CS 4860 Lecture 14 Thur March 15, 2018 

Topics

1. Smullyan's completeness proof for 1st Order tableaux rules. Please read pages 57–61.

2. Extending completeness to Refinement Logic. The rules without realizers and a sequencing rule.

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| | F rule | Sequent | T rule | Sequent |
|---|---|---|---|---|
| & | $\dfrac{F(X \& Y)}{FX \mid FY}$ | $H \vdash X \& Y$<br>$H \vdash X$<br>$H \vdash Y$ | $\dfrac{T(X \& Y)}{TX \quad TY}$ | $H, X \& Y, H' \vdash G$<br>$H, x{:}X, y{:}Y, H' \vdash G$ |
| $\vee$ | $\dfrac{F(X \vee Y)}{FX \quad FY}$ | $H \vdash X \vee Y$<br>$H \vdash X$<br>- - - - - - - -<br>$H \vdash X \vee Y$<br>$H \vdash Y$ | $\dfrac{T(X \vee Y)}{TX \mid TY}$ | $H, d{:}X \vee Y, H' \vdash G$<br>$H, x{:}X, H' \vdash G$<br>$H, y{:}Y, H' \vdash G$ |
| $\Rightarrow$ | $\dfrac{F(X \Rightarrow Y)}{TX \quad FY}$ | $H \vdash X \Rightarrow Y$<br>$H, x{:}X \vdash Y$ | $\dfrac{T(X \Rightarrow Y)}{FX \mid TY}$ | $H, f{:}X \Rightarrow Y, H' \vdash G$<br>" " " " $\vdash X$<br>" " ", $y{:}Y$ $H' \vdash G$ |
| $\sim$ | $\dfrac{F{\sim}X}{TX}$ | $H \vdash X \Rightarrow \bot$<br>$H, x{:}X \vdash \bot$ | $\dfrac{T({\sim}X)}{FX}$ | $H, f{:}X \Rightarrow \bot, H' \vdash G$<br>$H,$ " ", $H' \vdash X$<br>$H,$ " ", $y{:}\bot$ $H' \vdash G$ any(y) |

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Example

$$\frac{F(X \Rightarrow X)}{\begin{array}{c} TX \\ FX \\ \times \end{array}}$$

$H \vdash X \Rightarrow X$

$H, x{:}X \vdash X$

Tableaux proof of $X \vee \sim X$

$$\frac{F(X \vee (X \Rightarrow \bot))}{FX \;,\; \dfrac{F(X \Rightarrow \bot)}{\begin{array}{c} TX \\ F\bot \end{array}}}$$

$$\begin{array}{c} \times \\ FX, TX \end{array}$$

For refinement logic we need a new rule

$H \vdash A \vee (A \Rightarrow \bot)$
by magic(A)

Classical logic assumes a non-empty domain D, so in refinement logic we need

$H \vdash D$ by $d_0$ or $H \vdash D$ by $obj(d_0)$

In refinement logic we can add a sequencing rule (also called a <u>cut rule</u>).

$H \vdash G$ by seq $T$ ap( $\lambda$(x. __ ) ; __ )

$H \vdash T$ by $t$

$H, x{:}T \vdash G$ by $g(x)$

Exercise Give a tableaux proof of

$$\exists y{:}D.\,(P(y) \Rightarrow \forall x{:}D.\,P(x))$$