

Homework 2

Reading Please read Smullyan p. 1-18 and the *Story of Logic* on Frege and Brouwer.

- (1) Solve exercise 4 on page 14 of Smullyan.
- (2)
 - (a) Do any of the definitions from problem (1) also serve as constructive definitions in the sense that the equivalence holds in constructive logic?
 - (b) If any definition “works”, prove that it does.
 - (c) Find one definition that fails to work and explain why it does not.
- (3) Prove item (6) on page 24 of Smullyan
 - (a) in classical tableau (Smullyan’s system).
 - (b) in refinement logic.
 - (c) explain the intuitive meaning of each proof in terms of the evidence it provides.
- (4) Prove the following propositions in refinement logic.
 - (a) $(P \vee \neg P) \Rightarrow (\neg\neg P \Rightarrow P)$
 - (b) $((P \vee \neg P) \wedge (Q \vee \neg Q)) \Rightarrow (\neg(P \& Q) \Rightarrow (\neg P \vee \neg Q))$
 - (c) $((P \vee \neg P) \wedge (Q \vee \neg Q)) \Rightarrow (\neg Q \Rightarrow \neg P) \Rightarrow (P \Rightarrow Q)$
 - (d) In (c) do you need both hypotheses that $(P \vee \neg P)$ and $(Q \vee \neg Q)$?
 - (e) Prove $(\neg Q \Rightarrow \neg P) \Rightarrow (P \Rightarrow Q)$ in Smullyan’s system.
- (5) Convert a truth table analysis for validity of the following formulas into evidence under the assumption that the atomic formulas are decidable.
 - (a) $(\neg Q \Rightarrow \neg P) \Rightarrow (P \Rightarrow Q)$
 - (b) $((P \Rightarrow Q) \Rightarrow P) \Rightarrow P$ (Pierce’s Law)
- (6) Given any program P in (Java, Lisp, ML or your favorite general purpose programming language) which computes a function from Booleans to Booleans, let $P(\text{“true”})$ Halts mean that program P halts on the value, “true” (the Boolean value for true in the programming language). Is it constructively plausible to assert: $P(\text{“true”})$ Halts $\vee \neg(P(\text{“true”})$ Halts) for all P ?
Explain.
- (7) What might Frege say about the meaning of $0/0 = 1$ based on your reading in the *Story of Logic*?

HW1 Make Up

Problems for Homework 1 only for students receiving less than 14 points on HW1.

Give the evidence expressions for these valid IPC formulas and for the problems that were completely wrong on your HW1. You can get the correct answers from the posted solutions.

(1) $P \Rightarrow \neg\neg P$

(2) $(P \Rightarrow P \wedge P) \wedge ((P \wedge P) \Rightarrow P)$

(3) $(P \wedge Q) \Rightarrow (Q \wedge P)$

(4) $(P \vee Q) \Rightarrow (Q \vee P)$

(5) $\neg P \vee \neg Q \Rightarrow \neg(P \wedge Q)$