Properties of send(m) and receive(m)

Benign failures:

Validity If p sends m to q, and p , q, and the link between them are correct, then q eventually receives m

Uniform* Integrity For any message m, q receives m at most once from p, and only if p sent m to q

* A property is uniform if it applies to both correct and faulty processes

Properties of send(m) and receive(m)

Arbitrary failures:

Integrity For any message m , if p and q are correct then q receives m at most once from p, and only if p sent m to q

Questions, Questions...

- Are these problems solvable at all?
- Can they be solved independent of the failure model?
- Does solvability depend on the ratio between faulty and correct processes?
- Does solvability depend on assumptions about the reliability of the network?
- Are the problems solvable in both synchronous and asynchronous systems?
- If a solution exists, how expensive is it?

Plan

- Benign Synchronous Systems
  - Consensus for synchronous systems with crash failures
  - Lower bound on the number of rounds
- Benign Asynchronous Systems
  - Impossibility of Consensus for crash failures
  - Failure detectors
  - PAXOS
- Byzantine (Synchronous and Asynchronous)
  - Reliable Broadcast for arbitrary failures
  - PBFT, Zyzzyva

Model

- Synchronous Message Passing
  - Execution is a sequence of rounds
  - In each round every process takes a step
    - sends messages to neighbors
    - receives messages sent in that round
    - changes its state
- Network is fully connected (an n-clique)
- No communication failures

A simple Consensus algorithm

Process p_i:

```
Initially $V = \{v_i\}$
To execute propose($v_i$)
1:    send $\{v_i\}$ to all
decide($x$) occurs as follows:
2:    for all $j$, $0 \leq j \leq n-1$, $j \neq i$ do
3:        receive $S_j$ from $p_j$
4:        $V := V \cup S_j$
5:    decide min($V$)
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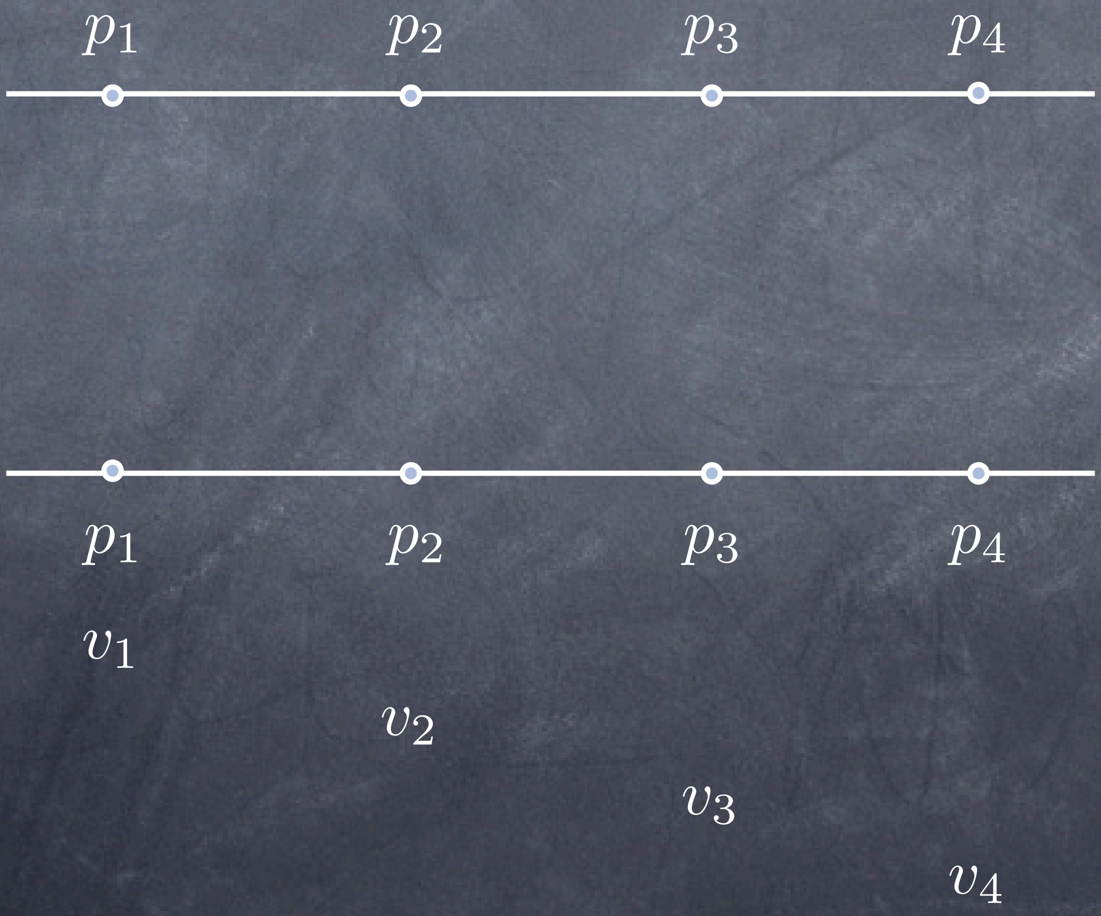
An execution



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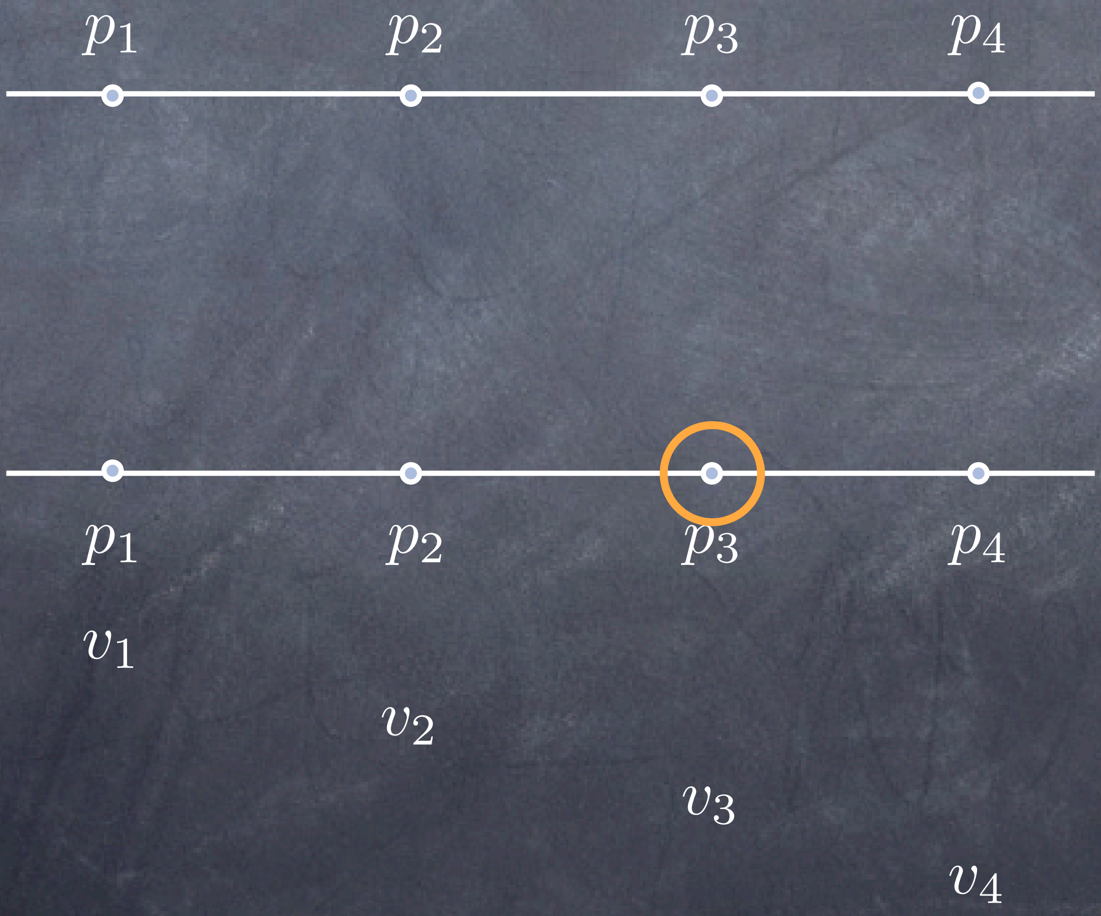
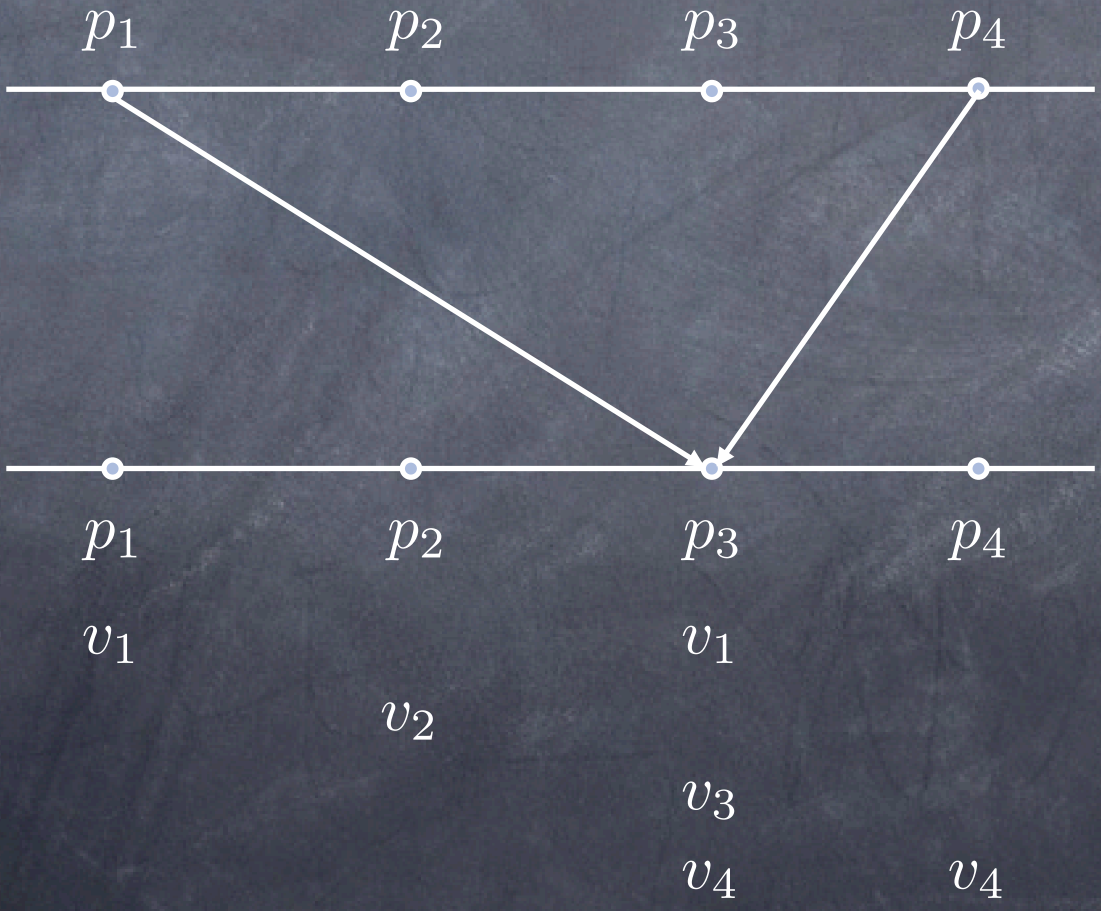


An execution

Suppose $v_1 = v_3 = v_4$ at the end of round 1

Can p_3 decide?



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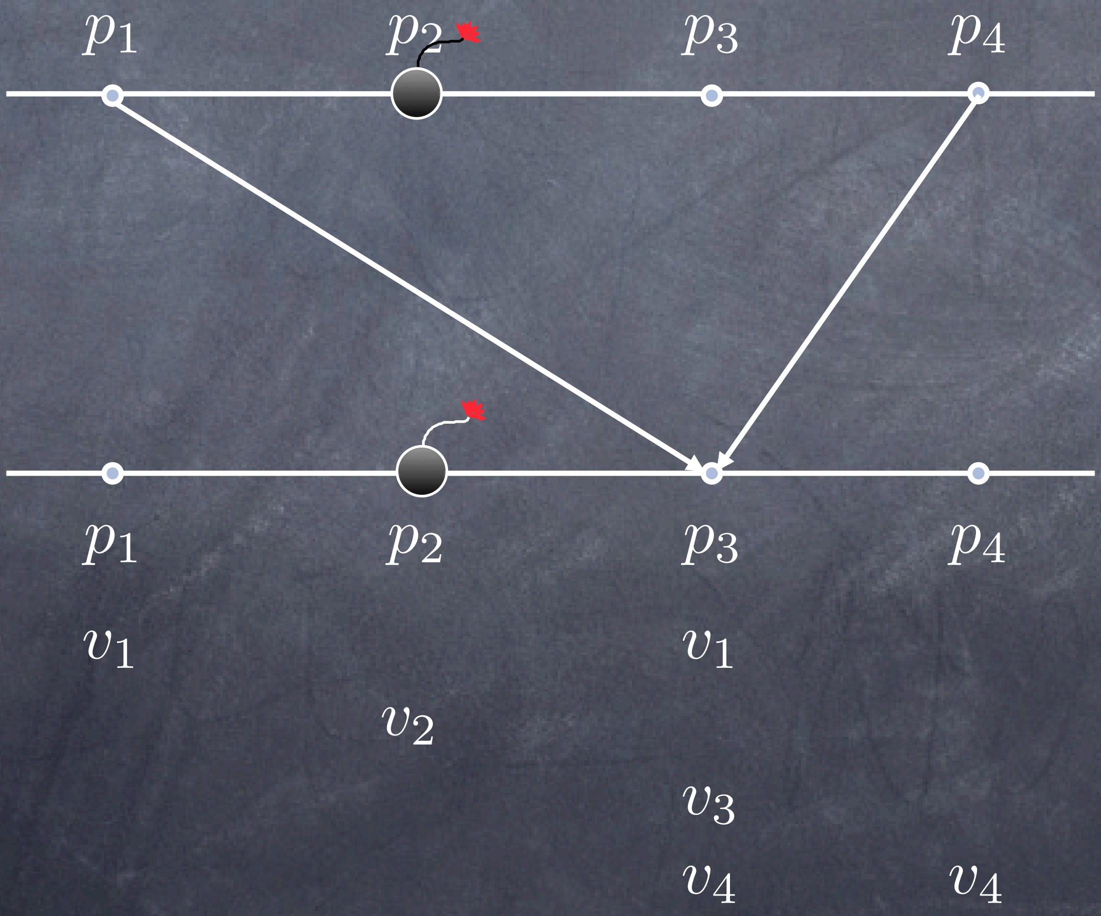

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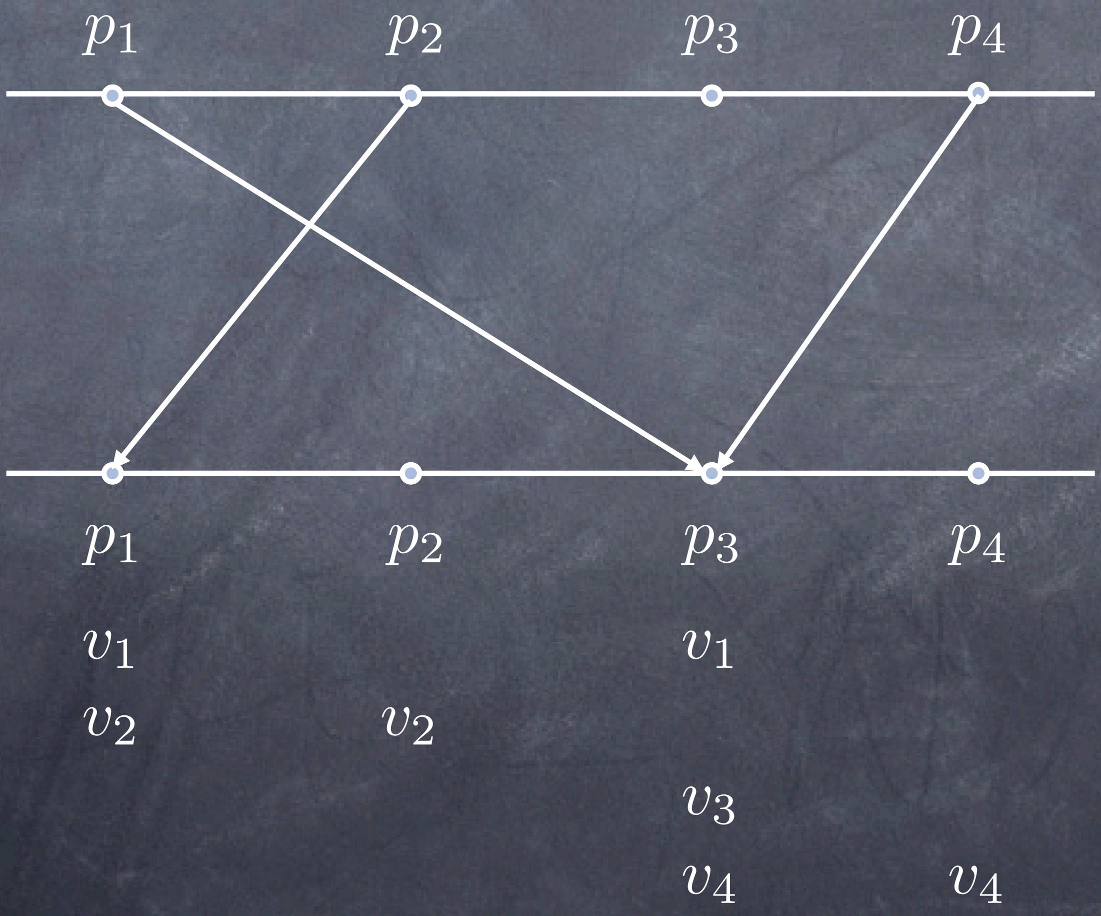

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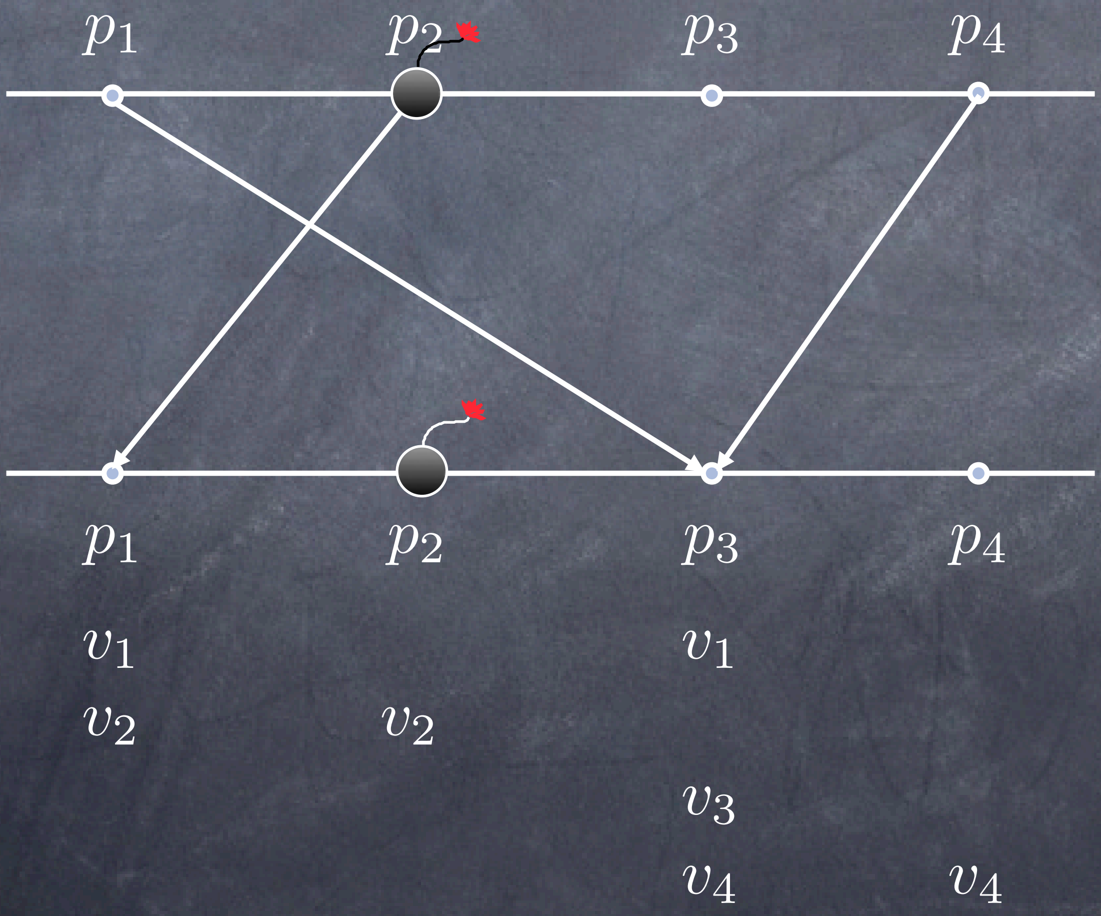

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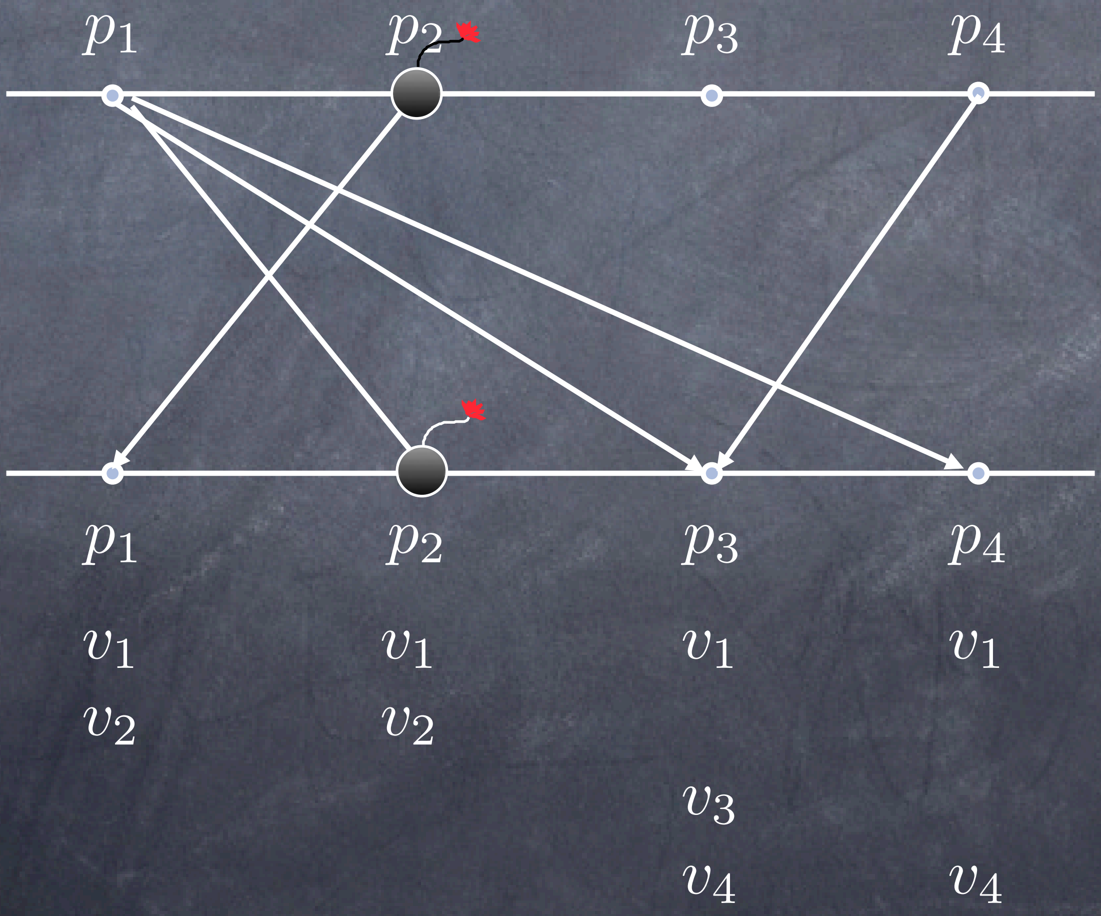

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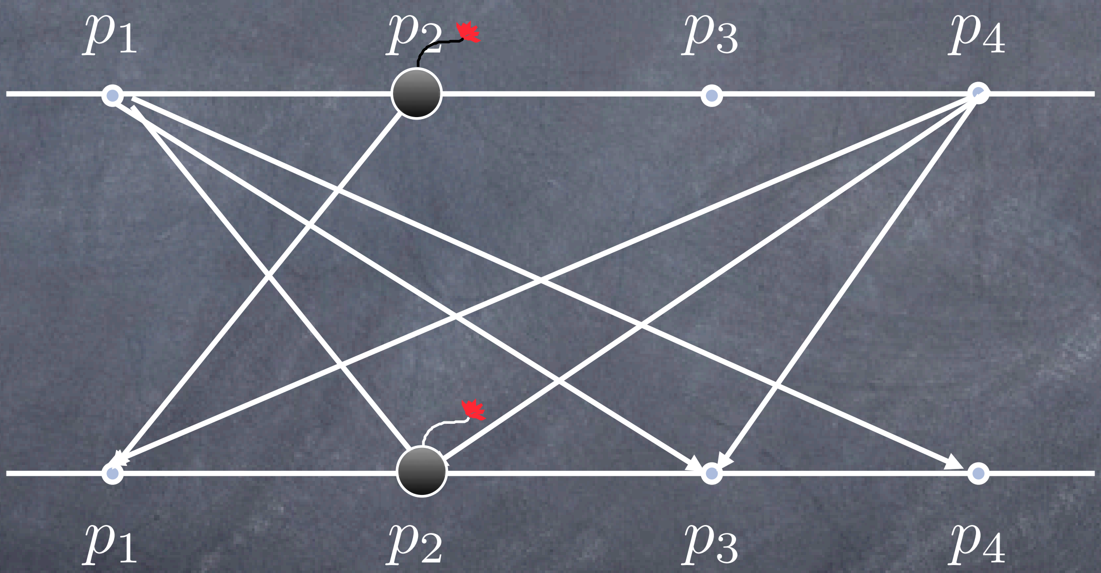

v_1	v_1	v_1	v_1
v_2	v_2		
		v_3	
v_4		v_4	v_4

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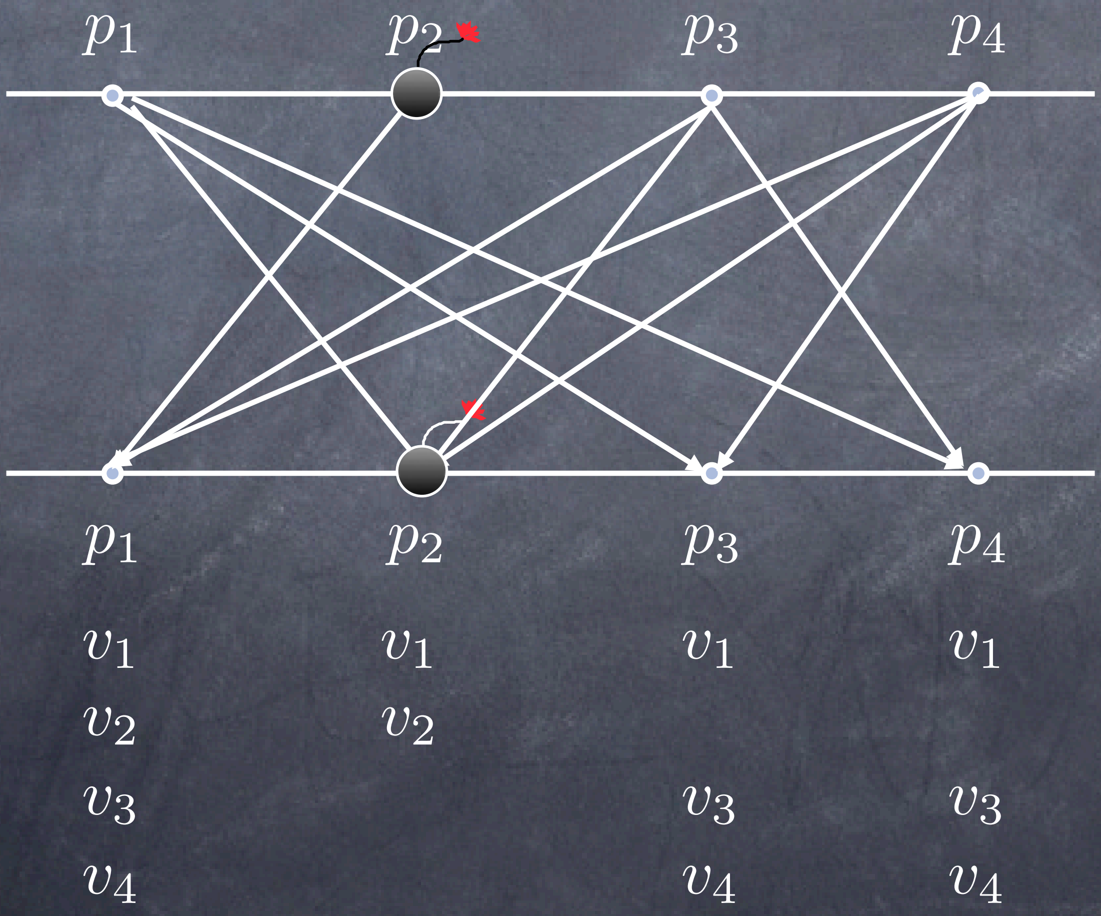

Echoing values

- A process that receives a proposal in round 1, relays it to others during round 2.

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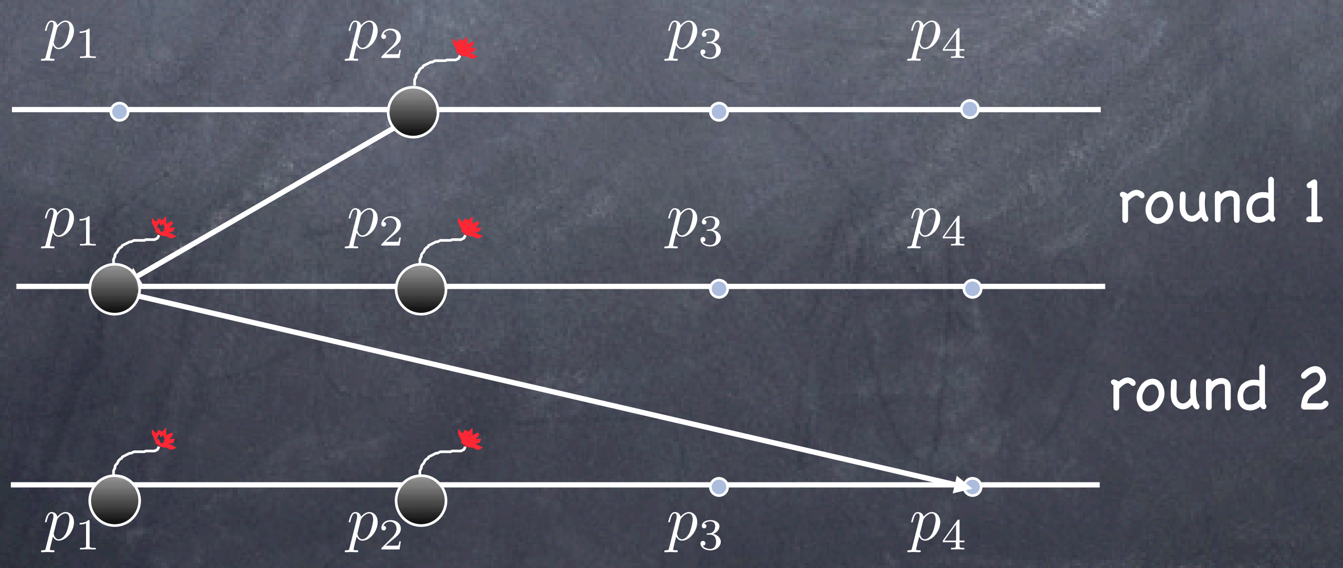


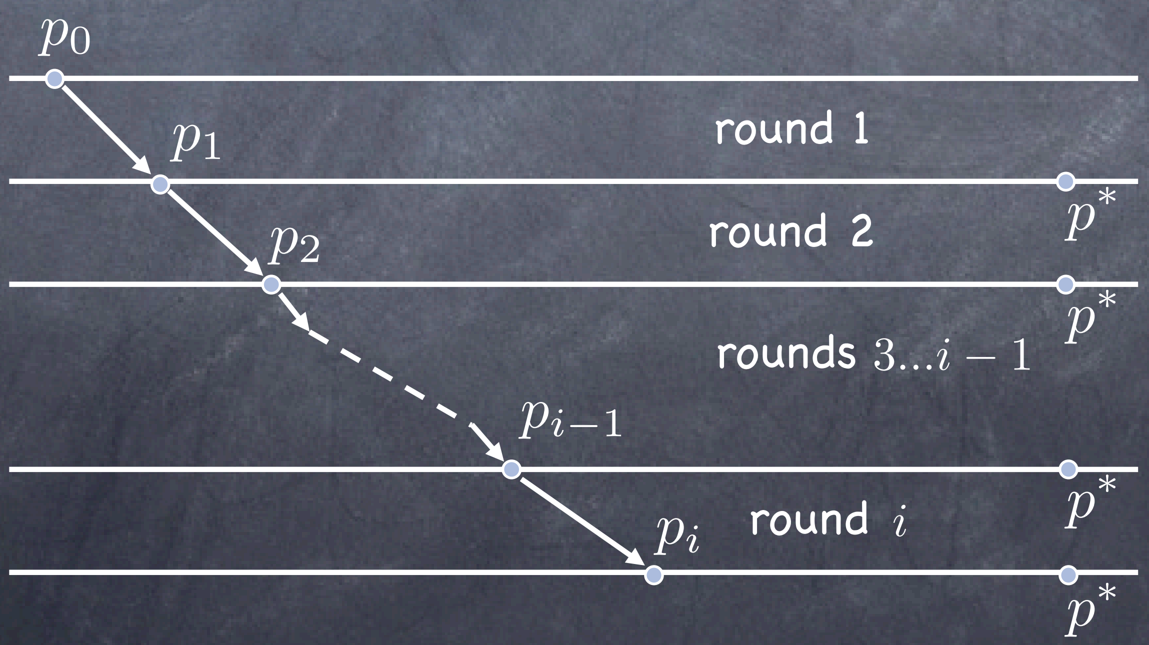
What is going on

- A correct process p^* has not received all proposals by the end of round i. Can p^* decide?
- Another process may have received the missing proposal at the end of round i and be ready to relay it in round $i+1$

Dangerous Chains

Dangerous chain

The last process in the chain is correct, all others are faulty



Living dangerously

How many rounds can a dangerous chain span?

- f faulty processes
- at most $f+1$ nodes in the chain
- spans at most f rounds

It is safe to decide by the end of round $f+1$!

The Algorithm

Code for process p_i :

```
Initially $V = \{v_i\}$
To execute propose($v_i$)
   round $k$, $1 \leq k \leq f+1$
1: send $\{v \in V : p_i$ has not already sent $v\}$ to all
2: for all $j$, $0 \leq j \leq n-1$, $j \neq i$ do
3:      receive $S_j$ from $p_j$
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Termination and Integrity

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Every correct process

- reaches round $f+1$
- Decides on min(V) --- which is well defined

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- reaches round $f+1$
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Integrity

At most one value:

- one decide, and min(V) is unique

Only if it was proposed:

- To be decided upon, must be in V at round $f+1$
- if value = v_i, then it is proposed in round 1
- else, suppose received in round k. By induction:
- $k = 1$:
  - by Uniform Integrity of underlying send and receive, it must have been sent in round 1
  - by the protocol and because only crash failures, it must have been proposed
- Induction Hypothesis: all values received up to round k = j have been proposed
- $k = j+1$:
  - sent in round j+1 (Uniform Integrity of send and synchronous model)
  - must have been part of V of sender at end of round j
  - by protocol, must have been received by sender by end of round j
  - by induction hypothesis, must have been proposed

Validity

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- Suppose every process proposes v^*
- Since only crash model, only v^* can be sent
- By Validity of send and receive, v^* sent by correct processes will be received by correct processes
- By Uniform Integrity of send and receive, only v^* can be received
- By protocol, $V = \{v^*\}$
- min(V) = v^*
- decide(v^*)

Agreement

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Lemma 1

For any $r \geq 1$, if a process p receives a value v in round r, then there exists a sequence of processes $p_0, p_1, \ldots, p_r$ such that $p_r = p$, p_0 is v's proponent, and in each round p_{k-1} sends v and p_k receives it. Furthermore, all processes in the sequence are distinct.

Proof

By induction on the length of the sequence

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Lemma 2:

In every execution, at the end of round $f+1$, $V_i = V_j$ for every correct processes p_i and p_j

Agreement follows from Lemma 2, since min is a deterministic function

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- Show that if a correct p has x in its V at the end of round $f+1$, then every correct p has x in its V at the end of round $f+1$

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Proof:

- Show that if a correct p has x in its V at the end of round $f+1$, then every correct p has x in its V at the end of round $f+1$
- Let r be earliest round x is added to the V of a correct p. Let that process be p^*
- If $r \le f$, then p^* sends x in round $r+1 \le f+1$; every correct process receives and adds x to its V in round $r+1$

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- What if $r = f+1$?

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- If $r \le f$, then p^* sends x in round $r+1 \le f+1$; every correct process receives and adds x to its V in round $r+1$
- What if $r = f+1$?
- By Lemma 1, there exists a sequence of distinct processes $p_0, \ldots, p_{f+1} = p^*$
- Consider processes $p_0, \ldots, p_f$
- $f+1$ processes; only f faulty
- one of $p_0, \ldots, p_f$ is correct, and adds x to its V before p^* does it in round r

CONTRADICTION!

TRB for benign failures

```
Sender in round 1:
1:  send m to all

Process p in round  k, 1 ≤ k ≤ f+1
1:  if delivered m in round k-1 then
2:    if p ≠ sender then
3:      send m to all
4:    halt
5:  receive round k messages
6:  if received m then
7:    deliver(m)
8:    if  k = f+1 then halt
9:  else if k = f+1
10:   deliver(SF)
11:   halt
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Terminates in $f+1$ rounds

How can we do better?

Find a protocol whose round complexity is proportional to t –the number of failures that actually occurred– rather than to f –the max number of failures that may occur

Early stopping: the idea

- Suppose processes can detect the set of processes that have failed by the end of round i
- Call that set $faulty(p, i)$
- How large must $faulty(p, i)$ be for a dangerous chain to exist in round i?

Early stopping: the idea

- Suppose processes can detect the set of processes that have failed by the end of round i
- Call that set $faulty(p, i)$
- If $|faulty(p, i)| < i$ there can be no active dangerous chains, and p can safely deliver SF

Early Stopping: The Protocol

$faulty(p,k) \equiv$ set of processes that failed to send a message to p in some round

```
1: if $p$ = sender then value := $m$ else value:= ?
```

Process p in round $k, 1 \leq k \leq f+1$

```
2:  send value to all
3:  if delivered in round $k-1$ then halt
4:  receive round $k$ values from all
5:  $faulty(p,k) := faulty(p,k-1) \cup \{ q \mid p$ received no value from $q$ in round $k \}$
6:  if received value $v \neq$ ? then
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Termination

Let $faulty(p,k)$ be the set of processes that have failed to send a message to p in any round $1,\ldots,k$

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- If in any round a process receives a value, then it delivers the value in that round
- If a process has received only "?" for $f+1$ rounds, then it delivers SF in round $f+1$

Validity

Let $faulty(p, k)$ be the set of processes that have failed to send a message to p in any round $1, \ldots, k$

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```

- If the sender is correct then it sends m to all in round 1
- By Validity of the underlying send and receive, every correct process will receive m by the end of round 1
- By the protocol, every correct process will deliver m by the end of round 1

Agreement - 1

Let $faulty(p, k)$ be the set of processes that have failed to send a message to p in any round $1, \ldots, k$

```
1:  if $p$ = sender then value := $m$ else value:= ?

Process $p$ in round $k, 1 \leq k \leq f+1$

2:  send value to all
3:  if delivered in round $k-1$ then halt
4:  receive round $k$ values from all
5:  $faulty(p,k) := faulty(p,k-1) \cup \{q \mid p$
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6:  if received value $v \neq$ ? then
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9:      if $p$ = sender then value := ?
10: else if $k = f+1$ or $|faulty(p,k)| < k$ then
11:     value := SF
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13:     if $k = f+1$ then halt
```

Lemma 1:

For any $r \geq 1$, if a process p delivers $m \neq$ SF in round r, then there exists a sequence of processes $p_0, p_1, \ldots, p_r$ such that p_0 = sender, $p_r = p$, and in each round $k, 1 \leq k \leq r$, p_{k-1} sent m and p_k received it. Furthermore, all processes in the sequence are distinct, unless $r = 1$ and $p_0 = p_1 =$ sender

Lemma 2:

For any $r \geq 1$, if a process p sets value to SF in round r, then there exist some $j \leq r$ and a sequence of distinct processes $q_j, q_{j+1}, \ldots, q_r = p$ such that q_j only receives "?" in rounds 1 to j, $|faulty(q_j, j)| < j$, and in each round $k, j+1 \leq k \leq r$, q_{k-1} sends SF to q_k and q_k receives SF

Agreement - 2

Let $\mathit{faulty}(p, k)$ be the set of processes that have failed to send a message to p in any round $1, \ldots, k$

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Lemma 3:

It is impossible for p and q, not necessarily correct or distinct, to set value in the same round r to m and SF, respectively

Agreement - 2

Let $faulty(p, k)$ be the set of processes that have failed to send a message to p in any round $1, \ldots, k$

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1:  if $p$ = sender then value := $m$ else value:= ?

Process $p$ in round $k, 1 \leq k \leq f+1$

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Lemma 3:

It is impossible for p and q, not necessarily correct or distinct, to set value in the same round r to m and SF, respectively

Proof

By contradiction

Suppose p sets value = m and q sets value = SF

By Lemmas 1 and 2 there exist

$$p_0, \ldots, p_r$$

$$q_j, \ldots, q_r$$

with the appropriate characteristics

Since q_j did not receive m from process p_{k-1} $1 \leq k \leq j$ in round k

q_j must conclude that $p_0, \ldots, p_{j-1}$ are all faulty processes

But then, $|faulty(q_j, j)| \geq j$

CONTRADICTION

Agreement - 3

Let $faulty(p, k)$ be the set of processes that have failed to send a message to p in any round $1, \ldots, k$

```
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Agreement - 3

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Proof

If no correct process ever receives m, then every correct process delivers SF in round $f+1$

Let r be the earliest round in which a correct process delivers value ≠ SF

$r \leq f$

- By Lemma 3, no (correct) process can set value differently in round r
- In round $r+1 \leq f+1$, that correct process sends its value to all
- Every correct process receives and delivers the value in round $r+1 \leq f+1$

$r = f+1$

- By Lemma 1, there exists a sequence $p_0, \ldots, p_{f+1} = p_r$ of distinct processes
- Consider processes $p_0, \ldots, p_f$
  - $f+1$ processes; only f faulty
  - one of $p_0, \ldots, p_f$ is correct-- let it be p_c
  - To send v in round $c+1, p_c$ must have set its value to v and delivered v in round $c < r$

CONTRADICTION

Integrity

Let $faulty(p, k)$ be the set of processes that have failed to send a message to p in any round $1, \ldots, k$

```
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Process p in round $k, 1 \leq k \leq f+1$

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3:   if delivered in round $k-1$ then halt
4:   receive round $k$ values from all
5:   $faulty(p, k) := faulty(p, k-1) \cup$ {$q$ | $p$
     received no value from $q$ in round $k$}
6:   if received value $v \neq$ ? then
7:       value := $v$
8:       deliver value
9:       if $p$ = sender then value := ?
10:  else if $k = f+1$ or $|faulty(p, k)| < k$ then
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Integrity

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```

- At most one m
  - Failures are benign, and a process executes at most one deliver event before halting
- If $m \neq$ SF, only if m was broadcast
  - From Lemma 1 in the proof of Agreement

What about the asynchronous model?

Theorem

FLP

There is no det[illegible]ministic proto[illegible]l that s[illegible]es [illegible]onsensus in [illegible]message-pas[illegible]g [illegible]ous[illegible]ystem in whi[illegible]at most [illegible]e [illegible]rocess may [illegible]l by crashin[illegible]

(F[illegible]er, Lynch, and Pa[illegible]son. Impossibility [illegible]distributed [illegible]nsensus with one[illegible]ulty process. JAC[illegible]Vol. 32, no. 2, April [illegible]85, pp. 374-382)

Around FLP in 80 Slides

How can one get around FLP?

Weaken the problem

- Weaken termination
  - use randomization to terminate with arbitrarily high probability
  - guarantee termination only during periods of synchrony
- Weaken agreement
  - ε - agreement
    - real-valued inputs and outputs
    - agreement within real-valued small positive tolerance ε
  - k-set agreement
    - Agreement: In any execution, there is a subset W of the set of input values, $|W| = k$, s.t. all decision values are in W
    - Validity: In any execution, any decision value for any process is the input value of some process

How can one get around FLP?

Constrain input values

- Characterize the set of input values for which agreement is possible

Strengthen the system model

- Introduce failure detectors to distinguish between crashed processes and very slow processes