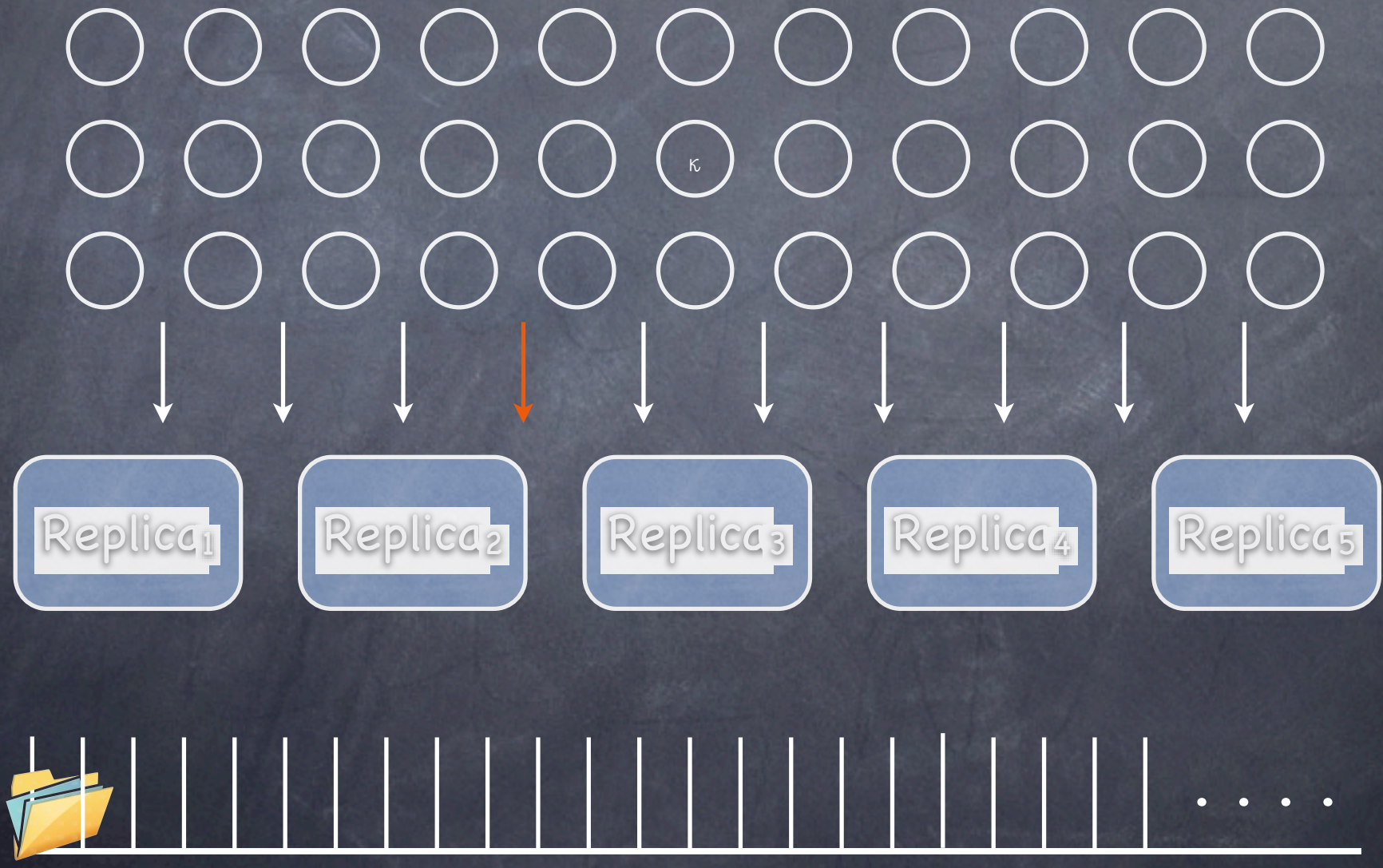
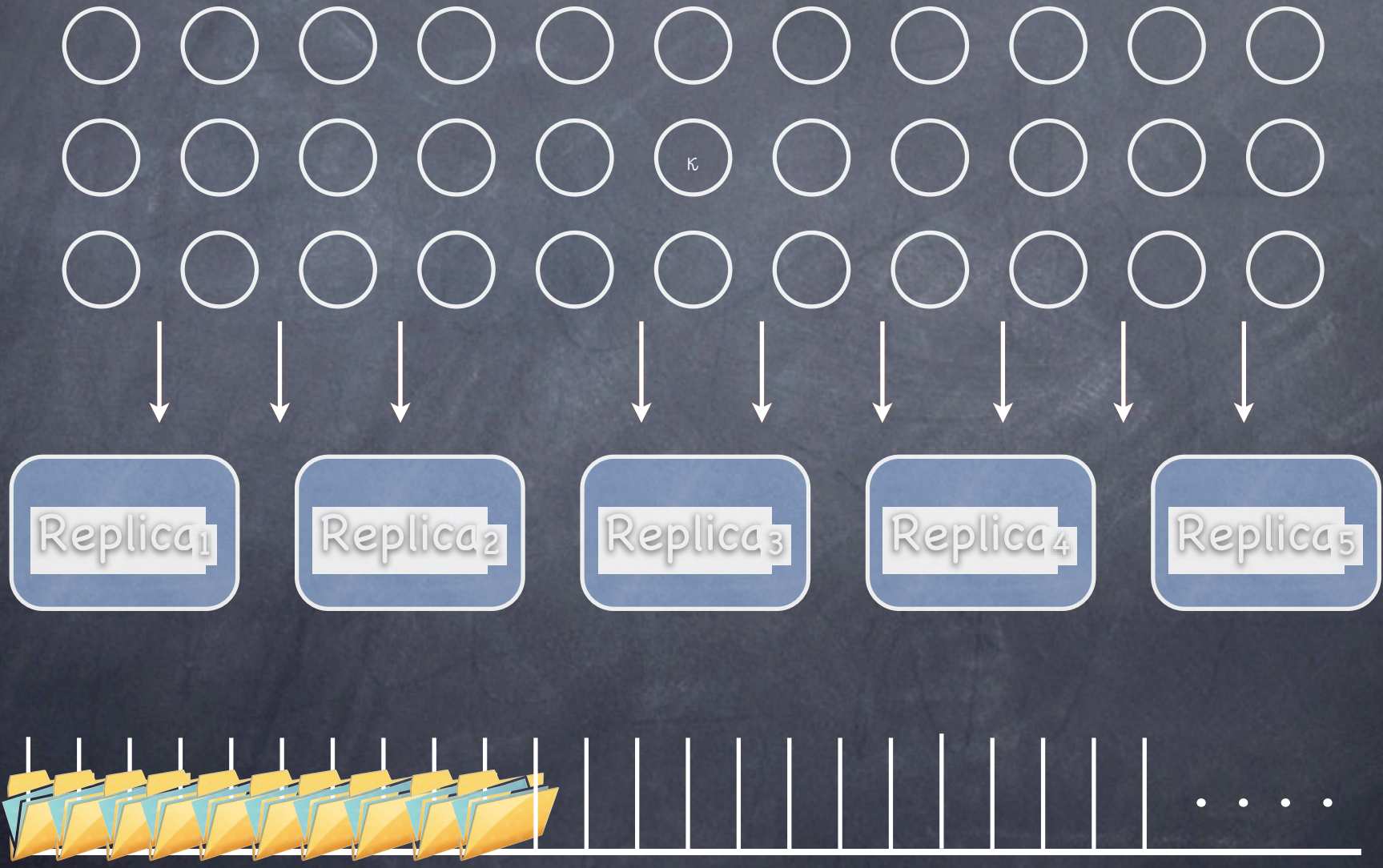


The big picture



The big picture



The Cast

- Replicas

- Leaders

- Acceptors

- Commanders

- Scout

Replicas

- Receive client requests
- Propose command for lowest unused slot to leaders
- Upon decision, execute commands in slot order
- Return result to clients
- Not necessarily identical at any time!

- Each replica ρ maintains four variables:
  - $\rho.state$: the application state
  - $\rho.slot\_num$: next slot for which ρ does not know a decision
  - $\rho.proposals$: set of <slot number, command> pairs for past proposals
  - $\rho.decisions$: set of <slot number, command> pairs for decided slots

Replica

```
process Replica(leaders, initial_state)
  var state := initial_state, slot_num := 1, proposals := ∅; decisions := ∅
   for ever
     switch receive()
       case <request, p> :
         propose(p);
       case <decision, s, p> :
           decisions := decisions ∪ {(s, p)}
           while ∃p' : ⟨slot_num, p'⟩ ∈ decisions do
             if ∃p'' : ⟨slot_num, p''⟩ ∈ proposals ∧ p'' ≠ p' then
                propose(p'');
             end if
             perform(p');
           end while
     end switch
   end for
end process
```

```
function perform(⟨κ, cid, op⟩)
    if ∃s : s < slot_num ∧
            ⟨s, ⟨κ, cid, op⟩⟩ ∈ decisions then
        slot_num := slot_num+1
    else
        ⟨next, result⟩ := op(state);
        atomic
          state := next;
          slot_num := slot_num+1
        end atomic
        send(κ, ⟨response, cid, result⟩);
    end if
end function
```

```
function propose(p)
    if ∄s : ⟨s, p⟩ ∈ decisions then
     s' := min{s | s ∈ ℕ+ ∧ ∄p' : ⟨s, p'⟩ ∈ proposal ∪ decisions};
      proposals := proposals ∪ {(s', p)};
      ∀λ ∈ leaders : send(λ, ⟨propose, s', p⟩);
    end if
end function
```

R4. For each ρ, the variable $\rho.slot\_num$ never decreases

Replica

```
process Replica(leaders, initial_state)
  var state := initial_state, slot_num := 1, proposals := ∅; decisions := ∅
   for ever
     switch receive()
       case <request, p> :
         propose(p);
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         decisions := decisions ∪ {(s, p)}
         while ∃p' : ⟨slot_num, p'⟩ ∈ decisions do
           if ∃p'' : ⟨slot_num, p''⟩ ∈ proposals ∧ p'' ≠ p' then
             propose(p'');
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         end while
     end switch
   end for
end process
```

```
function perform(⟨κ, cid, op⟩)
   if ∃s : s < slot_num ∧
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      slot_num := slot_num+1
   else
      ⟨next, result⟩ := op(state);
      atomic
        state := next;
        slot_num := slot_num+1
      end atomic
      send(κ, ⟨response, cid, result⟩);
   end if
end function
```

```
function propose(p)
   if ∄s : ⟨s, p⟩ ∈ decisions then
     s' := min{s | s ∈ ℕ+ ∧ ∄p' : ⟨s, p'⟩ ∈ proposal ∪ decisions};
     proposals := proposals ∪ {(s', p)};
     ∀λ ∈ leaders : send(λ, ⟨propose, s', p⟩);
   end if
end function
```

R3. For all replicas ρ, $\rho.state$ is the result of applying the operations in $\rho.decisions$ to the initial state in increasing order of slot number s :

$$1 \leq s < slot\_num$$

Replica

```
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  var state := initial_state, slot_num := 1, proposals := ∅; decisions := ∅
   for ever
     switch receive()
       case <request, p> :
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         decisions := decisions ∪ {(s, p)}
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           if ∃p'' : ⟨slot_num, p''⟩ ∈ proposals ∧ p'' ≠ p' then
             propose(p'');
           end if
           perform(p');
         end while
     end switch
   end for
end process
```

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function perform(⟨κ, cid, op⟩)
   if ∃s : s < slot_num ∧
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      slot_num := slot_num+1
   else
      ⟨next, result⟩ := op(state);
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        state := next;
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      end atomic
      send(κ, ⟨response, cid, result⟩);
   end if
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function propose(p)
   if ∄s : ⟨s, p⟩ ∈ decisions then
    s' := min{s | s ∈ ℕ+ ∧ ∄p' : ⟨s, p'⟩ ∈ proposal ∪ decisions};
    proposals := proposals ∪ {(s', p)};
    ∀λ ∈ leaders : send(λ, ⟨propose, s', p⟩);
   end if
end function
```

R2. All commands up to $slot\_num$ are in the set of decisions

Replica

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  var state := initial_state, slot_num := 1, proposals := ∅; decisions := ∅
   for ever
     switch receive()
       case <request, p> :
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         while ∃p' : ⟨slot_num, p'⟩ ∈ decisions do
           if ∃p'' : ⟨slot_num, p''⟩ ∈ proposals ∧ p'' ≠ p' then
             propose(p'');
           end if
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         end while
     end switch
   end for
end process
```

```
function perform(⟨κ, cid, op⟩)
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         state := next;
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       end atomic
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    if ∄s : ⟨s, p⟩ ∈ decisions then
      s' := min{s | s ∈ ℕ+ ∧ ∄p' : ⟨s, p'⟩ ∈ proposal ∪ decisions};
      proposals := proposals ∪ {(s', p)};
      ∀λ ∈ leaders : send(λ, ⟨propose, s', p⟩);
    end if
end function
```
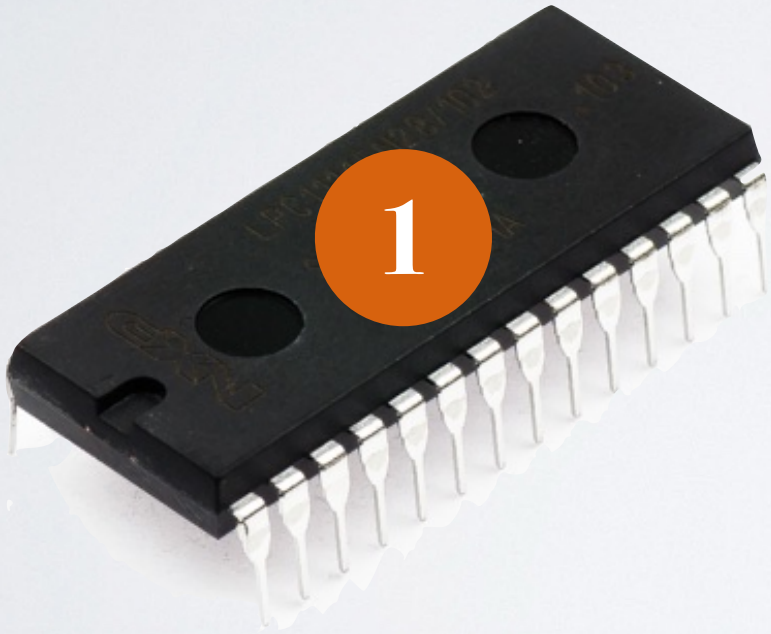

R1. For any given slot, replicas decide the same command

THE BASIC IDEA: THE WRITE-ONCE REGISTER



Leaders compete to create a permanent mapping between slot numbers and proposals

Participants forever

read register

if register contains a mapping then

decide that mapping and halt

else

attempt to write own mapping