# CS 522 Assignment 1 Due Date: Feb., 13, 2001

This assignment is designed to familiarize you with Matlab computing, option basics, and the binomial method for European option pricing. The assignment should be done alone; however, discussions with fellow students are encouraged. Matlab programs should be sufficiently documented.

1. Options can be used to create different payoff functions.

   (a) Consider two European puts $P_1$ and $P_2$ with the same time to maturity $T$ but different strikes $E_1 > E_2 > 0$. Form a portfolio of a long position in $P_1$ and a short position in $P_2$, i.e., $\{P_1, -P_2\}$ (this is a *spread*).

      i. Make a table illustrating the payoff of this portfolio at the expiry;

      ii. Write a Matlab file to graph the payoffs of the portfolio and the two puts in one plot;

      iii. What is the view of the investor, who holds a long position of this portfolio, on the underlying price movement?

   (b) A *butterfly spread* of puts can be formed by taking positions in puts with three different strike prices, e.g., $\{P(E+\delta E, T), -2P(E,T), P(E-\delta E, T)\}$ for $E > 0$, $\delta E > 0$ and $E - \delta E > 0$. Make a table illustrating the payoff of this portfolio.

2. Let $P(E,T)$ denote the time $t$ price of a call option with strike $E$ and time to maturity $T \geq 0$ (you may assume that it is a smooth function). Under the assumption of no arbitrage, show that a vertical spread of puts must have nonnegative value, i.e.,

$$\frac{\partial P}{\partial E} \geq 0, \qquad E > 0.$$

   In addition, prove that a butterfly spread of puts must also have nonnegative value,

$$\frac{\partial^2 P}{\partial E^2} \geq 0, \qquad E > 0.$$

3. Assume that a stock price follows a binomial model,

$$S_{t+1} = \begin{cases} uS_t & \text{with a probability of } \frac{1}{2} \\ dS_t & \text{with a probability of } \frac{1}{2} \end{cases}$$

   with $u > 1 > d > 0$ and $t = 0, 1, \cdots, M$ (imagine the scenario when whether the stock price goes up or down is decided by tossing a coin).

(a) What is the conditional expected rate of return $\mathcal{E}(R_t|S_t)$ where $R_t = \frac{S_{t+1}-S_t}{S_t}$ of this stock? What is the standard deviation $\sqrt{\text{var}(R_t|S_t)}$ of the return $R_t$?

(b) Write a Matlab program to simulate a price path of this stock using the Matlab function **rand**. You may assume that $S_0 = 100$, $u = 1.1$, $d = \frac{1}{u}$, and $M = 250$. Graph three simulated price paths in one plot.

(c) Using 1000 simulated price paths generated with $S_0 = 100$, $u = 1.1$, $d = \frac{1}{u}$, and $M = 250$, graph the frequency distribution of the total return $\sum_{t=0}^{M-1} R_t$. Compare/comment the distribution with that generated from $u = 1.5$ (and $d = \frac{1}{u}$).

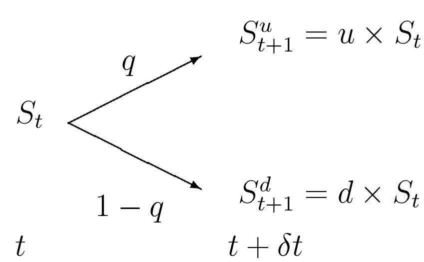
(d) Assume that $r > 0$ is the constant risk free interest rate per annum with continuously compounding. Write a **Matlab** function

$$\textbf{BinomialPrice}(S0, E, T, r, u, d, M, \text{flag})$$

to compute the price of a European option with a $M$-period binomial method using $O(M)$ space and $O(M^2)$ time ( Given two functions $f(M)$ and $g(M)$, $f(M) = O(g(M)$ means that there exists a constant $c > 0$ such that $f(M) \le cg(M)$ when $M \to +\infty$) ). The function **BinomialPrice** returns the call price when flag equals one and put price when flag equals zero.

In one plot, graph the the call option prices computed using **BinomialPrice** for $u = 1.01 : 0.01 : 1.06$ and $d = \frac{1}{u}$ with $S_0 = 100$, $r = 4\%$, $E = S_0$, $T = 1$, and $M = 250$. Comment on the effect of $u$ on the option curves.

4. Assume that a stock price movement is described by a binomial model




where $u > 1 > d > 0$ and $0 < q < 1$. Assume that the stock pays no dividend, it has an expected rate of annual return of $\mu > 0$, and its volatility (sqareroot of the variance of the annual return) is $\sigma > 0$. Assume that the continuous compounding interest rate is $r > 0$.

(a) In order for the binomial model to accurately describe the price movement, the parameters $u$, $d$ and $q$ should be chosen such that the expected rate of the return and the standard deviation of the return of the binomial model convergences to $\mu$ and $\sigma$ respectively as the number of periods goes to $+\infty$. One popular choice is

$$u = e^{r\delta t}(1 + \sqrt{e^{\sigma^2 \delta t} - 1}),$$
$$d = e^{r\delta t}(1 - \sqrt{e^{\sigma^2 \delta t} - 1}).$$

This leads to the risk neutral probability $p = 1 - p = \frac{1}{2}$. Write a Matlab function **MyBSPrice**(S0,E,T,r,$\sigma$,M,flag) to implement this binomial method using your **BinomialPrice**.

(b) In one plot, graph $\max(E - S_0, 0)$ as a function of the strike $E$ and the put option prices $P(E,T)$ computed using **MyBSPrice** for $E = 50 : 1 : 150$ with $S_0 = 100$, $r = 4\%$, $\sigma = 30\%$, $M = 250$, and $T = 1$.

Comment on the change of the option price and the relationship of the option price curve with the payoff function $\max(E - S_0, 0)$.

(c) Plot the call option prices computed using **MyBSPrice** for $\sigma = 10\% : 10\% : 100\%$. with $S_0 = 100$, $r = 4\%$, $E = 100$, $M = 250$, and $T = 1$. Comment on the change of the call option price with the volatility.