CS 4860 Notes on Lecture 15

Thursday Oct 18, 2012

This lecture was devoted to discussing answers to the Prelim questions and responding to questions from students. We also discussed possible project topics and aspects of FOL.

Question 1 on implication

One good answer is to assume that there is evidence for $(P \supset Q)$ and see what that means if we assume that true cases for P, Q mean there is evidence, say $p \in [P]$, $q \in [Q]$ or that $[P]$, $[Q]$ are empty evidence types.

Another possible answer is to analyze the truth table in terms of the assumptions $P \vee \sim P$, $Q \vee \sim Q$ as we did in lecture when we defined the dimension of a propositional proof or evidence term.

Question 2 on extended truth tables

The key point is that even $P \supset P$ has value u (for unknown) in any sensible truth table using t, f, u.

Question 3 on counter models

When a formula is not a tautology, we can use truth value counter models or particular values for P, Q, R, etc.

Question 4. on König's lemma

We discussed one common bogus proof attempt.

Question 5. on syllogisms

We looked at a realizer and a very simple counter model where Socrates is a dog or a robot.

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1. Solutions to Prelim questions, discussion of grading and projects. (See page 1 for details.)

2. Aspects of First-Order Logic (FOL)

There is a sense in which FOL is a logic sufficient for expressing all of classical mathematics. It is also the logic that has been proposed as the basis of the Semantic Web (perhaps an interesting project topic).

The reason it is considered adequate for classical mathematics is that we can express the axioms for classical ZFC set theory in FOL and we can express the finitely axiomatizable Bernays Gödel set theory as well. (Note, it is not possible to get an intuitionistic IZFC set theory by just using ZFC axioms in iFOL because the Axiom of Choice allows us to derive $P \vee \sim P$ in iFOL.

In this lecture I will sketch how we can define a theory of arithmetic in FOL and iFOL. We see, in order, how to define 0 (zero), then successor $S(x,y)$, then equality Eq, then unique successor and unique zero. We can then define $Add(x,y,z)$ to mean $x+y=z$ and $Mult(x,y,z)$ to mean $x * y = z$ (where $*$ is the multiplication operator).