Implementing State Machine Replication

- Implement a sequence of separate instances of consensus, where the value chosen by the i^{th} instance is the i^{th} message in the sequence.
- Each server assumes all roles in each instance of the algorithm.
- Assume that the set of servers is fixed

The role of the leader

- In normal operation, elect a single server to be a leader. The leader acts as a distinguished proposer in all instances of the consensus algorithm.
  - Clients send commands to the leader, which decides where in the sequence each command should appear.
  - If the leader, for example, decides that a client command is the k^{th} command, it tries to have the command chosen as the value in the k^{th} instance of consensus.

What if a new λ is elected?

- Since λ serves also as a replica in all instances of consensus, it should know most of the commands that have already been chosen. For example, it might know commands for slots 1-10, 13, and 15.
  - It executes phase 1 for slots 11, 12, and 14 and of all slots 16 and larger.
  - λ may find that some value was already accepted for slots 14 and 16 and that slots 11, 12 and all slots after 16 have accepted no command.
  - λ then executes phase 2 of 14 and 16, using the value with the highest ballot it retrieved for those slots

What if II: Stop-gap measures

- All replicas now can execute commands 1-10, but not 13-16 because 11 and 12 haven't yet been chosen.
- λ can either take the next two commands it receives by clients to be commands 11 and 12, or can propose immediately that 11 and 12 be no-op commands.
  - this is what happens on "Olive Day"!
- λ runs phase 2 of consensus for slots 11 and 12.
- Once consensus is achieved, all replicas can execute all commands through 16.

To infinity, and beyond

- λ can efficiently execute phase 1 for infinitely many instances of consensus! (e.g. command 16 and higher)
  - λ just sends a message with a sufficiently high ballot number for all instances
  - An acceptor replies non trivially only for instances for which it has already accepted a value

Paxos and FLP

- Paxos is always safe–despite asynchrony
- Once a leader is elected, Paxos is live.
-  "Ciao ciao" FLP?
  - To be live, Paxos requires a single leader
  - "Leader election" is impossible in an asynchronous system (gotcha!)
- Given FLP, Paxos is the next best thing: always safe, and live during periods of synchrony

A Lower Bound

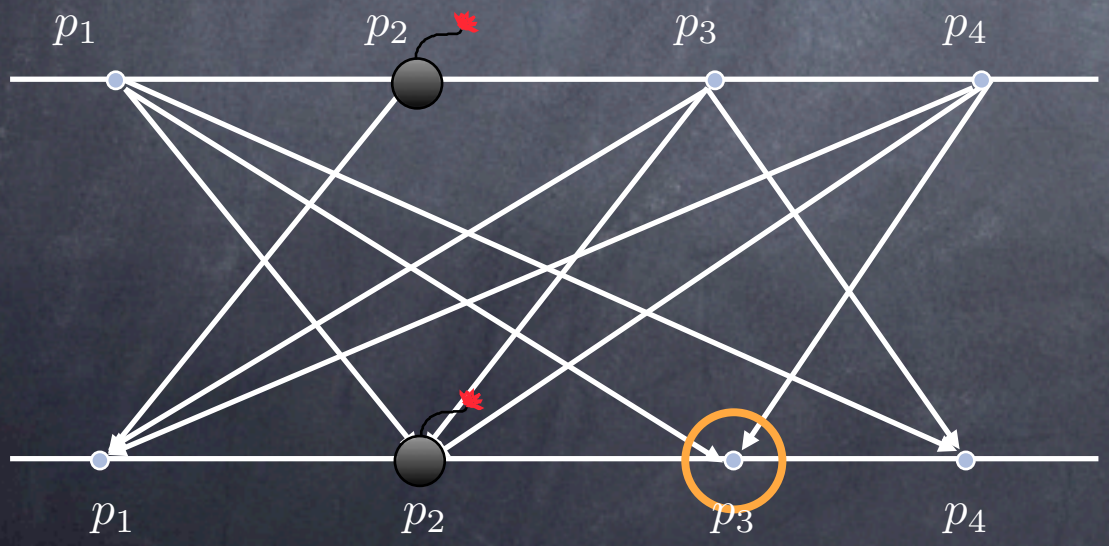
Theorem

There is no algorithm that solves the consensus problem in fewer than $f+1$ rounds in the presence of f crash failures, if $n \geq f+2$

We consider a special case $(f=1)$ to study the proof technique

Views

Let α be an execution. The view of process p_i in α, denoted by $\alpha|p_i$, is the subsequence of computation and message receive events that occur in p_i together with the state of p_i in the initial configuration of α



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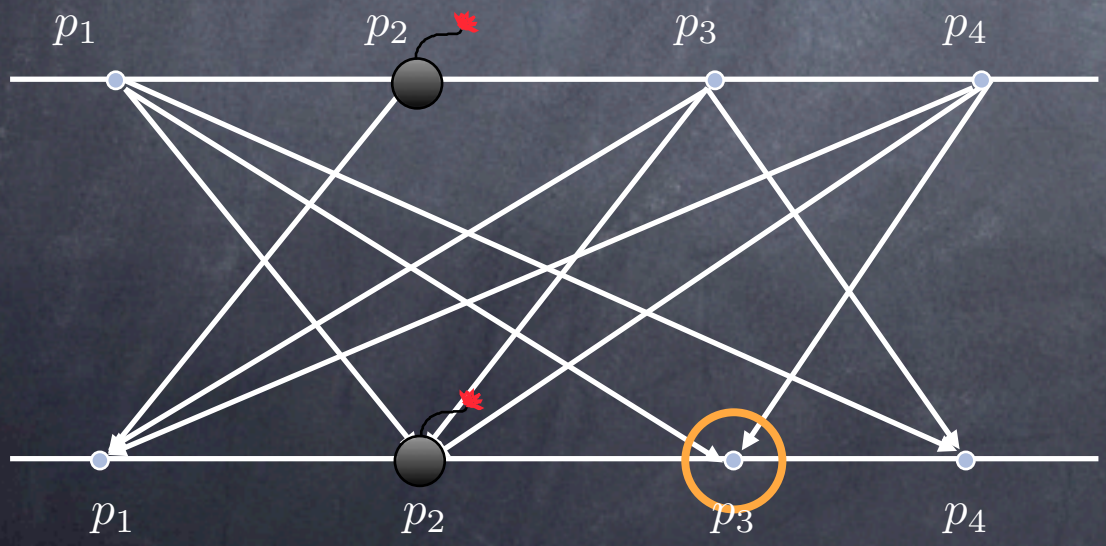

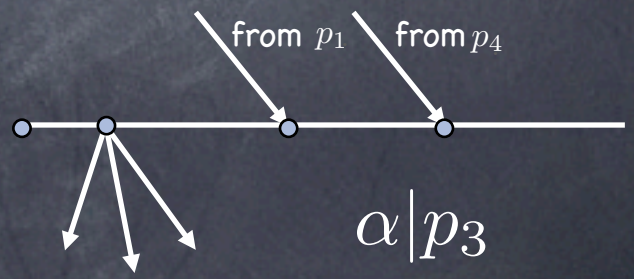

Similarity

Definition Let α_1 and α_2 be two executions of consensus and let p_i be a correct process in both α_1 and α_2.

α_1 is similar to α_2 with respect to p_i, denoted $\alpha_1 \sim_{p_i} \alpha_2$ if

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The transitive closure of $\alpha_1 \sim_{p_i} \alpha_2$ is denoted $\alpha_1 \approx \alpha_2$.

We say that $\alpha_1 \approx \alpha_2$ if there exist executions $\beta_1, \beta_2, \ldots, \beta_{k+1}$ such that

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Lemma If $\alpha_1 \approx \alpha_2$ then dec(α_1) = dec(α_2)

Single-Failure Case

There is no algorithm that solves consensus in fewer than two rounds in the presence of one crash failure, if $n \geq 3$

The Idea

By contradiction

- Consider a one-round execution in which each process proposes 0. What is the decision value?
- Consider another one-round execution in which each process proposes 1. What is the decision value?
- Show that there is a chain of similar executions that relate the two executions.

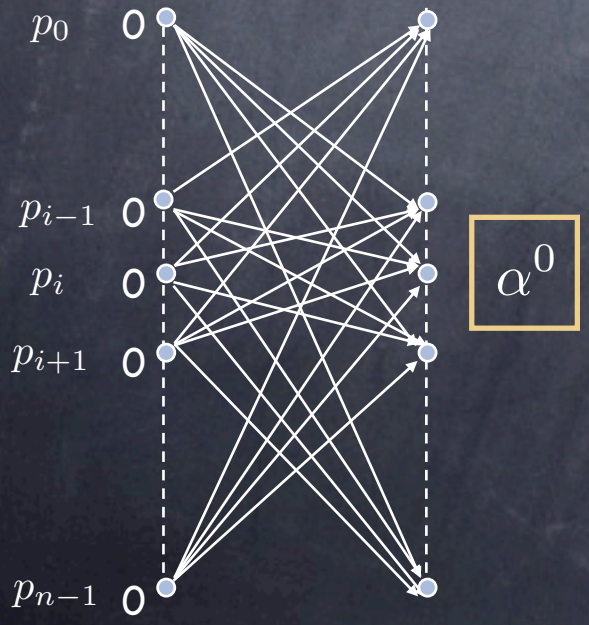
So what?

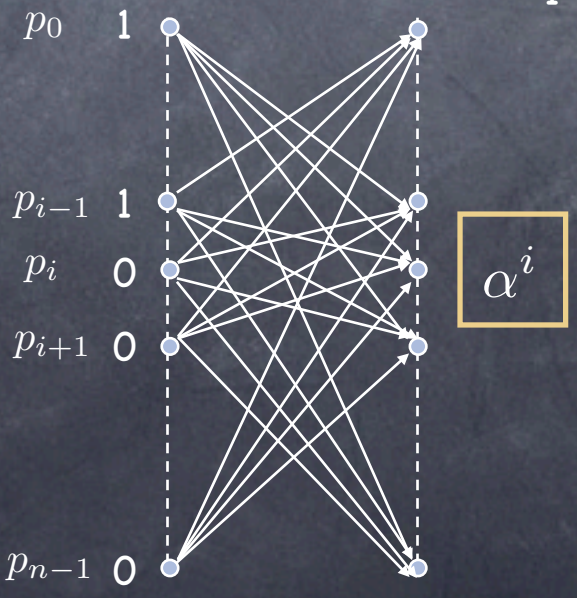
α^is

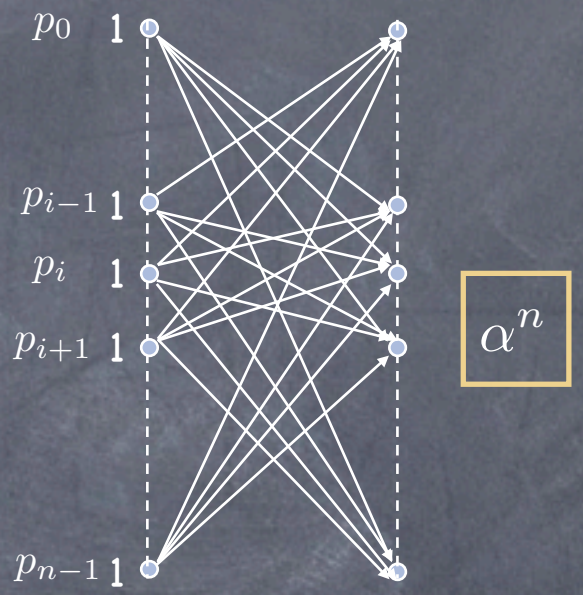
Definition

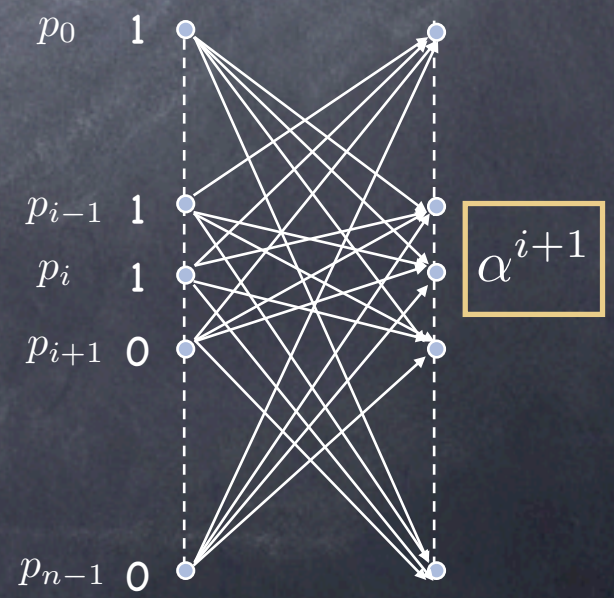
α^i is the execution of the algorithm in which

- no failures occur
- only processes $p_0, \ldots, p_{i-1}$ propose 1







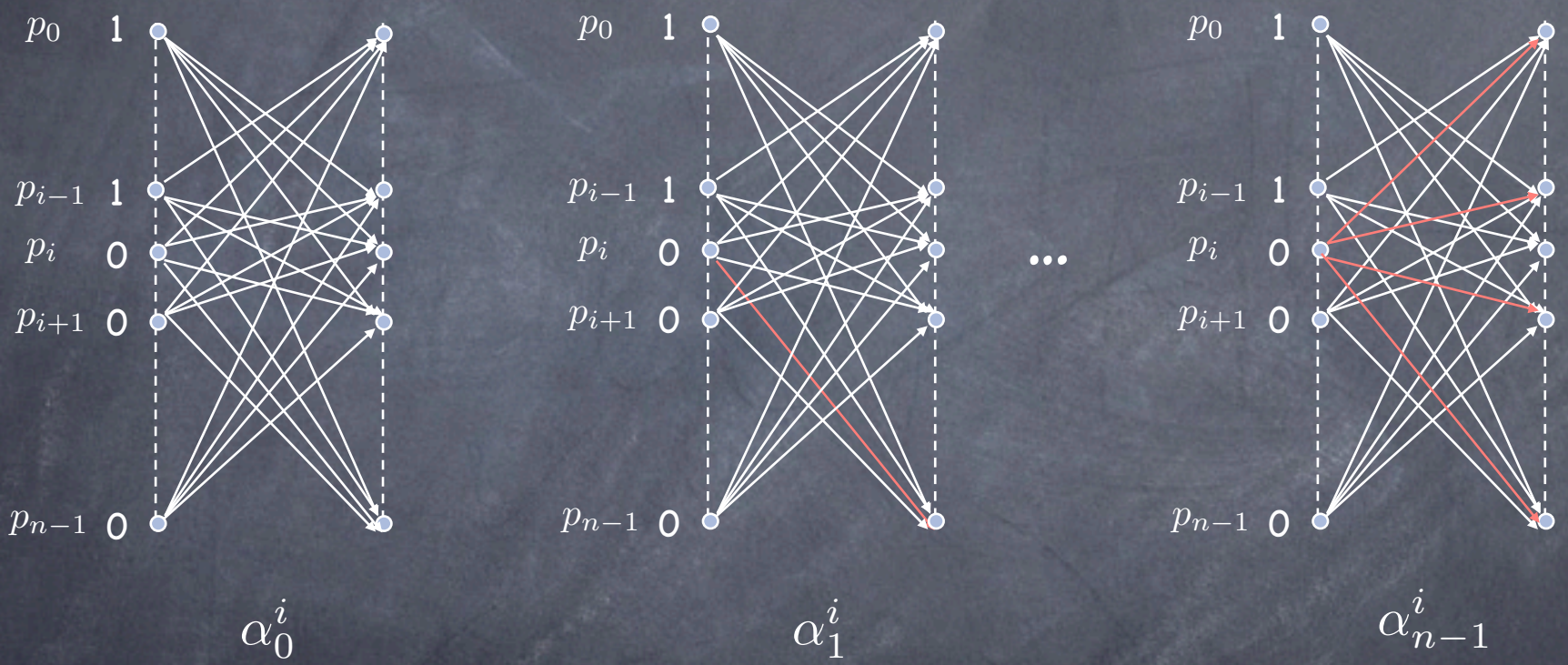


Adjacent α^is are similar!

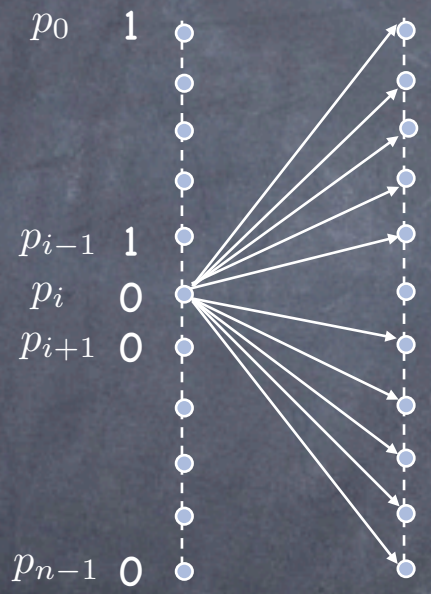
Starting from α^i, we build a set of executions α^i_j where $0 \leq j \leq n-1$as follows:

α^i_j is obtained from α^i after removing the messages that p_i sends to the j-th highest numbered processors (excluding itself)

The executions



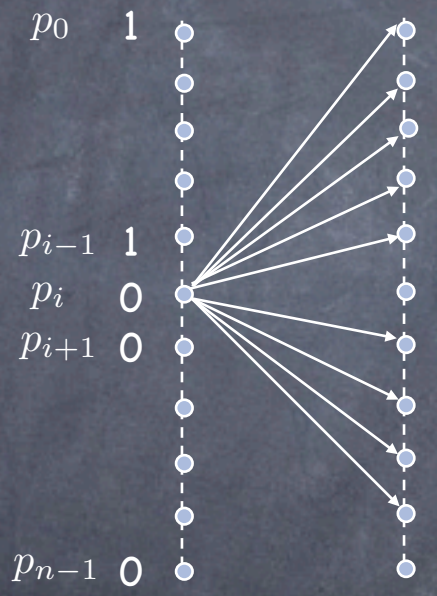
Indistinguishability



α^i

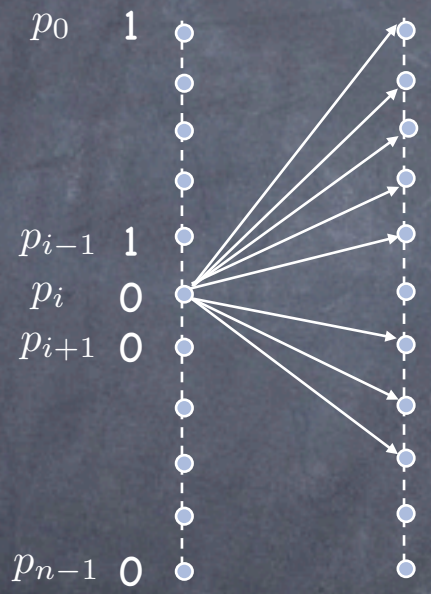
α_0^i

Indistinguishability



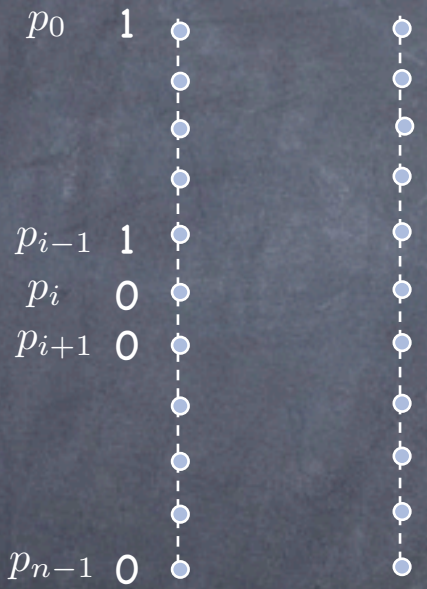
$$\alpha^i \approx \alpha_1^i$$

Indistinguishability



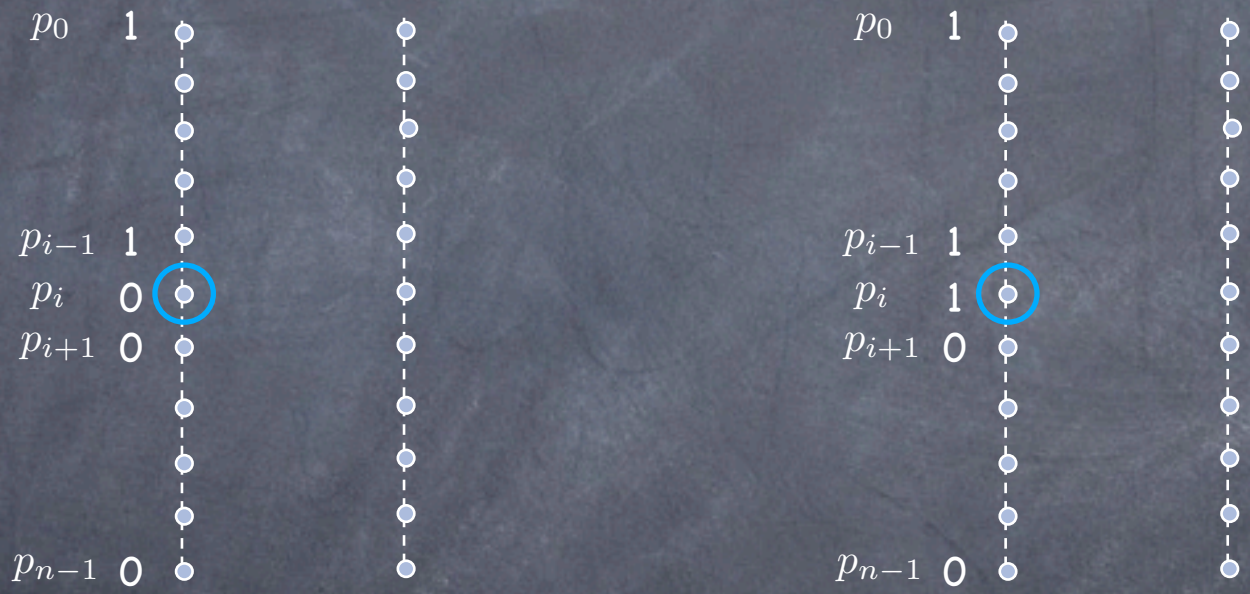
$$\alpha^i \approx \alpha_2^i$$

Indistinguishability



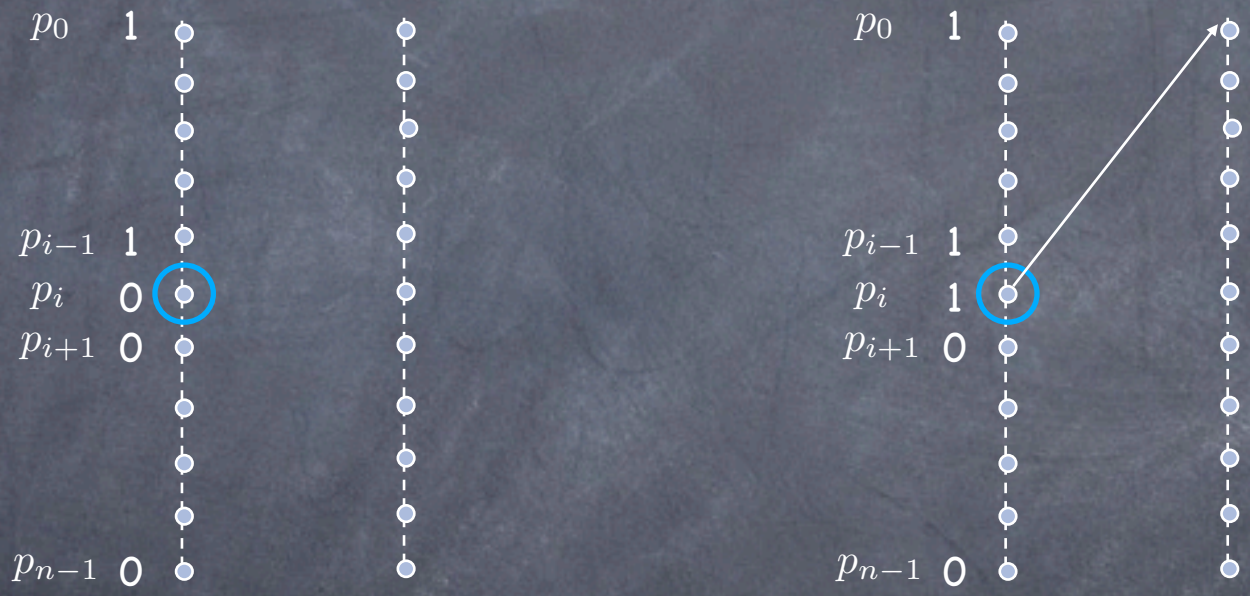
$$\alpha^i \approx \alpha^i_{n-1}$$

Indistinguishability



$$\alpha^i \approx \alpha^i_{n-1} \approx \beta^i_{n-1}$$

Indistinguishability



$$\alpha^i \approx \alpha^i_{n-1} \qquad \approx \qquad \beta^i_{n-2}$$

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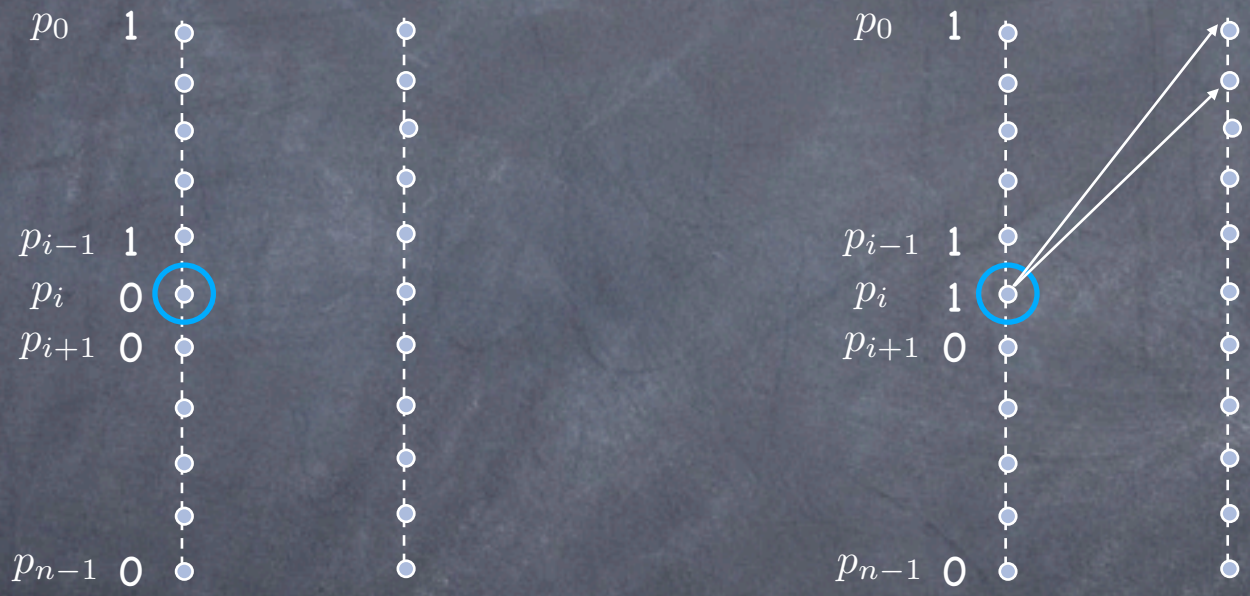

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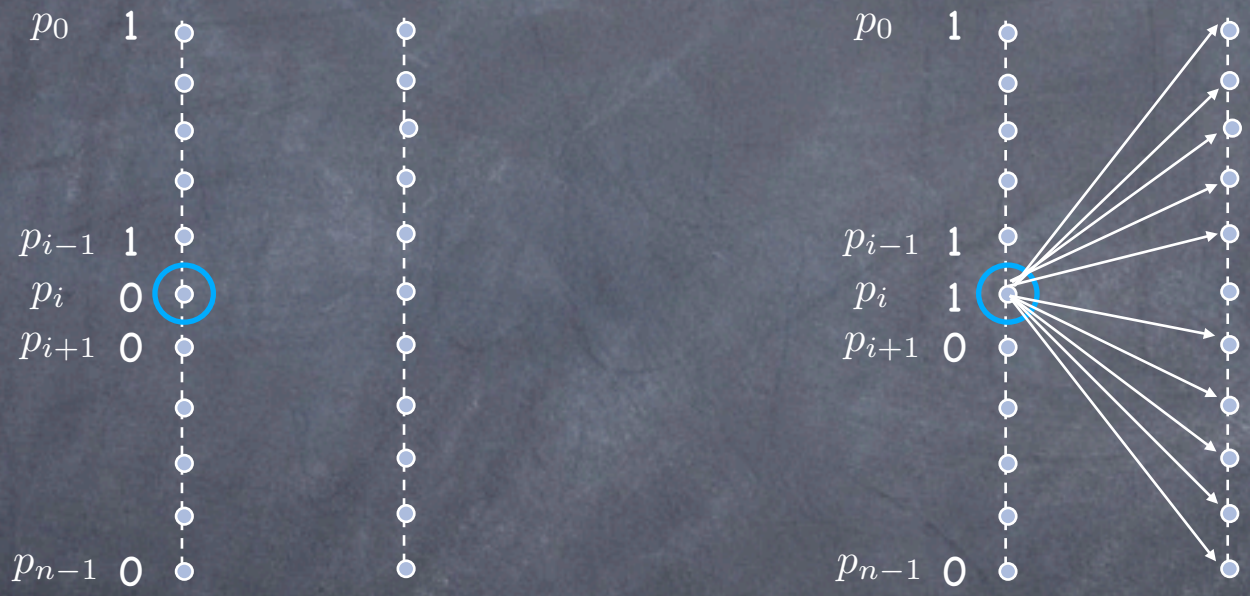
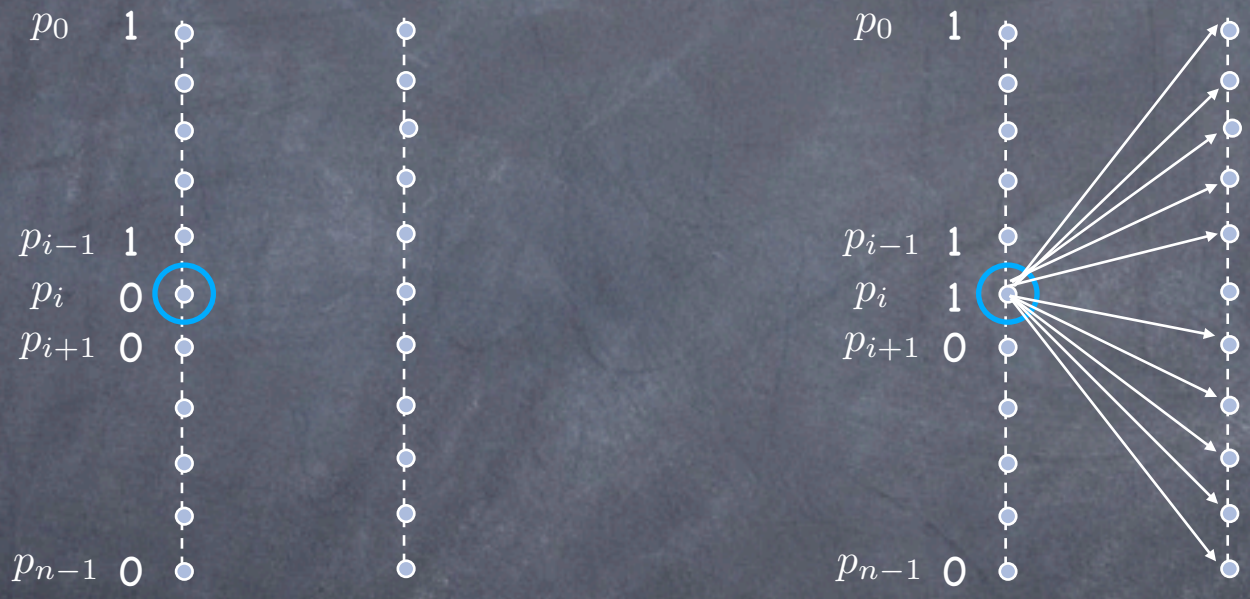
Indistinguishability



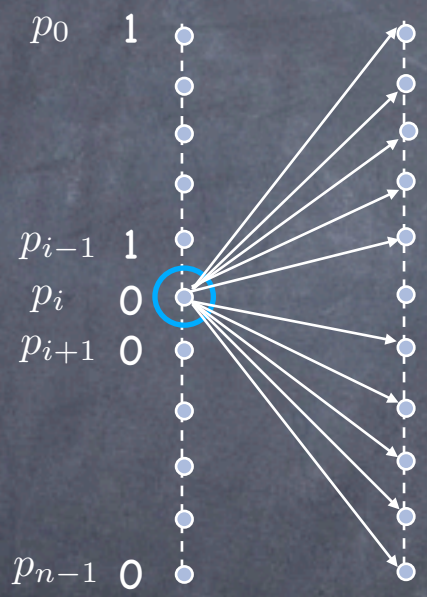
$$\alpha^i \wr \alpha^i_{n-1} \approx \beta^i_0$$

Indistinguishability

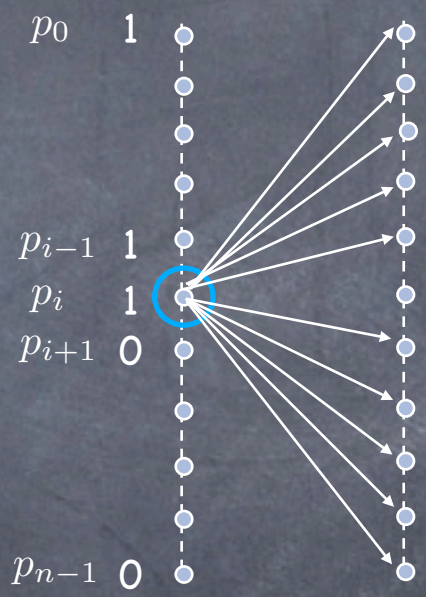


$$\alpha^i \wr \alpha^i_{n-1} \approx \alpha^{i+1} \wr \beta^i_0$$

Indistinguishability

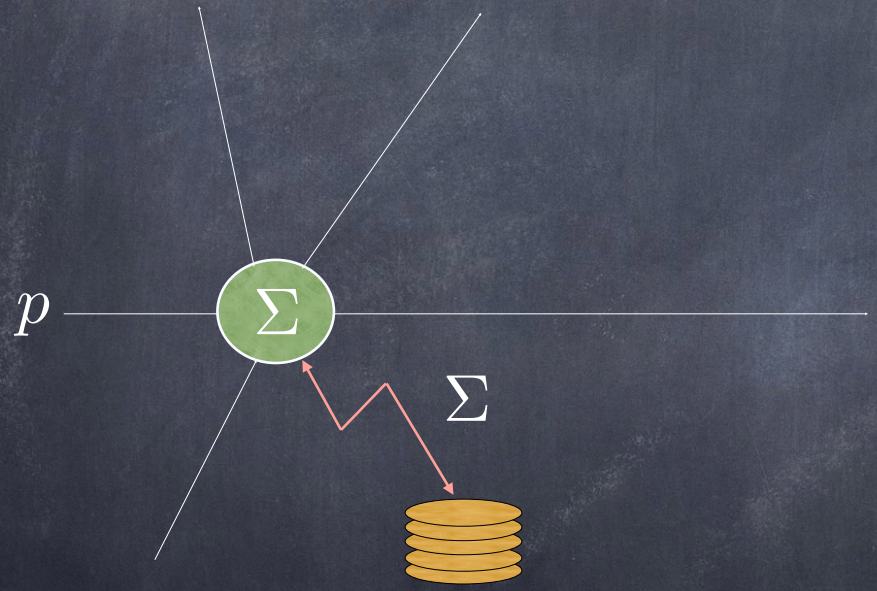


α^i $\approx$ α^{i+1}



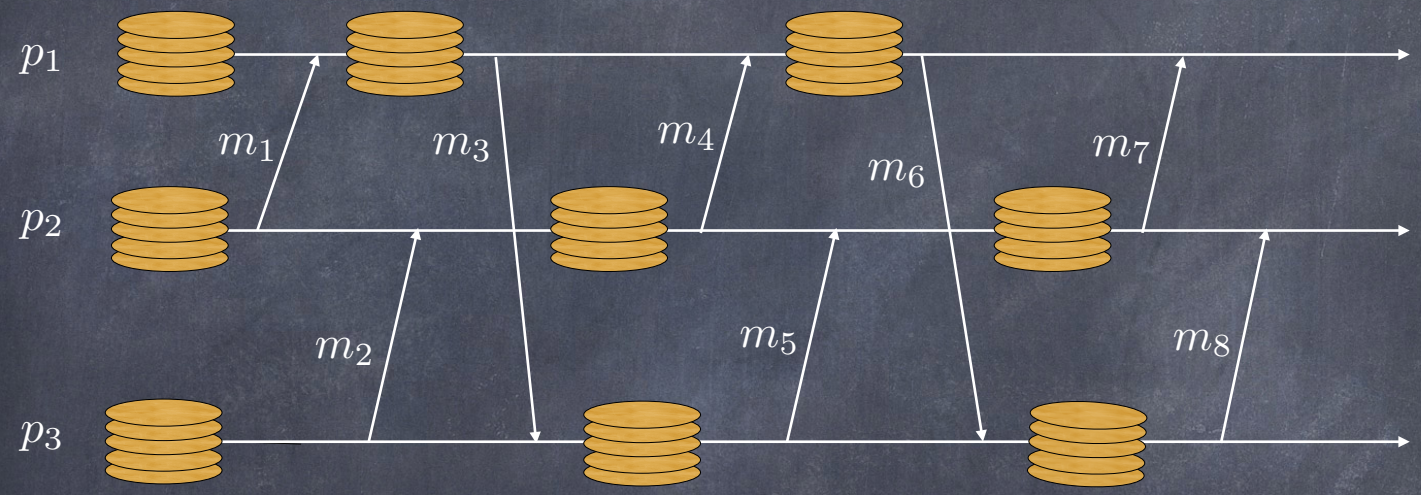
Rollback-Recovery

Uncoordinated Checkpointing



- Easy to understand
- No synchronization overhead
- Flexible
  - can choose when to checkpoint
- To recover from a crash:
  - go back to last checkpoint
  - restart

The Domino Effect



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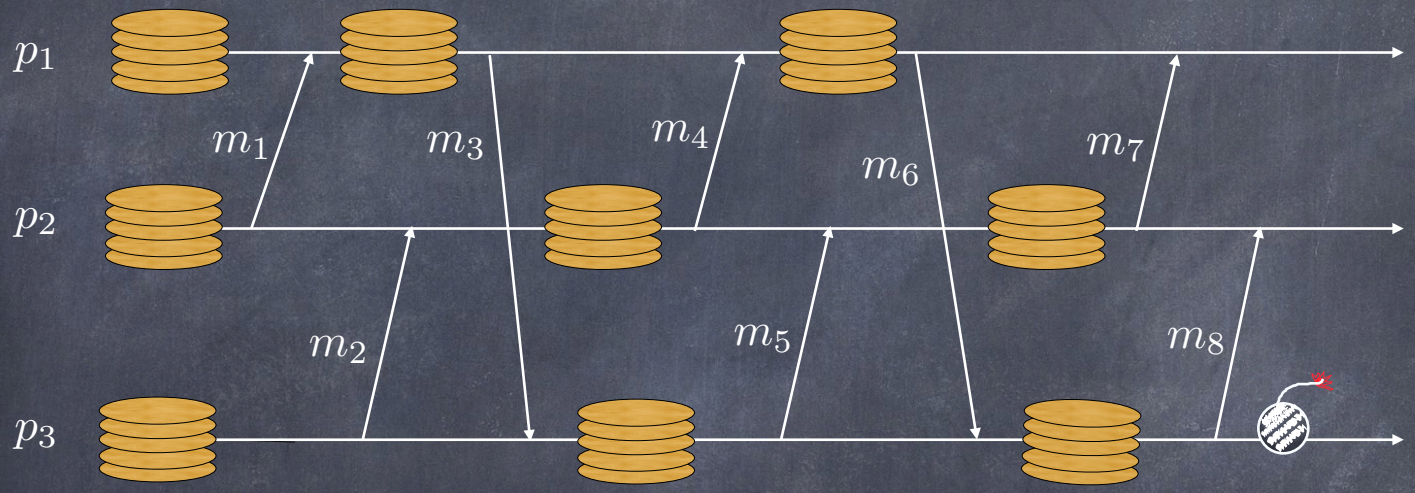

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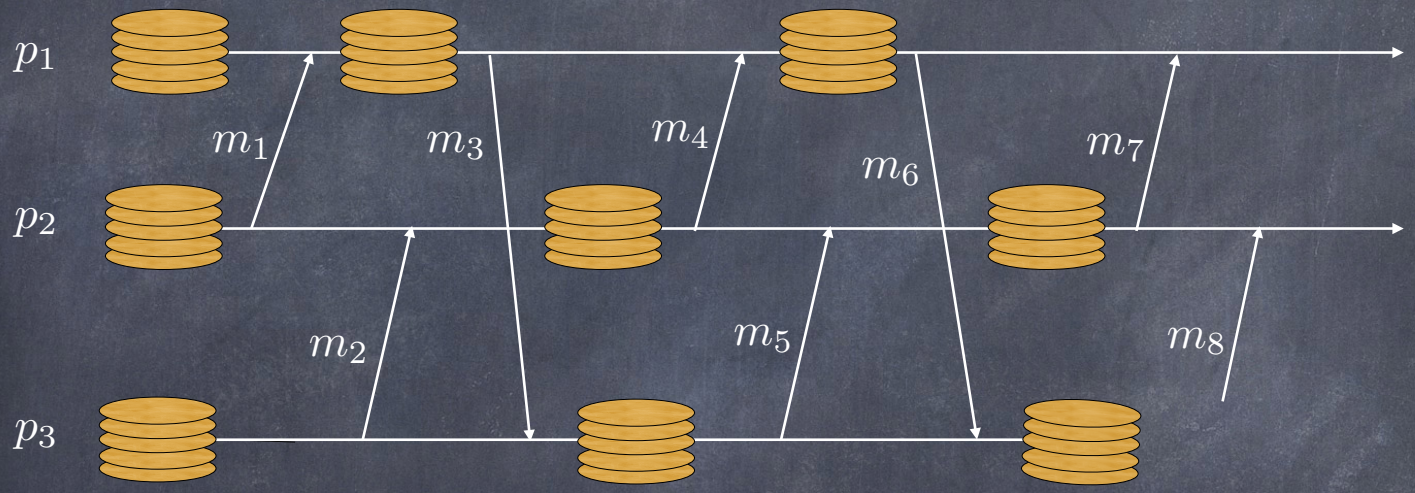

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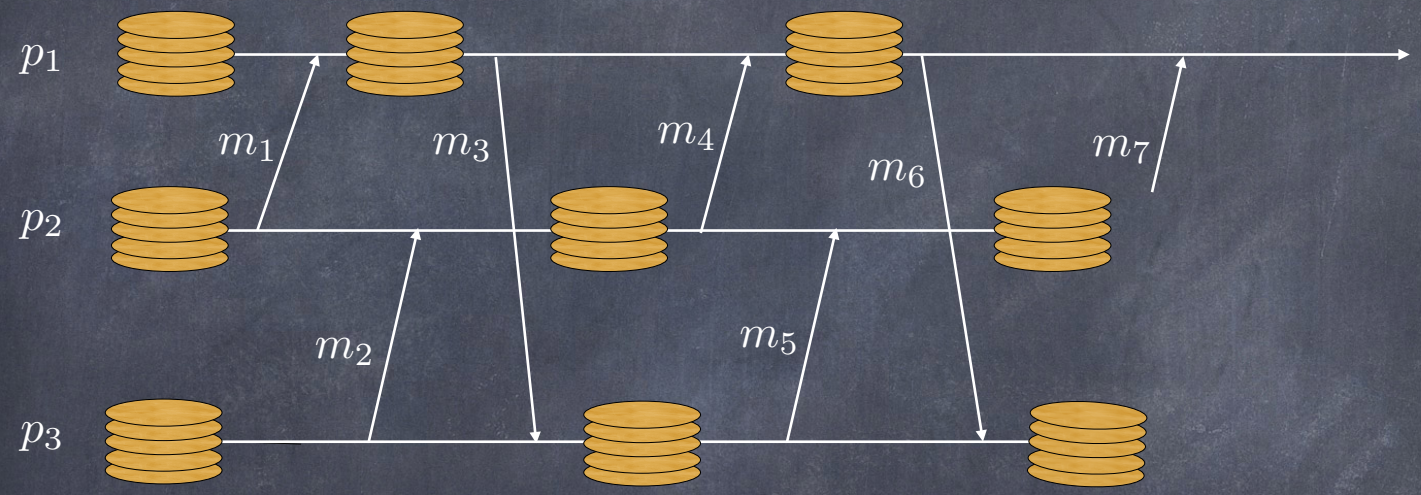

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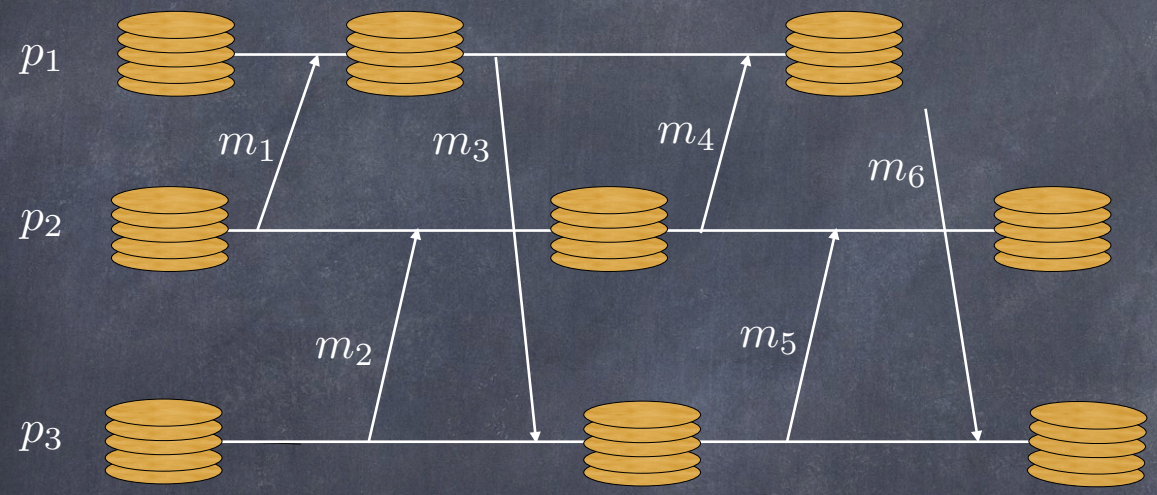

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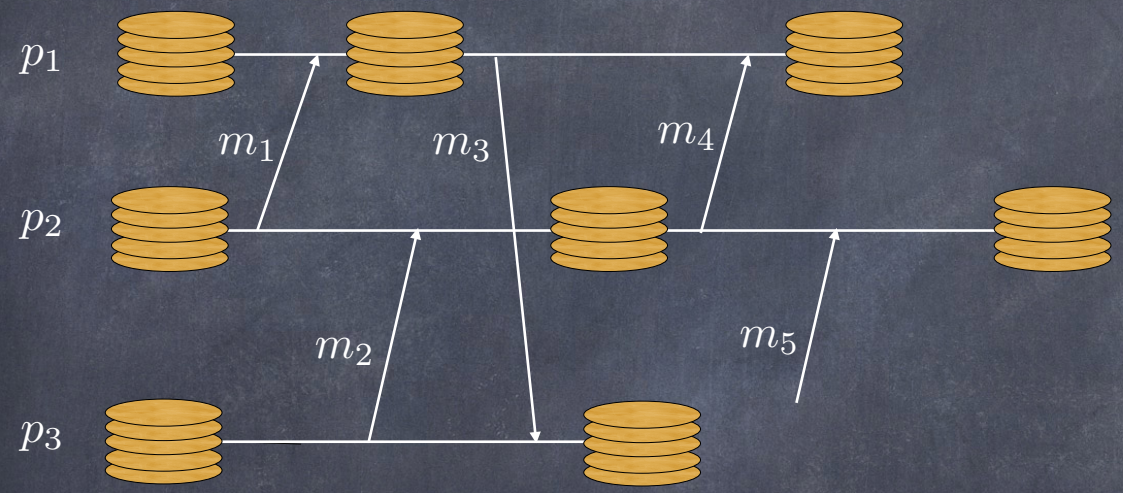

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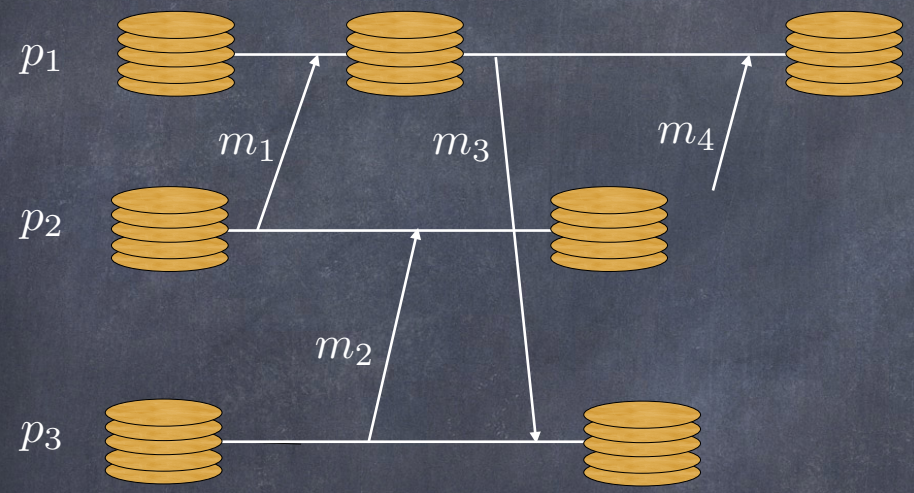

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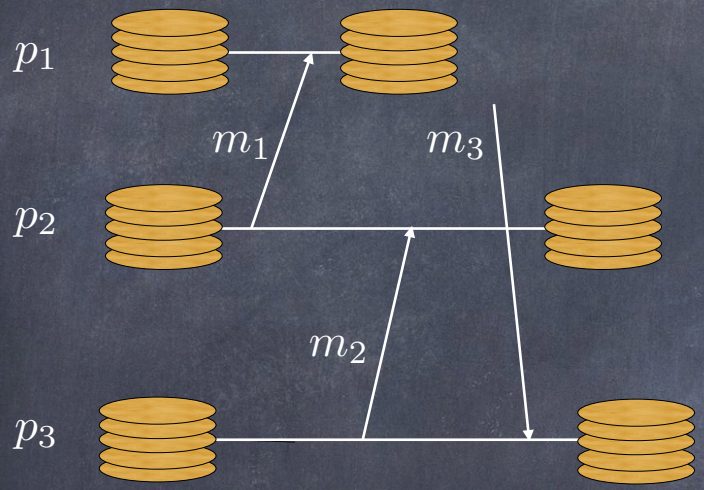

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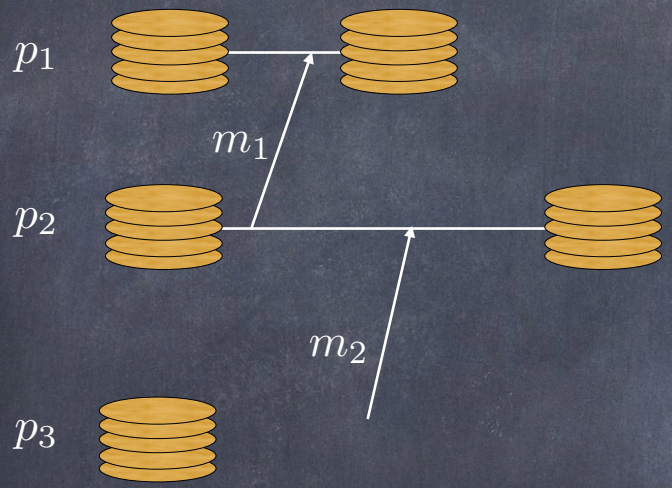

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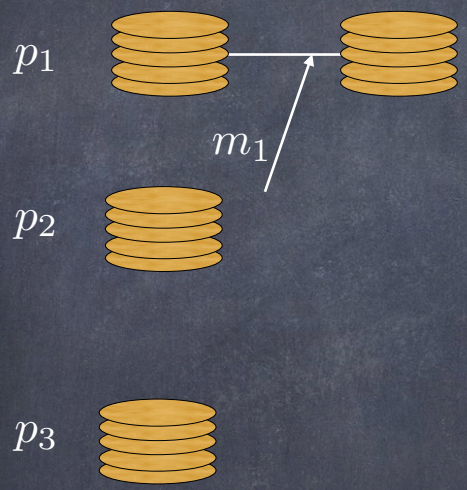

The Domino Effect

p_1 

p_2 

p_3