

CS 5154

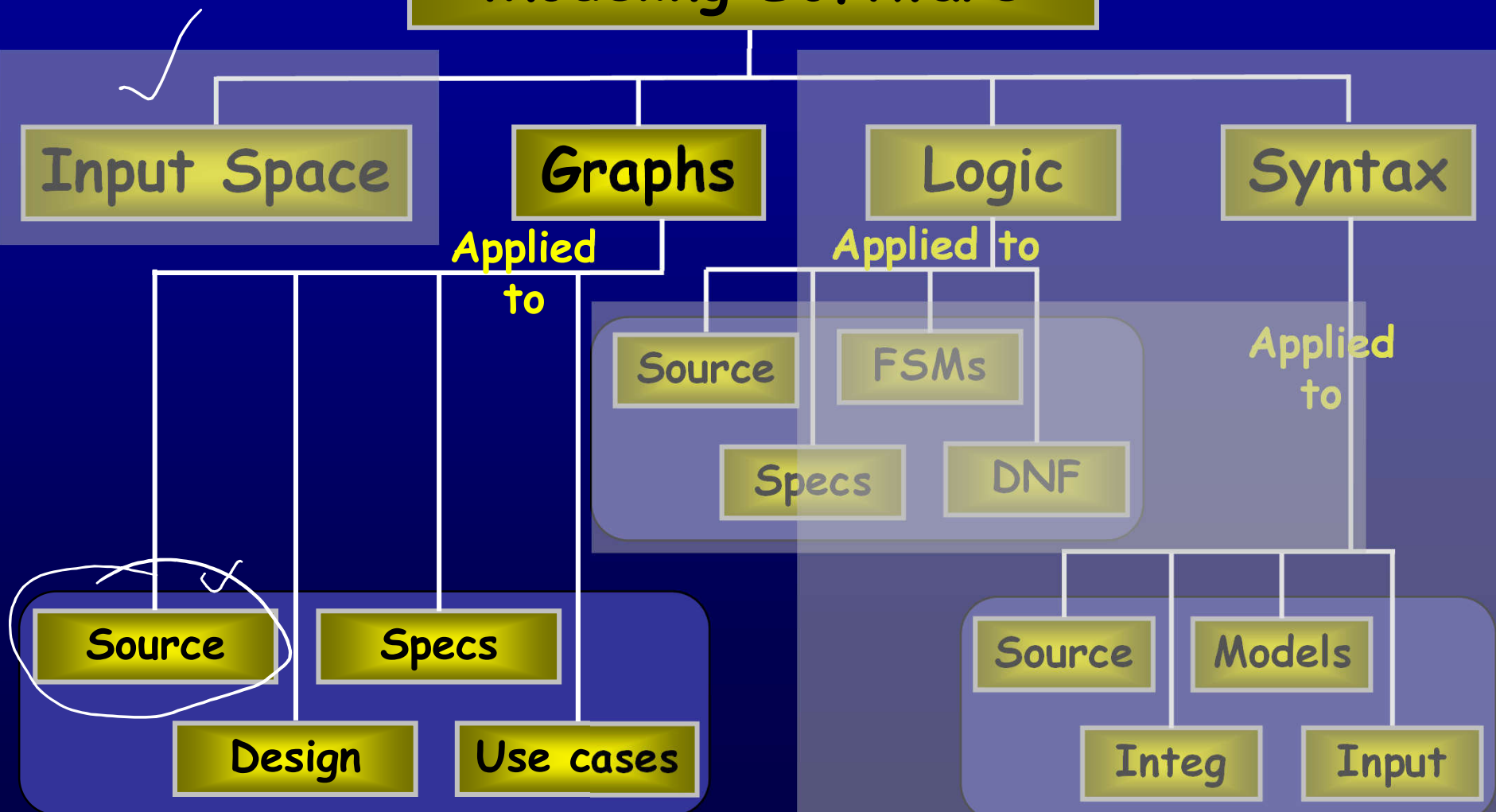
Graph Coverage Criteria

Owolabi Legunsen

**The following are modified versions of the publicly-available slides for Chapter 7
in the Ammann and Offutt Book, “Introduction to Software Testing”
(<http://www.cs.gmu.edu/~offutt/softwaretest>)**

Graph Coverage

Four Structures for Modeling Software



Covering Graphs

- Graphs are the most **commonly** used structure for testing
- Graphs can come from **many sources**
 - Control flow graphs
 - Design structure
 - FSMs and statecharts
 - Use cases
- Tests usually are intended to “**cover**” the graph somehow

Why Graph Coverage?

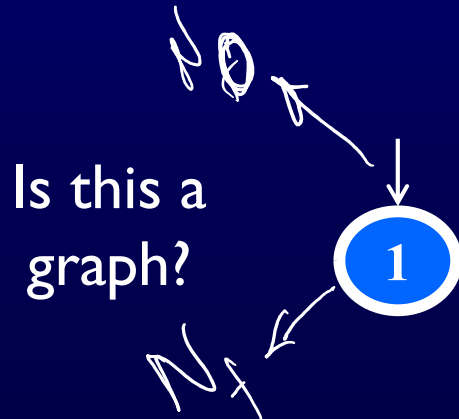
- Some of the most widely-used coverage criteria
- The “R” in the RIPR model
 - Graph coverage criteria help create tests that reach different parts of software

The next two classes

- Today:
 - Review of graph concepts
 - Coverage criteria defined over generic graphs
- Next class (depending on progress today):
 - Apply concepts learned in today's class to source code
- Not in CS 5154
 - Applying graph coverage criteria to design, specs, and use cases

Definition of a Graph

- A set N of **nodes**, N is not empty
- A set N_0 of **initial nodes**, N_0 is not empty
- A set N_f of **final nodes**, N_f is not empty
- A set E of **edges**, each edge from one node to another
 - (n_i, n_j) , i is **predecessor**, j is **successor**



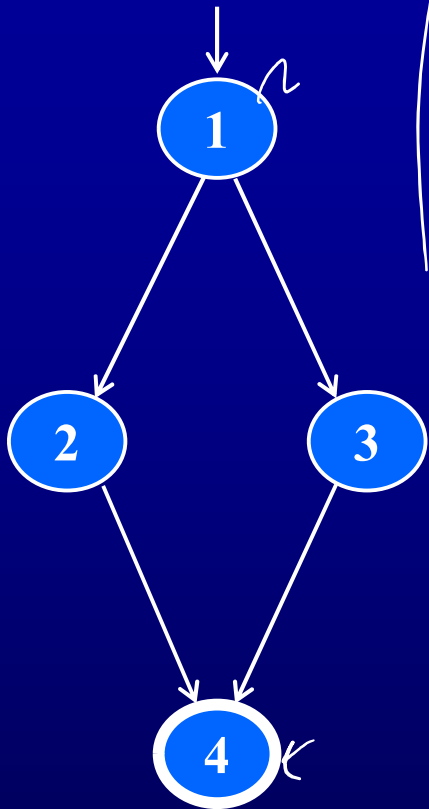
$$N_0 = \{1\}$$

$$N_f = \{1\}$$

$$E = \{\}$$



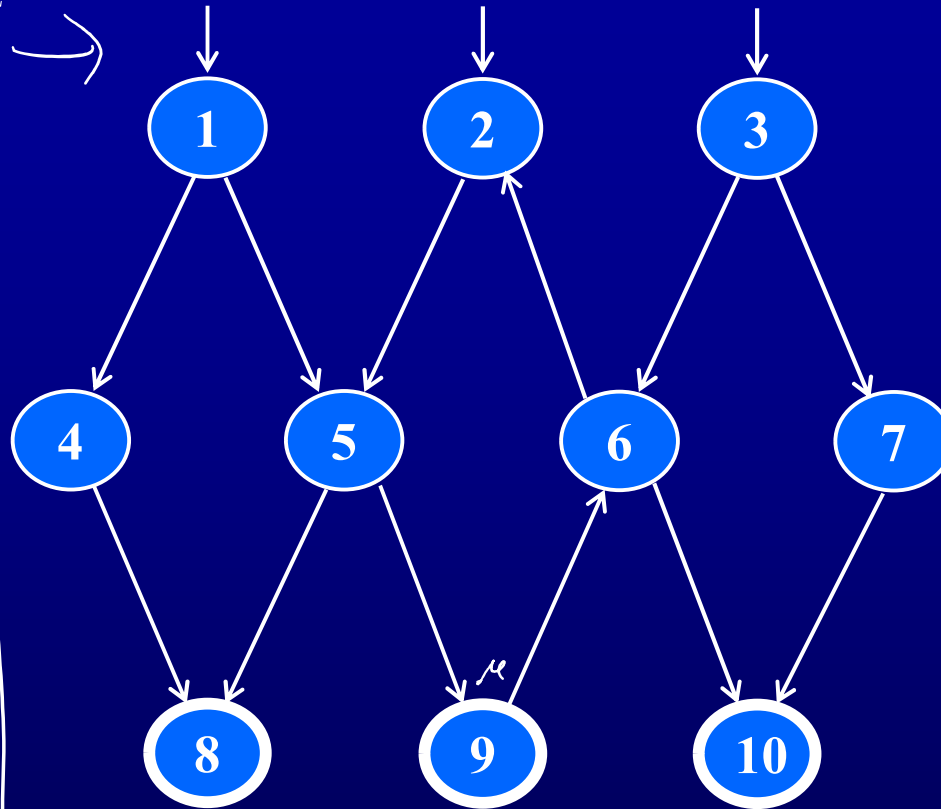
Example Graphs



$$N_0 = \{ 1 \}$$

$$N_f = \{ 4 \}$$

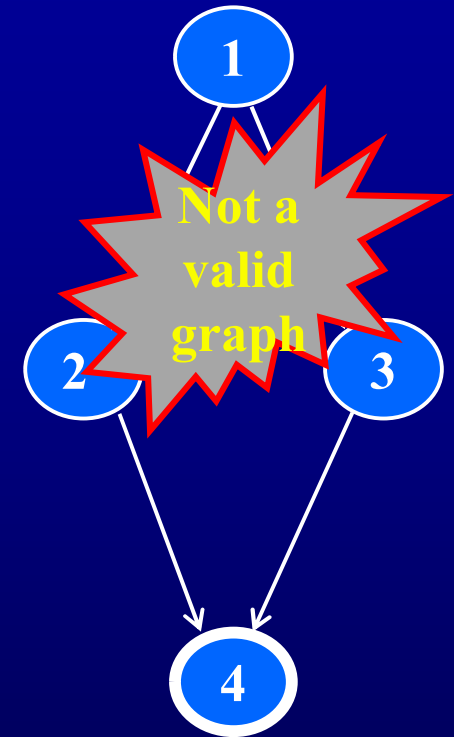
$$E = \{ (1, 2), (1, 3), (2, 4), (3, 4) \}$$



$$N_0 = \{ 1, 2, 3 \}$$

$$N_f = \{ 8, 9, 10 \}$$

$$E = \{ (1, 4), (1, 5), (2, 5), (3, 6), (3, 7), (4, 8), (5, 8), (5, 9), (6, 2), (6, 10), (7, 10), (9, 6) \}$$



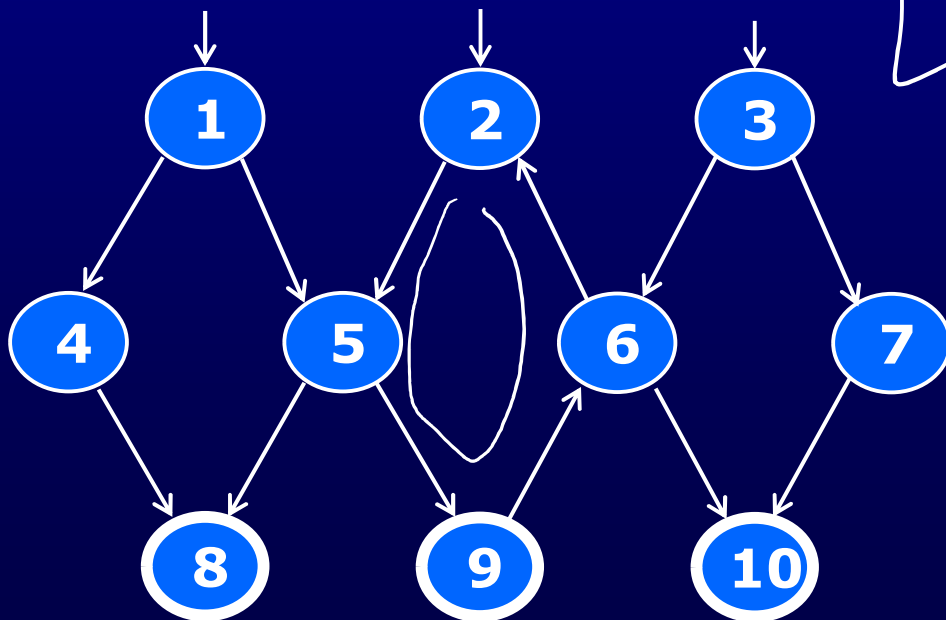
$$N_0 = \{ \}$$

$$N_f = \{ 4 \}$$

$$E = \{ (1, 2), (1, 3), (2, 4), (3, 4) \}$$

Paths in Graphs

- **Path** : A sequence, p , of nodes $[n_1, n_2, \dots, n_M]$ s.t. there is an edge between each pair of nodes in p
- **Length of a path** : The number of edges in p
 - A single node is a path of length 0
- **Subpath** : A subsequence of nodes in p is a subpath of p



A Few Paths

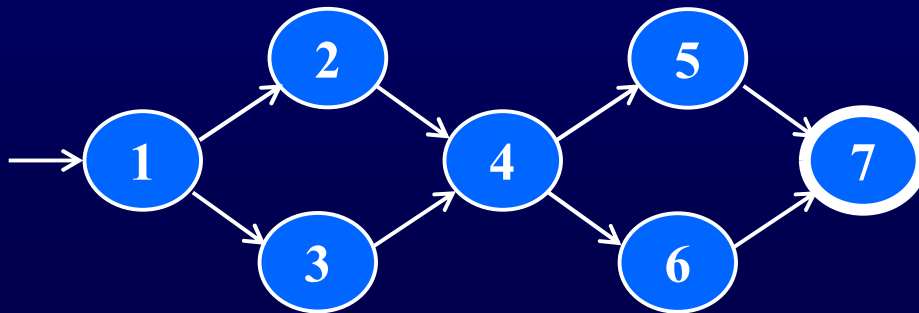
[1, 4, 8]

[2, 5, 9, 6, 2]

[3, 7, 10]

Test Paths and SESE graphs

- **Test Path** : A path that starts at an initial node and ends at a final node
- Test paths represent execution of test cases
 - Some test paths can be executed by many tests
 - Some test paths cannot be executed by any tests
- **SESE graphs** : All test paths start at a single node and end at another node
 - Single-entry, single-exit
 - N_0 and N_f have exactly one node



Double-diamond graph

Four test paths

[1, 2, 4, 5, 7]

[1, 2, 4, 6, 7]

[1, 3, 4, 5, 7]

[1, 3, 4, 6, 7]

Visiting and Touring

- **Visit** : A test path p *visits* node n if n is in p
A test path p *visits* edge e if e is in p
- **Tour** : A test path p *tours* subpath q if q is a subpath of p

Test path [1, 2, 4, 5, 7]

Visits nodes ? 1, 2, 4, 5, 7

Visits edges ? (1,2), (2,4), (4, 5), (5, 7)

**Tours subpaths ? [1,2,4], [2,4,5], [4,5,7], [1,2,4,5],
[2,4,5,7], [1,2,4,5,7]**

(Also, each edge is technically a subpath)

Tests and Test Paths

- **path** (t) : The test path executed by test t
- **path** (T) : The set of test paths executed by set of tests T
- Each test executes **one and only one** test path
 - Complete execution from a start node to a final node

Is the last bullet true?
- recursion

Tests and Test Paths (2)

- A location in a graph (node or edge) can be **reached** from another location if there is a sequence of edges from the first location to the second
 - *Syntactic reach* : A subpath exists in the graph
 - *Semantic reach* : A test exists that can execute that subpath
 - This distinction (semantic vs syntactic) is important when applied to source code

Tests and Test Paths (3)

test 1

many-to-one

test 2

test 3

Test Path

Deterministic software: a test always execute same test path

test 1

many-to-many

test 2

test 3

Test Path 1

Test Path 2

Test Path 3

Non-deterministic software: a test can execute >1 test paths

Testing and Covering Graphs

- We use graphs in testing as follows :
 - Develop a model of the software as a graph
 - Require tests to visit/tour sets of nodes, edges or sub-paths

Testing and Covering Graphs (2)

- **Test Requirements (TR)** : Describe properties of test paths
- **Test Criterion** : Rules that define test requirements
- **Satisfaction** : *Given a set TR of test requirements for a criterion C , a set of tests T satisfies C on a graph if and only if for every test requirement tr in TR , there is a test path in $path(T)$ that meets the test requirement tr*

Two kinds of graph coverage criteria

1. **Structural Coverage Criteria** : Defined on a graph just in terms of nodes and edges
2. **Data Flow Coverage Criteria** : Requires a graph to be annotated with references to variables

Node Coverage

- The first (and simplest) two criteria require that each node and edge in a graph be executed

Node Coverage (NC) : Test set T satisfies node coverage on graph G iff for every syntactically reachable node n in N , there is some path p in $path(T)$ such that p visits n .

- This statement is a bit cumbersome, so we abbreviate it in terms of the set of test requirements

Node Coverage (NC) : TR contains each reachable node in G .

Edge Coverage

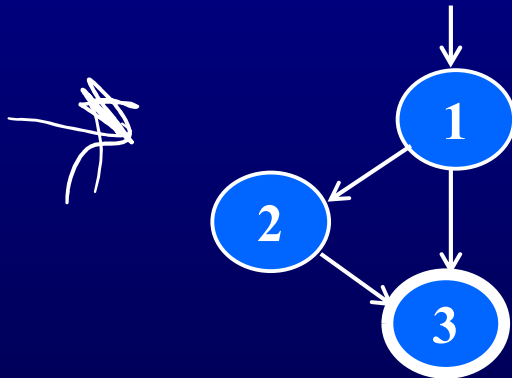
- Edge coverage is slightly stronger than node coverage

Edge Coverage (EC) : TR contains each reachable path of length up to l , inclusive, in G .

- The phrase “*length up to l* ” allows for graphs with one node and no edges

Node and Edge Coverage

- NC and EC are only different when there is an edge and another subpath between a pair of nodes (as in an “if-else” statement)



Node Coverage : ? TR = { 1, 2, 3 }

Test Path = [1, 2, 3]

Edge Coverage : ? TR = { (1, 2), (1, 3), (2, 3) }

Test Paths = [1, 2, 3] ✓

[1, 3]

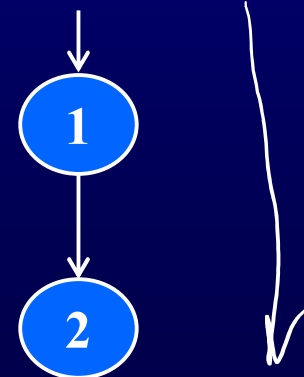
Paths of Length 1 and 0

- A graph with **only one node** will not have any edges



- It may seem trivial, but formally, Edge Coverage needs to require Node Coverage on this graph
- Else, Edge Coverage will not subsume Node Coverage
 - So, we define “**length up to 1**” instead of simply “length 1”

- We have the same issue with graphs that only have **one edge** – for Edge-Pair Coverage ...

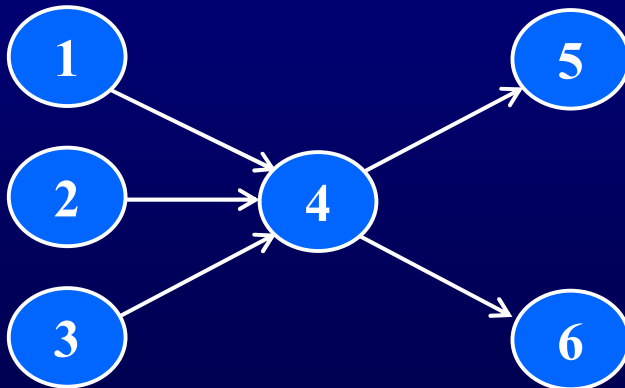


Covering Multiple Edges

- Edge-pair coverage requires **pairs of edges**, or subpaths of length 2

Edge-Pair Coverage (EPC) : TR contains each reachable path of length up to 2, inclusive, in G.

- The phrase “**length up to 2**” is used to include graphs that have less than 2 edges



Edge-Pair Coverage : ?

TR = { [1,4,5], [1,4,6], [2,4,5], [2,4,6], [3,4,5], [3,4,6] }

- A logical extension is to require covering **all paths** ...

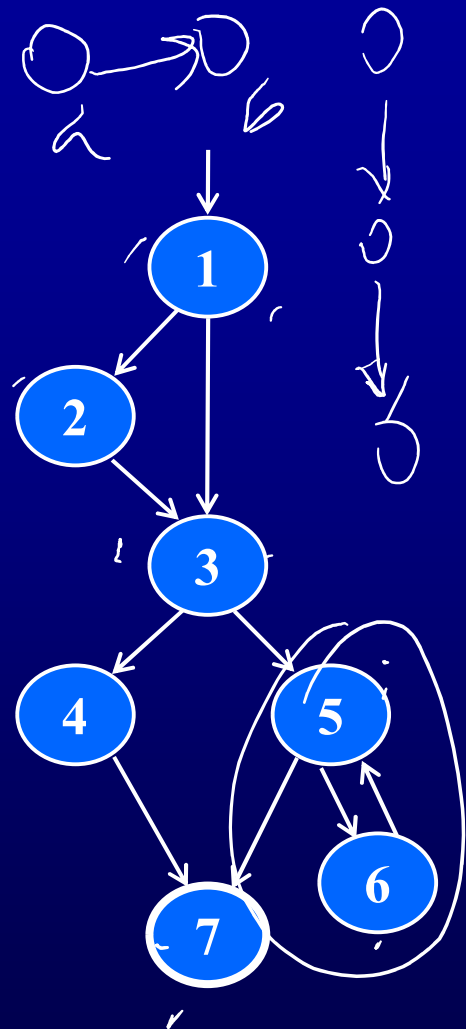
Covering Multiple Edges

Complete Path Coverage (CPC) : TR contains all paths in G.

Unfortunately, this is **impossible** if the graph has a loop, so a weak compromise makes the tester decide which paths:

Specified Path Coverage (SPC) : TR contains a set S of test paths, where S is supplied as a parameter.

Structural Coverage Example



Node Coverage
 TR = { 1, 2, 3, 4, 5, 6, 7 }
 Test Paths: [1, 2, 3, 4, 7] [1, 2, 3, 5, 6, 5, 7]

Edge Coverage
 TR = { (1,2), (1,3), (2,3), (3,4), (3,5), (4,7), (5,6), (5,7), (6,5) }
 Test Paths: [1, 2, 3, 4, 7] [1, 3, 5, 6, 5, 7]

Edge-Pair Coverage
 TR = { [1,2,3], [1,3,4], [1,3,5], [2,3,4], [2,3,5], [3,4,7], [3,5,6], [3,5,7], [5,6,5], [6,5,6], [6,5,7] }
 Test Paths: [1, 2, 3, 4, 7] [1, 2, 3, 5, 7] [1, 3, 4, 7]
 [1, 3, 5, 6, 5, 6, 5, 7] [1, 3, 5, 7]

Complete Path Coverage
 Test Paths: [1, 2, 3, 4, 7] [1, 2, 3, 5, 7] [1, 2, 3, 5, 6, 5, 7]
 [1, 2, 3, 5, 6, 5, 6, 5, 7] [1, 2, 3, 5, 6, 5, 6, 5, 6, 5, 7] ...

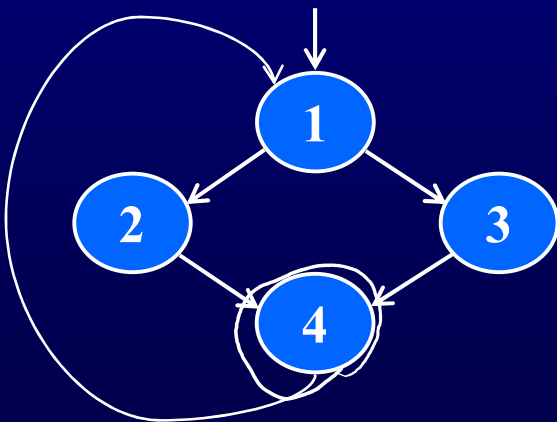
Handling Loops in Graphs

- If a graph contains a loop, it has an **infinite** number of paths
- Thus, Complete Path Coverage is **not feasible**
- SPC is not satisfactory because the results are **subjective** and vary with the tester
- Attempts to “deal with” **loops**:
 - **1970s** : Execute cycles once ([5, 6, 5] in previous example, informal)
 - **1980s** : Execute each loop, exactly once (formalized)
 - **1990s** : Execute loops 0 times, once, more than once (informal description)
 - **2000s** : Prime paths (touring, sidetrips, and detours)

Simple Paths and Prime Paths

- **Simple Path** : A path from node n_i to n_j is simple if no node appears more than once, except possibly the first and last nodes are the same
 - No internal loops
 - A loop is a simple path
- **Prime Path** : A simple path that does not appear as a proper subpath of any other simple path

$[2, 5, 6, 5, 7]$ ✗
 $[5, 6, 5]$ ✓



Simple Paths : [1,2,4,1], [1,3,4,1], [2,4,1,2], [2,4,1,3], [3,4,1,2], [3,4,1,3], [4,1,2,4], [4,1,3,4], [1,2,4], [1,3,4], [2,4,1], [3,4,1], [4,1,2], [4,1,3], [1,2], [1,3], [2,4], [3,4], [4,1], [1], [2], [3], [4]

Prime Paths : [2,4,1,2], [2,4,1,3], [1,3,4,1], [1,2,4,1], [3,4,1,2], [4,1,3,4], [4,1,2,4], [3,4,1,3]

Prime Path Coverage

- A simple, elegant and finite criterion that requires **loops** to be executed as well as skipped

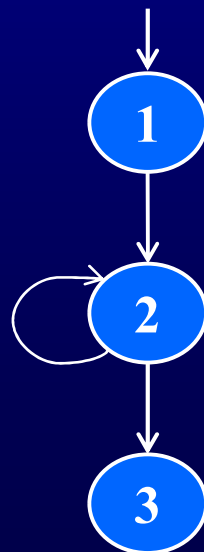
Prime Path Coverage (PPC) : TR contains each prime path in G.

- Will tour all paths of length 0, 1, ...
- That is, it **subsumes** node and edge coverage
- PPC almost, but **not quite**, subsumes **EPC** ...

Why does PPC not subsume EPC?

PPC Does Not Subsume EPC

- If a node n has an edge to itself (self edge), **EPC** requires $[n, n, m]$ and $[m, n, n]$
- Neither $[n, n, m]$ nor $[m, n, n]$ are simple paths (not prime)



EPC Requirements : ?

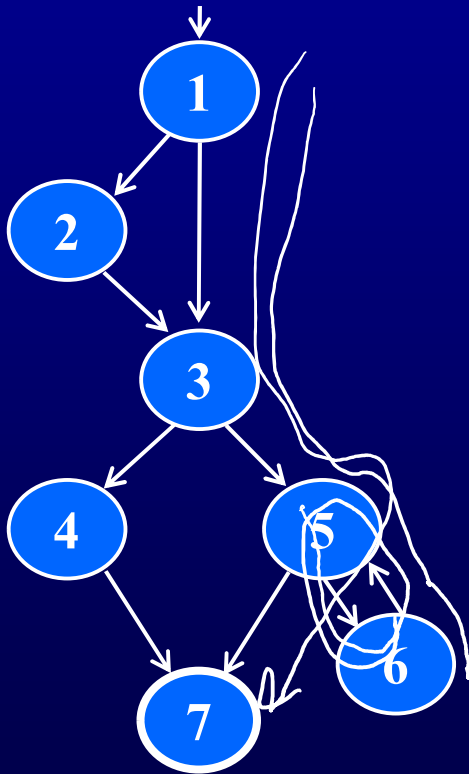
TR = { [1,2,3], [1,2,2], [2,2,3], [2,2,2] }

PPC Requirements : ?

TR = { [1,2,3], [2,2] }

Prime Path Example

- The previous example has 38 **simple** paths
- Only **nine** *prime paths*



Prime Paths

[1, 2, 3, 4, 7]

[1, 2, 3, 5, 7]

[1, 2, 3, 5, 6]

[1, 3, 4, 7]

[1, 3, 5, 7]

[1, 3, 5, 6]

[6, 5, 7]

[6, 5, 6]

[5, 6, 5]

Execute
loop 0 times

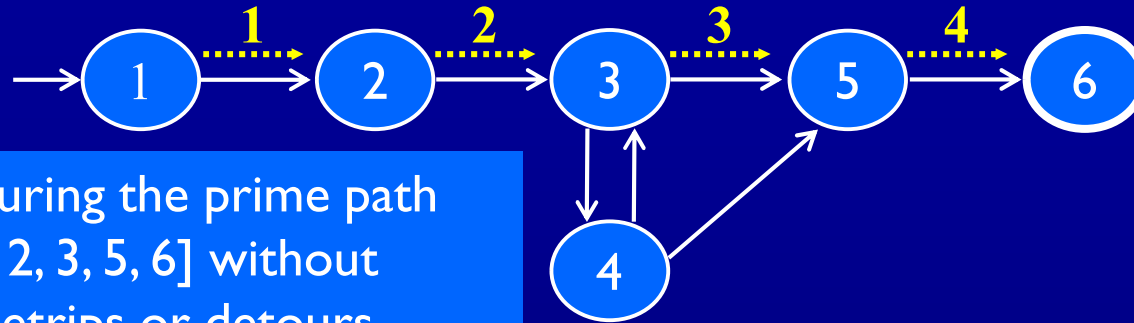
Execute
loop once

Execute loop
more than once

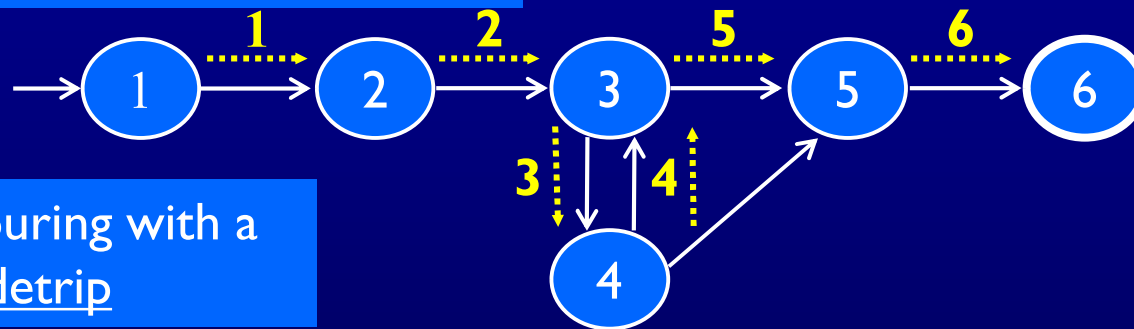
Touring, Sidetrips, and Detours

- Prime paths have no **internal loops** ... test paths might
- **Tour** : A test path p tours subpath q if q is a subpath of p
- **Tour With Sidetrips** : A test path p tours subpath q with sidetrips iff every edge in q is also in p in the same order
 - Tour can have a sidetrip if it comes back to the same node
- **Tour With Detours** : A test path p tours subpath q with detours iff every node in q is also in p in the same order
 - Tour can have a detour from node n_i , if it returns to the prime path at a successor of n_i

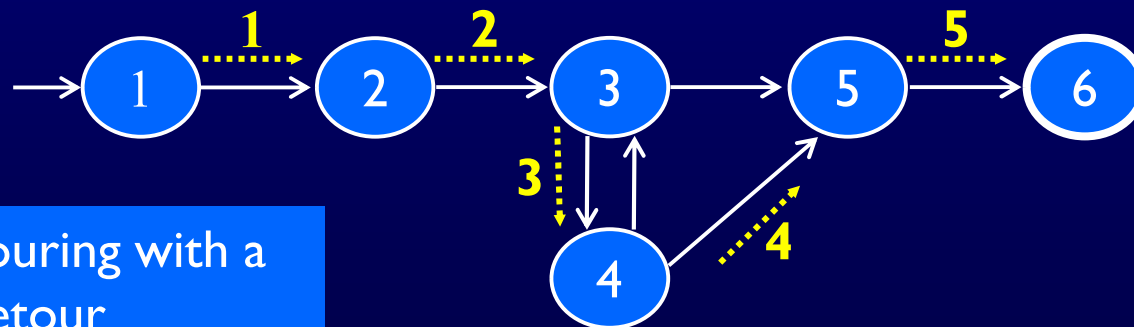
Sidetrips and Detours Example



Touring the prime path
[1, 2, 3, 5, 6] without
sidetrips or detours



Touring with a
sidetrip



Touring with a
detour

Dealing with Infeasible TRs

- An **infeasible** test requirement cannot be satisfied
 - Unreachable statement (dead code)
 - Subpath that can only be toured if a contradiction holds, e.g., if $(X > 0 \text{ and } X < 0)$
- Most test **criteria** have some infeasible test requirements
- It is usually **undecidable** whether all test requirements are feasible

Infeasible TRs and Sidetrips

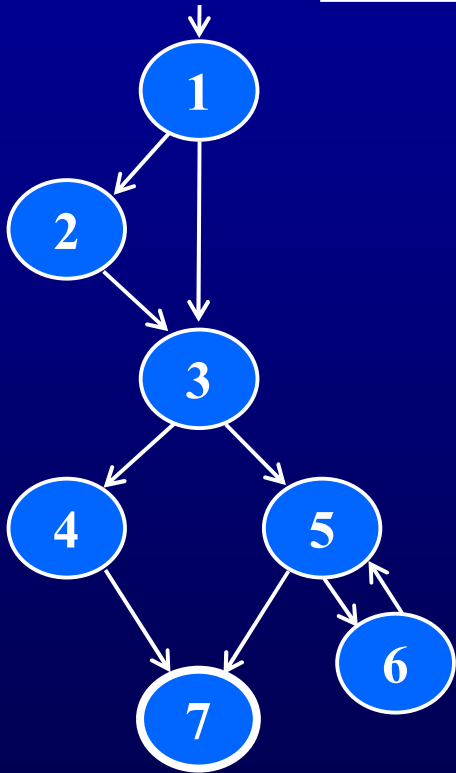
- When sidetrips are not allowed, many structural criteria have **more infeasible test requirements**
- However, always allowing **sidetrips weakens** the test criteria

Practical recommendation—Best Effort Touring

- First, satisfy as many test requirements as possible without sidetrips
- Then, allow sidetrips to try to satisfy remaining test requirements

Simple path & prime path example

“!” Means “cannot be extended to a simple path”



Simple paths

- Len 0
- [1]
 - [2]
 - [3]
 - [4]
 - [5]
 - [6]
 - [7] !

- Len 1
- [1, 2]
 - [1, 3]
 - [2, 3]
 - [3, 4]
 - [3, 5]
 - [4, 7] !
 - [5, 7] !
 - [5, 6]
 - [6, 5]

- Len 2
- [1, 2, 3]
 - [1, 3, 4]
 - [1, 3, 5]
 - [2, 3, 4]
 - [2, 3, 5]
 - [3, 4, 7] !
 - [3, 5, 7] !
 - [3, 5, 6] !
 - [5, 6, 5] *
 - [6, 5, 7] !
 - [6, 5, 6] *

- Len 3
- [1, 2, 3, 4]
 - [1, 2, 3, 5]
 - [1, 3, 4, 7] !
 - [1, 3, 5, 7] !
 - [1, 3, 5, 6] ! *
 - [2, 3, 4, 7] !
 - [2, 3, 5, 6] !
 - [2, 3, 5, 7] !

- Len 4
- [1, 2, 3, 4, 7] !
 - [1, 2, 3, 5, 7] !
 - [1, 2, 3, 5, 6] !

* means path cycles

Prime Paths ?

Round Trips

- **Round-Trip Path** : A *prime path* that starts and ends at the same node

Simple Round Trip Coverage (SRTC) : TR contains at least one round-trip path for each reachable node in **G** that begins and ends a round-trip path.

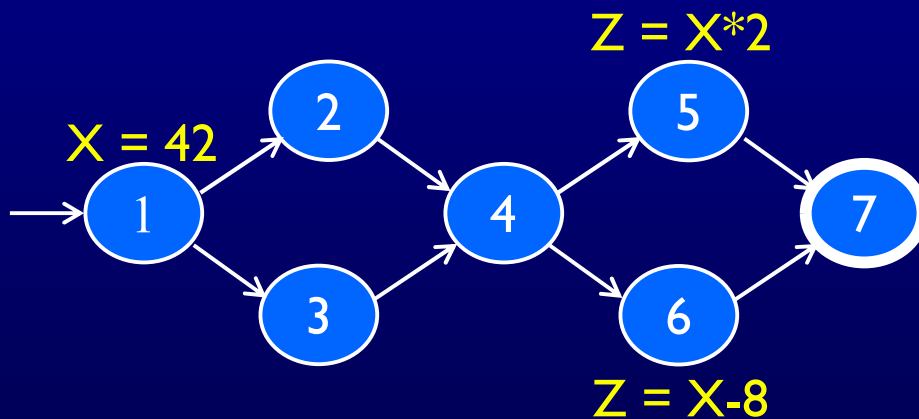
Complete Round Trip Coverage (CRTC) : TR contains all round-trip paths for each reachable node in **G**.

- The criteria **omit nodes & edges** that are not in round trips
- They do **not** subsume edge-pair, edge, or node coverage

Data Flow Criteria

Goal : Ensure that values are computed and used correctly

- **Definition (def)** : A location where a value for a variable is stored into memory
- **Use** : A location where a variable's value is accessed



Defs: def (1) = { **X** }

def (5) = { **Z** }

def (6) = { **Z** }

Uses: use (5) = { **X** }

use (6) = { **X** }

Fill in
these
sets

The values given in **defs** should **reach** at least one, some, or all possible **uses**

DU Pairs and DU Paths

- **def (n) or def (e)** :The set of variables that are defined by node n or edge e
- **use (n) or use (e)** :The set of variables that are used by node n or edge e
- **DU pair** :A pair of locations (l_i, l_j) such that a variable v is defined at l_i and used at l_j
- **Def-clear** :A path from l_i to l_j is *def-clear* with respect to variable v if v is not given another value on any of the nodes or edges in the path
- **Reach** : If there is a def-clear path from l_i to l_j with respect to v , the def of v at l_i reaches the use at l_j
- **du-path** :A simple subpath that is def-clear with respect to v from a def of v to a use of v
- **du (n_i, n_j, v)** – the set of du-paths from n_i to n_j
- **du (n_i, v)** – the set of du-paths that start at n_i

Touring DU-Paths

- A test path p *du-tours* subpath d with respect to v if p tours d and d is def-clear with respect to v
- **Sidetrips** can be used, just as with previous touring
- Three criteria
 - Use every def
 - Get to every use
 - Follow all du-paths

Data Flow Test Criteria

- First, we make sure every def reaches a use

All-defs coverage (ADC) : For each set of du-paths $S = du(n, v)$, TR contains at least one path d in S .

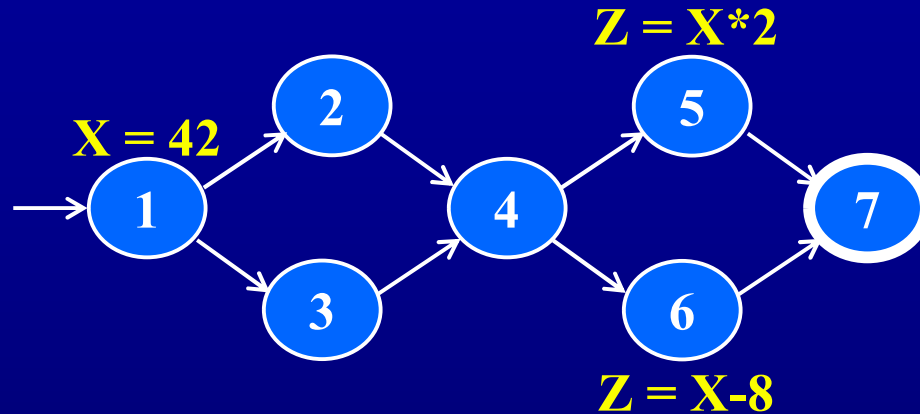
- Then we make sure that every def reaches all possible uses

All-uses coverage (AUC) : For each set of du-paths to uses $S = du(n_i, n_j, v)$, TR contains at least one path d in S .

- Finally, we cover all the paths between defs and uses

All-du-paths coverage (ADUPC) : For each set $S = du(n_i, n_j, v)$, TR contains every path d in S .

Data Flow Testing Example



All-defs for X

[1, 2, 4, 5]

All-uses for X

[1, 2, 4, 5]

[1, 2, 4, 6]

All-du-paths for X

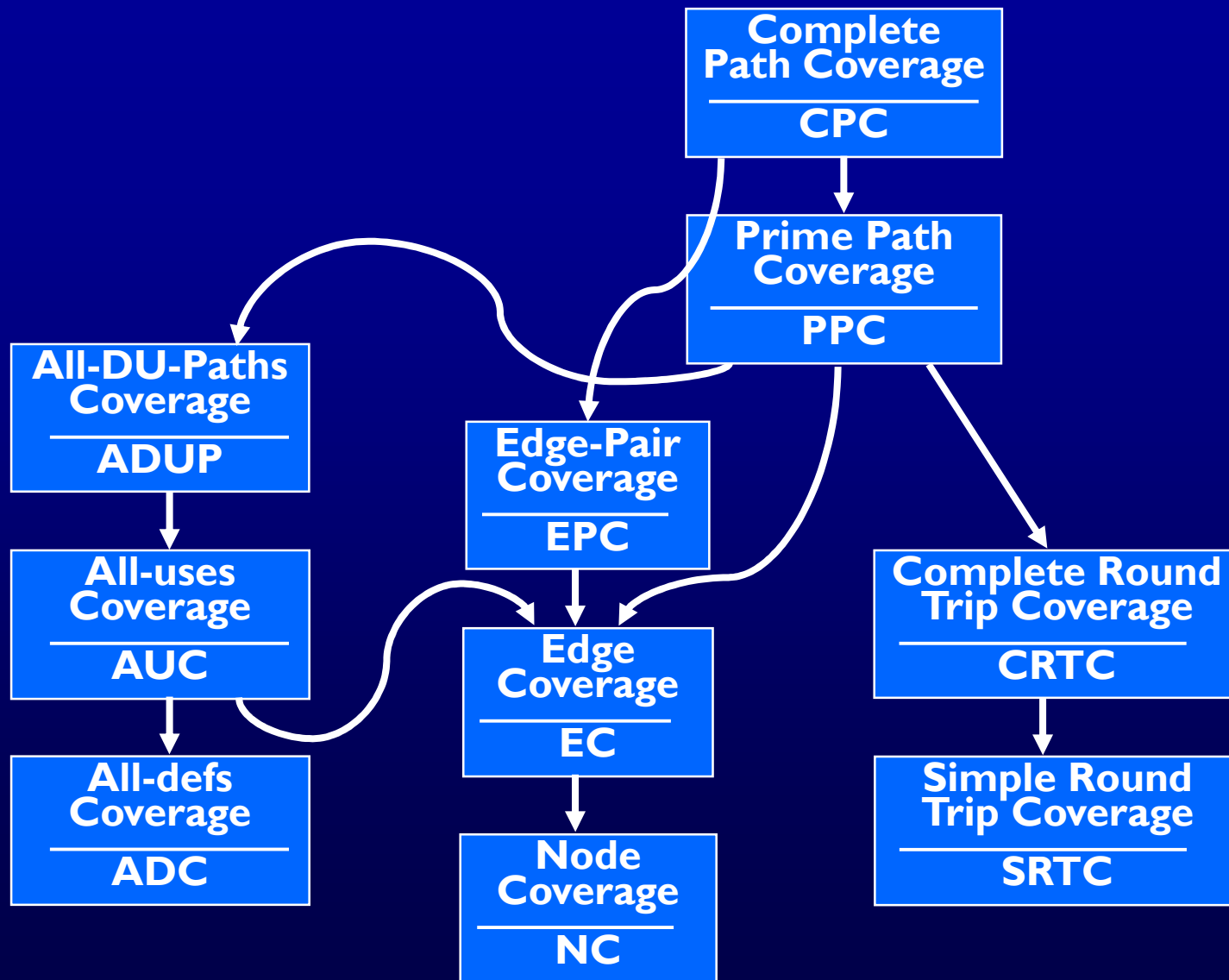
[1, 2, 4, 5]

[1, 3, 4, 5]

[1, 2, 4, 6]

[1, 3, 4, 6]

Graph-based criteria subsumption



Summary

- Graphs are a very **powerful abstraction** for designing tests
- The various criteria allow lots of **cost / benefit** tradeoffs
- These two sections are entirely at the “**design abstraction level**” from chapter 2
- Graphs appear in **many situations** in software
 - Next: we will apply these criteria to source code
 - Design, specs, and use cases are not covered in CS 5154