Applied Logic	**Homework 6**
CS 4860 Fall 2012	**Due Date: Thursday, November 29**

Projects are due Friday, Nov 30 by 5pm.

Questions

(1) Prove these theorems in HA

(a) $\forall x.(0 + x) = x$

(b) $\forall x, y.(x + y = y + x)$

(c) Define $x < y$ to mean $\exists z.(z \neq 0 \;\&\; y = x + z)$, prove $\forall x.\, \exists y.(x < y)$ and show the evidence term.

(2) If we apply the minimization operator to a function $f(x, y)$ that is always positive at x, e.g. $\forall y.\, f(x, y) \neq 0$, then it does not produce a value but "diverges," on some input x. The *domain* of such a function $\mu y.f(x, y) = 0$ is $\{x : \mathbb{N} \,|\, \exists y.\, f(x, y) = 0\}$.

Note, we can represent $\lambda x.\mu y.f(x, y) = 0$ in the logic $\mathcal{Q}$, say by the relation we used in proving that MinRec functions are representable.

In HA we might try to be more precise about these functions and ask whether HA can also prove $\neg\, \exists y.\, f(x, y) = 0$ when x is not in the domain.

Sketch a proof that if HA is consistent, then it cannot prove all such theorems (and is thus incomplete).

Additional problems may be added.